Addis Ababa institute of Technology

Reinforced Concrete Structures 2

(CEng-3122)

Chapter Five

Torsion

1

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School of Civil and Environmental Engineering
Concrete Material and Structures Chair

Presentation Outline

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Introduction

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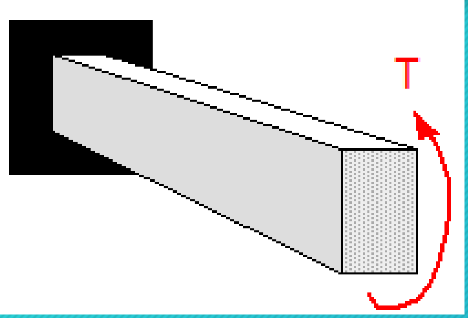
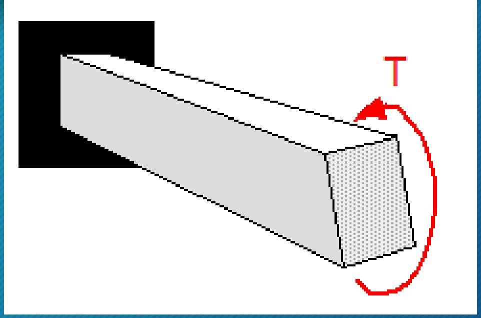
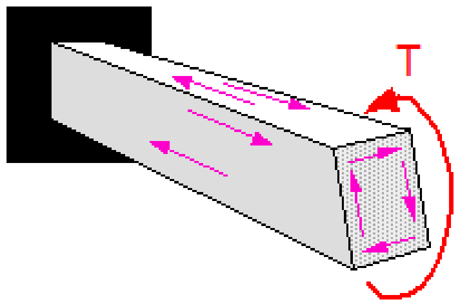
Introduction



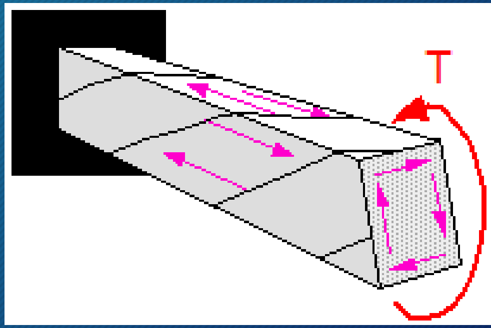
Torsion is the action of a moment T about the longitudinal axis of a member.



This twisting action induces shear stresses in both the transverse and longitudinal directions of the members.



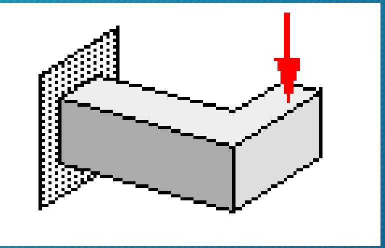
These shear stresses produce principal tensile stresses at 45^{o} to the longitudinal axis. When theses exceed the tensile strength of the concrete, diagonal cracks form and tend to 'spiral' around the member.



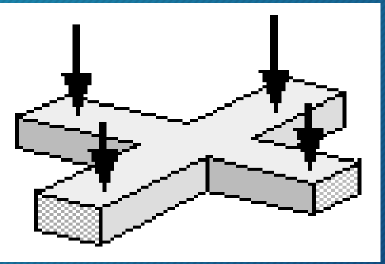
Introduction

In structures there are considered to be two types of torsion:-

- Equilibrium torsion, or primary torsion, which is statically determinate and essential in maintaining stability in a structure .



- Compatibility torsion, or secondary torsion, which is statically indeterminate and maintains compatibility between members of a structure.



Equilibrium torsion requires a full design at both the ultimate and serviceability limit states. Generally, compatibility torsion may be disregarded, but in some arrangement of structural members excessive cracking could arise, and must, therefore, be controlled by the provision of sufficient reinforcement.

Torsional Resistance

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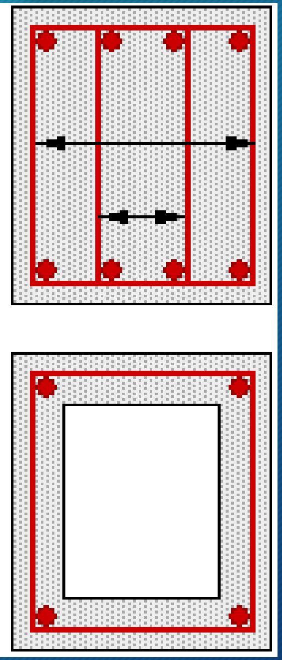
Torsional Resistance



In reinforced concrete structures-1, it was explained how all components of a reinforced concrete section contribute to its shear resistance. (Cracked, Uncracked concrete section and the reinforcement provide for flexure)

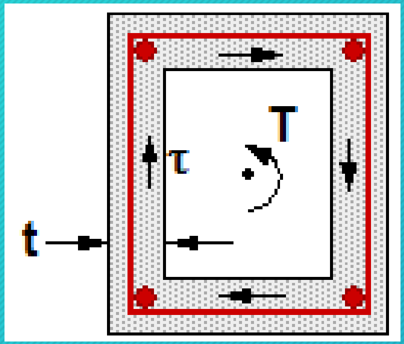
However, the torsional resistance of the section is provided almost entirely by the outer portion of the section. Longitudinal reinforcement without an enclosing link contributes very little to the torsional resistance, but is essential in providing an anchorage for the links.

For design purposes the torsional resistance of the concrete is ignored..

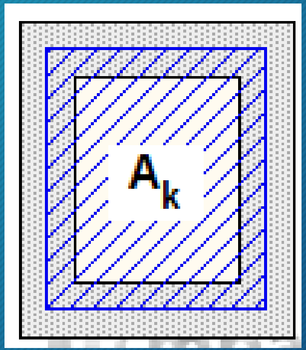
Because the center of the section contributes little to the torsional resistance, the section can be modelled as a thin walled, closed section.

Torsional Resistance

Bredt's formula can then be used to provide a relationship between the applied torsional moment, T and the induced shear stress, τ in terms of the wall thickness, t and the area within the center line of the section, A_k.



$$\tau = \frac{T}{2 A_k t}$$



The wall thickness, t can be any value between $2c$ and A/u, provided $A/u > 2c$,

where, c is the cover to the longitudinal bars,

A is the area of the section, and

u is the perimeter of the section

This allows the designer to select a value of t to give the maximum value of $A_k t$ (A_k decreases as t increases), thus giving the minimum value of t, but generally the maximum value of t is used.

Analysis

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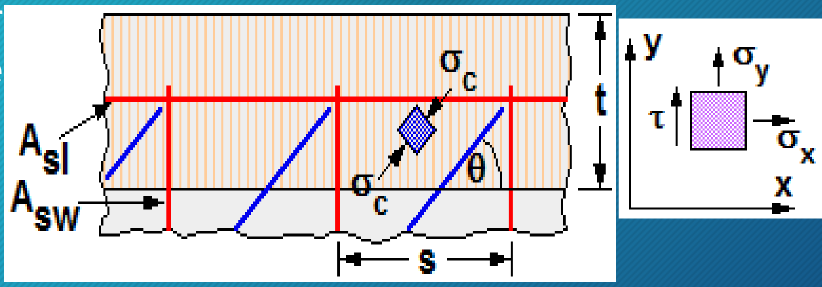
Analysis

The analysis is similar to that of the truss analogy for shear. The concrete forms the notional' compressive struts, inclined at an angle θ to the longitudinal axis.



The stress in the concrete is treated as homogenous, consisting of uniaxial compressive stress σ_c. The reinforcement acts as the ties. The circumferential reinforcement is always placed right angles to the longitudinal axis.

The stresses, σ_x, σ_y and τ in the concrete are given by:

$$\sigma_x = -\sigma_c \cos^2\theta + A_{sx} f_{yld} / t \qquad 1$$

$$\sigma_y = -\sigma_c \sin^2\theta + A_{sy} f_{ywd} / t \qquad 2$$

$$\tau = \sigma_c \sin\theta \cos\theta \qquad 3$$

Where, A_{sx}, A_{sy} are the reinforcement areas per unit length in the x and y directions respectively.

Bredt's formula provides a relationship between the constant shear stress, τ and the torsional moment, T thus:

$$\tau = T / (2A_k t) \qquad 4$$

At the section boundary σ_x and σ_y are zero.

$$\sigma_x = \sigma_y = 0 \qquad 5$$

If A_{sl} is the total area of longitudinal reinforcement, U_k is the perimeter of the center line of the thin-walled section, and A_{sw} is the area of the one link leg, then:

$$A_{sx} = A_{sl} / u_k \qquad 6$$

$$A_{sy} = A_{sw} / s \qquad 7$$

Analysis

From these equation three relationships can be established in terms of the applied torsional moment T.

1. The area of link reinforcement is given by:

$$T = 2 A_k (A_{sw} / s) f_{ywd} \cot\theta$$

2. The compressive stress in the struts is given by:

$$T = 2 A_k t \sigma_c \sin\theta \cos\theta$$

3. The area of longitudinal reinforcement is given by:

$$T = 2 A_k (A_{sl} / u_k) f_{yld} \tan\theta$$

Design

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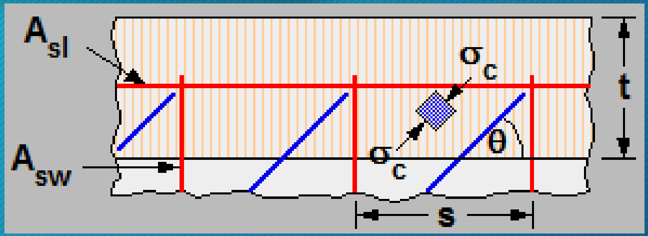
Design

The Variable Strut Inclination (VSI) method, which allows the designer to vary the strut angle, between defined limits (the same as for shear) must be used. If the member is designed for shear as well (the usual case), then the angle chosen must be the same for both shear and torsion. The torsional resistance of the concrete is ignored.

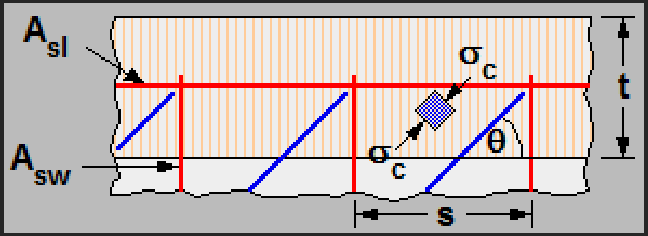


The design must consider 3 conditions:

- The compressive stress, σ_c in the notional concrete struts should not exceed its maximum value.
- Sufficient circumferential reinforcement (torsional links), A_{sw}/s should be provided to resist the design torsional moment. This reinforcement combined with that for shear should also satisfy the serviceability requirements of minimum percentage and crack control.
- Sufficient longitudinal reinforcement, A_{sl} should be provided.

Design

The compressive stress, σ_c



From the analysis the relationship between T_{Ed} and σ_c is

$$T_{Ed} = 2 A_k t \sigma_c \sin\theta \cos\theta$$

Clearly for any value T_{Ed} , σ_c can be found and checked against its maximum permissible value. Alternative, σ_c can be replaced in the relationship by its maximum value, to give the maximum design torsional moment that can be applied to the section; this is the method in the Code.

The maximum permissible value of σ_c is the Plastic strength of the concrete, which is related to the design compressive strength, f_{cd} by an effectiveness factor, ν, thus;

$$\sigma_{c,max} = \nu f_{cd}$$

$$\nu = 0.6 (1 - f_{ck} / 250)$$

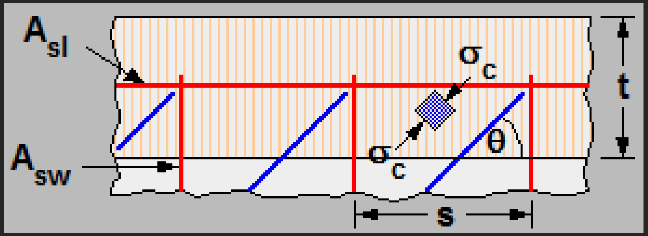
Thus, the maximum design torsional moment, $T_{Rd,max}$ that can be carried without crushing of the concrete struts is:

$$T_{Rd,max} = 2 A_k t \alpha_c \nu f_{cd} \sin\theta \cos\theta$$

Where, α_c is a coefficient which allows for the effects of an axial compressive force.

Design

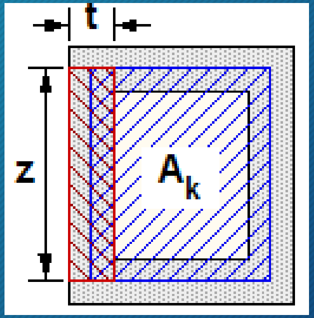
A_{sw}/s



The method adopted in the Code is to calculate A_{sw}/s from the combined effects of torsion and shear using the method described in the discussion of Shear.

The shear force V_{Ed} on the shaded part of the thin walled section is given by:-

$$V_{Ed,i} = \tau t z \quad \dots 1$$



The Bredit formula showed that:-

$$\tau = \frac{T_{Ed}}{2 A_k t} \quad \dots 2$$

Substituting 2 into 1 gives ;-

$$V_{Ed,i} = \frac{T_{Ed}\, z}{2 A_k}$$

To find the total A_{sw}/s required the beam is designed for a combined shear force, $V_{Ed,r}$.

$$V_{Ed,r} = V_{Ed} + \Sigma V_{Ed,i}$$

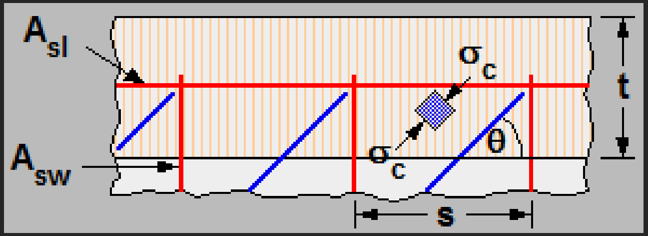
Note: $\sum V_{Ed,i}$ must act in the same direction as the beam shear force, V_{Ed}.

For the rectangular beam shown opposite,

$$\Sigma V_{Ed,i} = \frac{2\, T_{Ed}\, z}{2 A_k} = \frac{T_{Ed}\,(h-t)}{A_k}$$

Design

A_{sw}/s

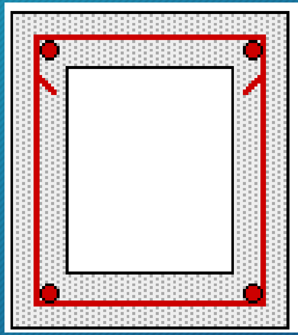


However, from the analysis the relationship between T_{Ed} and A_{sw}/s is :-

$$T_{Ed} = 2 A_k (A_{sw}/s) f_{ywd} \cot\theta$$

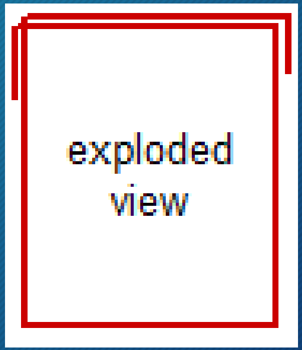
Thus,

$$A_{sw}/s = \frac{T_{Ed}}{2 A_k f_{ywd} \cot\theta}$$



Therefore, this amount of reinforcement should be provided in the outer section of the beam in the form of torsion links.

Torsion links must always be closed, anchored and placed at 90º to the longitudinal axis of the member. This is an example of a typical torsion link.

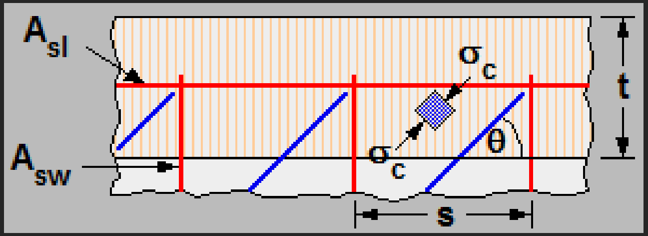


In addition to the spacing limitations imposed for shear links (0.75d), the longitudinal spacing of torsion links should also not exceed -

- The lesser dimension of the beam cross-section, and
- $U_k/8$, where u_k is the perimeter of the center line of the equivalent thin-walled section

Design

A_{sl}



From the relationship between T_{Ed} and A_{sl} is :-

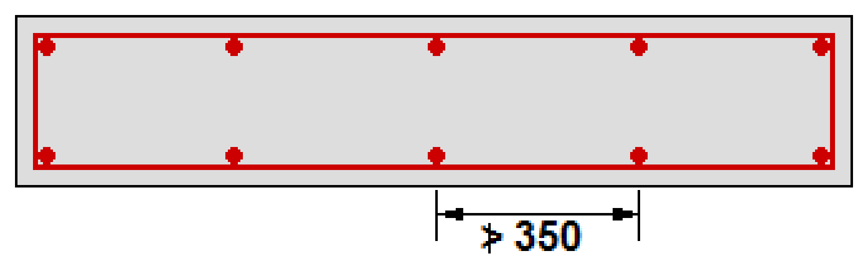
$$T_{Ed} = 2 A_k (A_{sl} / u_k) f_{yld} \tan\theta$$

Therefore, the force in the longitudinal tension reinforcement is:-

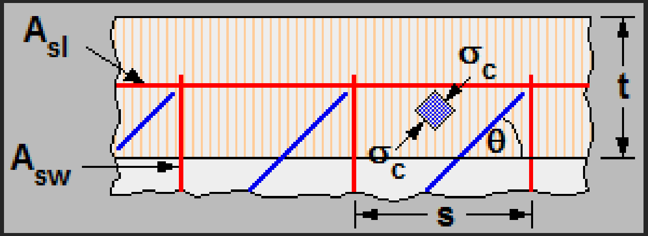
$$A_{sl} f_{yld} = T_{Ed} (u_k / 2 A_k) \cot\theta$$

Longitudinal reinforcement should be uniformly distributed around he inner periphery of the torsion links satisfy the following conditions:

- There must be at least one bar at each corner, and
- The center to center spacing of the bars should not exceed 350 mm.



Design
cotθ



The same value of cotθ must be used for both shear and torsion.

Permissible values are in the range 1.0-2.5

Non-Convex Sections

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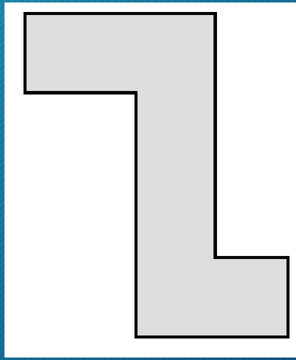
Non-Convex Sections

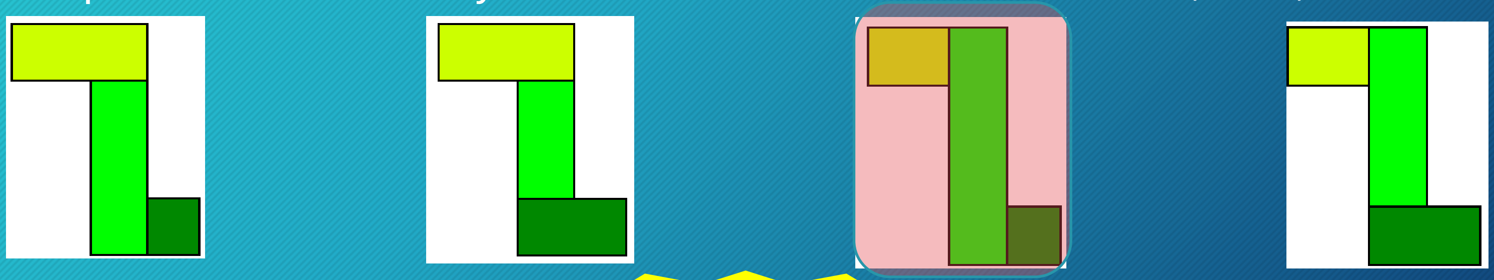


So far we have looked at the design of closed or convex sections, but many structural sections are non-convex. Consider this Zed beam..

A thin walled analogy could be developed for this shape, but a simpler method would be to divide the section into a number of rectangles, and design each as an equivalent thin-walled section subjected to its apportionment of the applied torsional moment. This is the method adopted by The Code.

The rectangles should be chosen to maximize the total torsional stiffness of the section.

In this example there are four ways in which the section can be divided, thus;



Only one of these is the correct way. Which one , do you think, it is?

Non-Convex Sections

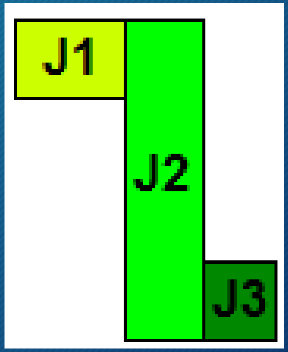
The elastic torsional stiffness (the St Venant values), J for a rectangular section is given by the expression:

$$J = \beta \, h_2^3 \, h_1$$

Where, h_1 and h_2 are the dimensions of the rectangle, h_2 being the smaller dimension, and β is a coefficient whose value is given by the following table.

Values of coefficient β						
h_1 / h_2	1	1.5	2	3	5	>5
β	0.14	0.20	0.23	0.26	0.29	0.33

If the torsional stiffness's of the 3 rectangular sections are J1, J2, J3, then the proportion the applied torsional moment, T_{Ed} resisted by each section



$$T1 = J1 \, T_{Ed} / (J1 + J2 + J3)$$
$$T2 = J2 \, T_{Ed} / (J1 + J2 + J3)$$
$$T3 = J3 \, T_{Ed} / (J1 + J2 + J3)$$

Combined Effects

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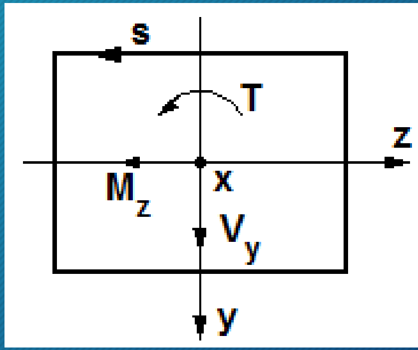
Combined Effects

The analysis and design procedure described on the previous page is for a member subject to pure torsion. In structures, torsion generally acts simultaneously with flexure and shear.

For a thin-walled closed section, a statically admissible stress field can be found for the normal stresses σ_x due to the bending moment, the shear stresses τ_{xs} due to the shear force, and the shear stresses τ_{xs} due to the torsional moment.

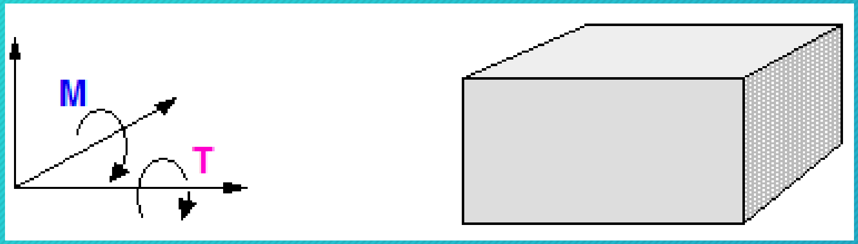


Having determined the distribution of these stresses, the areas of reinforcement required to maintain equilibrium can be found.

The Code also permits a simplified design procedure for combined action, which involves designing the section for each action and then using simple checking procedures to ensure that the section provides an adequate resistance to the combined effects.

Combined Effects
Flexure

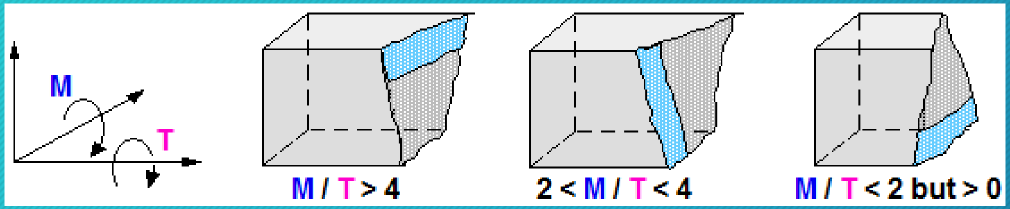
At failure, the ratio of M to T determines the position of the compression zone

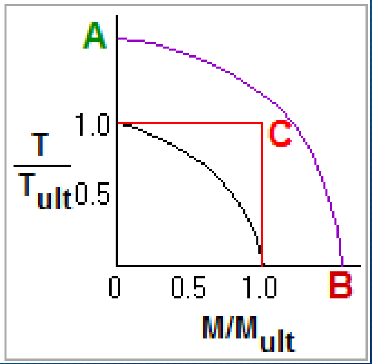


If there was NO torsion, the compression zone would be at the top of the section.

If there was NO moment, i.e. pure torsion, there would be NO compression zone.

At failure, the ratio of M to T determines the position of the compression zone. It is always skewed.



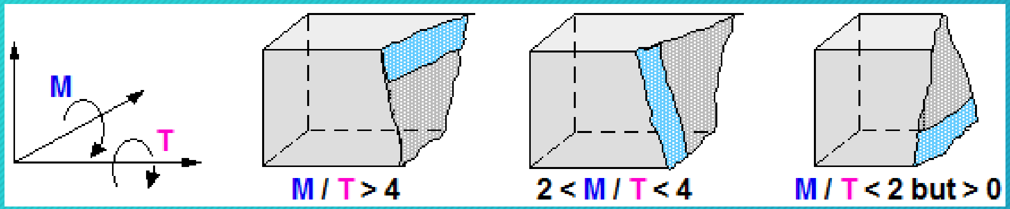


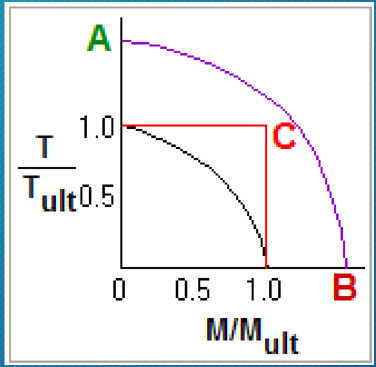
Where the longitudinal reinforcement is symmetrical about both axes, the interaction of torsion and flexure is represented by this form of relationship:

The simplified design procedure of designing for torsion and flexure separately, represented by point **C**, is justified since the reinforcement provided for flexure increases the strength in pure torsion, point **A**, and vice versa for pure flexure, point **B**.

Combined Effects
Flexure

At failure, the ratio of M to T determines the position of the compression zone. It is always skewed.





When using the simplified design procedure, 3 conditions must be satisfied:-

- In the flexural tension zone, the longitudinal torsion steel must be additional to that required to resist flexural and axial forces.
- In the flexural compression zone, if the tensile stress due to torsion is less than the concrete compression stress due to flexure, no additional longitudinal steel is required.
- The principal stress in the compression zone found from the mean longitudinal, flexural compressive stress and the torsional tangential stress ($\tau = T/(2A_k t)$), must be limited to f_{cd}.

Combined Effects

Shear

The interaction of torsion and shear is not fully understood, but a circular interaction curve is generally used, such that the design torsional moment T_{Ed} and the design shear force V_{Ed} should satisfy one of these conditions:-.

For solid sections -

$$(T_{Ed} / T_{Rd,max})^2 + (V_{Ed} / V_{Rd,max})^2 \leq 1$$

For hollow sections -

$$T_{Ed} / T_{Rd,max} + V_{Ed} / V_{Rd,max} \leq 1$$

The design torsional resistance moment $T_{Rd,max}$ and the design shear resistance $V_{Rd,max}$ must be based on the same value of $\cot\theta$.

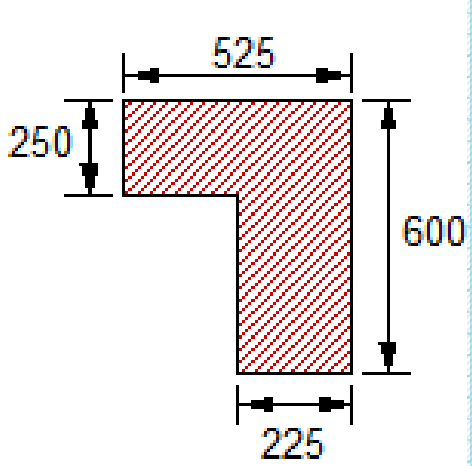
For approximately rectangular solid sections only minimum shear and torsion reinforcement is required, provided both these two conditions are satisfied.

$$T_{Ed} \leq \frac{V_{Ed}\, b_w}{4.5}$$

$$V_{Ed}\left[1 + \frac{4.5\, T_{Ed}}{V_{Ed}\, b_w}\right] \leq V_{Rd,ct}$$

Design example with questions

Example 5.1: Calculate the area of steel required in the web of the edge beam below by addressing the questions I to V sequentially.



USE

- C40/50 and S-500 bars for stirrup and longitudinal reinforcement.
- Assume cover to longitudinal bars to be 35mm
- Assume, $\cot\theta = 2.5$
- $T_{Ed} = 45$ kNm

I. What is the value of the torsional moment to be resisted by the web, $T_{ED,web}$ (kNm)?
II. What is the thickness of the equivalent thin-walled section for the web, t(mm)?
III. What is the maximum torsional moment that can be resisted by the compressive struts in the web, $T_{RD,max}$ (kNm)?
IV. What area of longitudinal reinforcement for torsion is required in the web, A_{sl} (mm^2)?
V. What is the minimum amount of link reinforcement required to resist the design torsional moment in the web, A_{sw}/s (mm^2/mm)?

Solution:

Step1: Summarize the given parameters

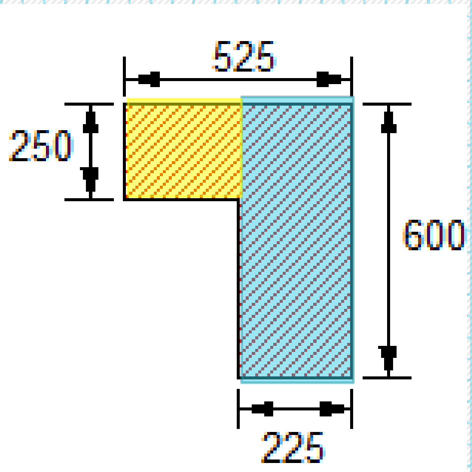
Material C40/50

f_{ck}=40MPa; f_{cd}=22.66MPa;
$f_{ctk,0.05}$=2.5MPa; f_{ctd}=1.4MPa
E_{cm}=35,000MPa
S-500
f_{yk}=500MPa;
f_{yd}=434.78MPa;
E_s=200,000MPa; ε_y=2.17‰

Action T_{Ed}=45 kNm

Step2: Compute the value of the torsional moment to be resisted by the web.

The T_{Ed} is divided between the web and the flange according to their torsional stiffness, J.



For the web:

h_2=225, h_1=600, β=0.25, J=1709x10^6

Values of coefficient β						
h_1/h_2	1	1.5	2	3	5	>5
β	0.14	0.20	0.23	0.26	0.29	0.33

For the flange:

h_2=250, h_1=300, β=0.164, J=769x10^6

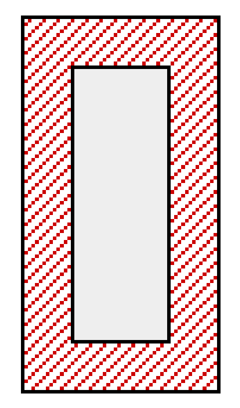
Thus:

$T_{Ed,web}$ = 45 x 1709/(1709+769)=31kNm

Step3: Compute the thickness of the equivalent thin- walled section for the web.

Thickness = area/perimeter:

t=A/u



Therefore,

t=600x225/1650= 81mm

Minimum is twice the cover to the longitudinal bars

2x35= 70mm

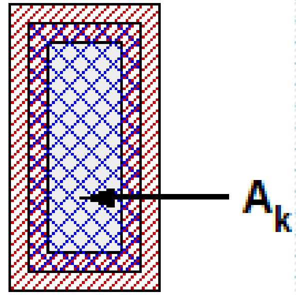
Therefore, t=81mm

Step4: Compute the maximum torsional moment that can be resisted by the compressive struts in the web.

$$T_{Rd,max} = 2 A_k t \alpha_c \nu f_{cd} \sin\theta \cos\theta$$

$$\nu = 0.6 (1 - f_{ck} / 250)$$

$$\alpha_c = 1$$



Thus, $\nu = 0.6 (1 - 40/250) = \mathbf{0.5}$

$t = \mathbf{81}$ mm

$A_k = (600-81) \times (225-81) = \mathbf{74736}\ mm^2$

$f_{cd} = 0.85 \times 40/1.5 = \mathbf{22.67}$ MPa

Therefore,

$T_{RD,max}$=2x74736x81x0.5x22.67x0.3714x0.9285

=47.7kNm

This exceeds $T_{Ed,web}$ (**31**kNm), therefore section is OK!

Step5: Compute the area of longitudinal reinforcement for torsion is required in the web.

$$A_{sl} f_{yld} = T_{Ed} (u_k / 2 A_k) \cot\theta$$

To satisfy the design torsional moment, $T_{Ed,web}$

$$A_{sl} = T_{Ed,web}\, u_k \cot\theta / (2 A_k f_{yd})$$

= 31 x 10^6 x 1326 x 2.5 / (2 x 74736 x 500/1.15)

= 1583mm^2

Step6: Compute the minimum amount of link reinforcement required to resist the design torsional moment in the web.

Minimum area of torsional link reinforcement is:-

$$A_{sw} / s = \frac{T_{Ed}}{2 A_k f_{ywd} \cot\theta}$$

$A_{sw/s}$ = 31 x10^6 / (2 x 74736x500/1.15 x 2.5)

= 0.19 mm^2/mm

Shear force due to torsional, $\sum V_{Ed,I}$) to be added to beam shear, V_{Ed}) is for a rectangular section-

$$\Sigma V_{Ed,i} = T_{Ed}(h\text{-}t)/A_k$$

$\sum V_{Ed,I}$ = 31 x (600-81)/(74736/1000) kN = **216kN**

Maximum spacing of torsion links is:

- at least of $u_k/8$ (166) and,
- the lesser beam dimension (225) and 0.75d (413) mm

Summary

Summary

The following are important concepts that have been discussed in this chapter:-

- The ***nature of torsion*** and its effect on a reinforced concrete member – the shear stresses induced in the member and the crack pattern this causes.
- The resistance of a member to torsion – mainly by circumferential reinforcement.
- The idealization of a convex section as an equivalent thin-walled section, and the use of Bredt's formula to express the induced shear stress in terms of the applied torsional moment.
- The analysis of the stress field (truss model) to develop expressions for the concrete compressive stress in the notional strut, and the areas of circumferential and longitudinal reinforcement in terms of the applied torsional moment.
- The Variable Strut Inclination (VSI) method of design which uses just the resistance of the torsional reinforcement with a varying angle for the concrete struts.
- The design of non-convex sections by dividing them into a number of rectangles.
- The combined effects of torsion, flexure and shear.

Thank you for the kind attention!

Questions?

RC-2 is over!

Good luck!