

FOUNDATION ENGINEERING I (CONTD...)

CEng 3204

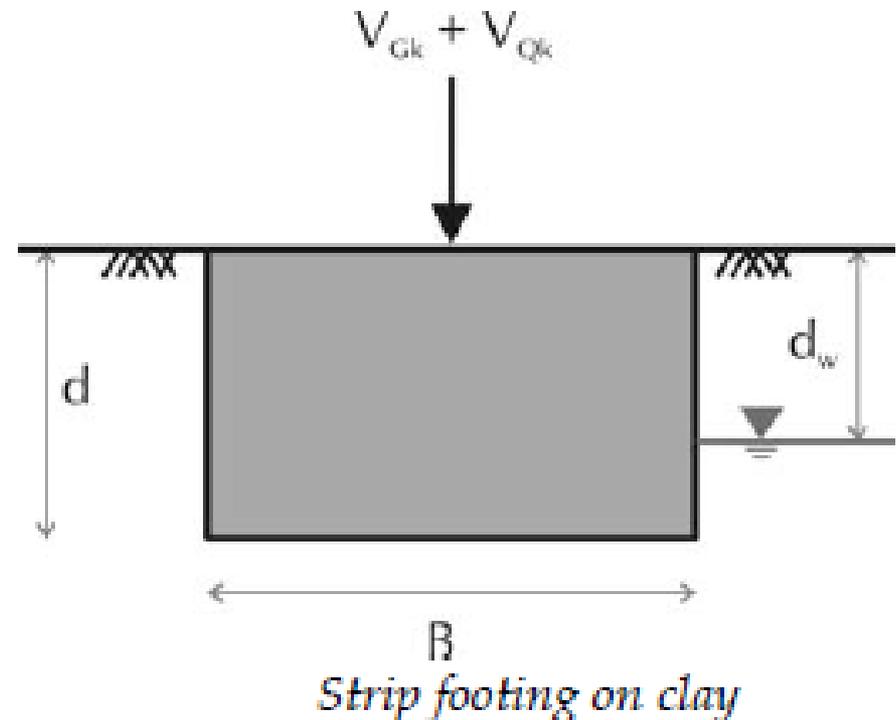
CHAPTER THREE

Design of Shallow Foundations: **ISOLATED FOOTINGS**

Worked Examples

3. considers the design of a strip footing on clay, as shown in Figure. Groundwater is at a depth d_w below ground surface.

This example demonstrates the use of partial factors for undrained and drained parameters. The inclusion of groundwater above the base of the footing illustrates the complications in applying partial factors to water pressures.



Worked Examples

Design situation

Consider an infinitely long footing of breadth $B = 2.5\text{m}$ and depth $d = 1.5\text{m}$, which is required to carry an imposed permanent action $V_{Gk} = 250 \frac{\text{kN}}{\text{m}}$ and an imposed variable action $V_{Qk} = 110 \frac{\text{kN}}{\text{m}}$. The footing is founded on a medium strength clay **1** with characteristic undrained strength $c_{uk} = 45\text{kPa}$, angle of shearing resistance $\varphi_k = 25^\circ$, effective cohesion $c'_k = 5\text{kPa}$, and weight density $\gamma_k = 21 \frac{\text{kN}}{\text{m}^3}$. The water table is currently at a depth $d_w = 1\text{m}$. The weight density of groundwater is $\gamma_w = 9.81 \frac{\text{kN}}{\text{m}^3}$ and of reinforced concrete $\gamma_{ck} = 25 \frac{\text{kN}}{\text{m}^3}$ (EN 1991-1-1 Table A.1).

Worked Examples

Design Approach 1

Geometrical parameters

Design depth of water table $d_{w,d} = 0\text{m}$ ②

Actions and effects

Characteristic self-weight of footing is $W_{Gk} = \gamma_{ck} \times B \times d = 93.8 \frac{\text{kN}}{\text{m}}$

Characteristic pore pressure underneath base

$$u_{k,b} = \gamma_w \times (d - d_{w,d}) = 14.7 \text{ kPa}$$

Partial factors, Set $\begin{pmatrix} A1 \\ A2 \end{pmatrix}$: $\gamma_G = \begin{pmatrix} 1.35 \\ 1 \end{pmatrix}$, $\gamma_{G, fav} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, and $\gamma_Q = \begin{pmatrix} 1.5 \\ 1.3 \end{pmatrix}$

Design vertical action: $V_d = \gamma_G \times (W_{Gk} + V_{Gk}) + \gamma_Q \times V_{Qk} = \begin{pmatrix} 629.1 \\ 486.8 \end{pmatrix} \frac{\text{kN}}{\text{m}}$

Design bearing pressure (total stress): $q_{Ed} = \frac{V_d}{B} = \begin{pmatrix} 251.6 \\ 194.7 \end{pmatrix} \text{ kPa}$

Worked Examples

Design upthrust (favourable): $u_d = \gamma_{G, fav} \times u_{k,b} = \begin{pmatrix} 14.7 \\ 14.7 \end{pmatrix} \text{ kPa} \textcircled{3}$

Design bearing pressure (effective stress): $q'_{Ed} = q_{Ed} - u_d = \begin{pmatrix} 236.9 \\ 180 \end{pmatrix} \text{ kPa}$

Material properties and resistance

Partial factors, Set $\begin{pmatrix} M1 \\ M2 \end{pmatrix}$: $\gamma_{cu} = \begin{pmatrix} 1 \\ 1.4 \end{pmatrix}$, $\gamma_{\varphi} = \begin{pmatrix} 1 \\ 1.25 \end{pmatrix}$ and $\gamma_c = \begin{pmatrix} 1 \\ 1.25 \end{pmatrix}$

Design undrained strength is $c_{ud} = \frac{c_{uk}}{\gamma_{cu}} = \begin{pmatrix} 45 \\ 32.1 \end{pmatrix} \text{ kPa}$

Design shearing resistance is $\varphi_d = \tan^{-1} \left(\frac{\tan(\varphi_k)}{\gamma_{\varphi}} \right) = \begin{pmatrix} 25 \\ 20.5 \end{pmatrix} \text{ }^\circ$

Design cohesion is $c'_d = \frac{c'_k}{\gamma_c} = \begin{pmatrix} 5 \\ 4 \end{pmatrix} \text{ kPa}$

Worked Examples

Drained bearing capacity factors

$$\text{For overburden: } N_q = \overrightarrow{\left[e^{(\pi \times \tan(\varphi_d))} \times \left(\tan \left(45^\circ + \frac{\varphi_d}{2} \right) \right)^2 \right]} = \begin{pmatrix} 10.7 \\ 6.7 \end{pmatrix}$$

$$\text{For cohesion: } N_c = \overrightarrow{\left[(N_q - 1) \times \cot(\varphi_d) \right]} = \begin{pmatrix} 20.7 \\ 15.3 \end{pmatrix}$$

$$\text{For self-weight: } N_\gamma = \overrightarrow{\left[2(N_q - 1) \times \tan(\varphi_d) \right]} = \begin{pmatrix} 9 \\ 4.3 \end{pmatrix}$$

Worked Examples

Depth and shape factors

Salgado's depth factor for undrained loading: $d_c = 1 + 0.27 \sqrt{\frac{d}{B}} = 1.21$ ④

Ignore depth factors for drained loading

Salgado's shape factor for undrained loading: $s_c = 1 + 0.17 \sqrt{\frac{d}{B}} = 1.13$ ④

Depth factors are all 1.0 for drained loading and so can be ignored

Worked Examples

Undrained bearing resistance **Short-term Analysis**

Total overburden at foundation base is $\sigma_{vk,b} = \gamma_k \times d = 31.5 \text{ kPa}$

Partial factors from Sets $\begin{pmatrix} R1 \\ R1 \end{pmatrix}$: $\gamma_{Rv} = \begin{pmatrix} 1.0 \\ 1.0 \end{pmatrix}$

Ultimate resistance is $q_{ult} = (\pi + 2) \times c_{ud} \times d_c \times s_c + \sigma_{vk,b} = \begin{pmatrix} 348.1 \\ 257.6 \end{pmatrix} \text{ kPa}$

Design resistance is $q_{Rd} = \frac{q_{ult}}{\gamma_{Rv}} = \begin{pmatrix} 348.1 \\ 257.6 \end{pmatrix} \text{ kPa}$

Worked Examples

Long-term Analysis

Drained bearing resistance

Effective overburden at foundation base is $\sigma'_{vk,b} = \sigma_{vk,b} - u_{k,b} = 16.8 \text{ kPa}$

$$\text{From overburden } q'_{ult_1} = \overrightarrow{(N_q \times \sigma'_{vk,b})} = \begin{pmatrix} 179 \\ 112.5 \end{pmatrix} \text{ kPa}$$

$$\text{From cohesion } q'_{ult_2} = \overrightarrow{(N_c \times c'_d)} = \begin{pmatrix} 103.6 \\ 61.1 \end{pmatrix} \text{ kPa}$$

$$\text{From self-weight } q'_{ult_3} = \overrightarrow{\left[N_\gamma \times (\gamma_k - \gamma_w) \times \frac{B}{2} \right]} = \begin{pmatrix} 126.1 \\ 59.5 \end{pmatrix} \text{ kPa}$$

$$\text{Total resistance } q'_{ult} = \sum_{i=1}^3 q'_{ult_i} = \begin{pmatrix} 408.7 \\ 233 \end{pmatrix} \text{ kPa}$$

$$\text{Design resistance is } q'_{Rd} = \frac{q'_{ult}}{\gamma_{Rv}} = \begin{pmatrix} 408.7 \\ 233 \end{pmatrix} \text{ kPa}$$

Worked Examples

Verification of undrained bearing resistance

$$\text{Degree of utilization } \Lambda_{\text{GEO},1} = \frac{q_{\text{Ed}}}{q_{\text{Rd}}} = \left(\frac{72}{76} \right) \% \quad \text{⑤}$$

Design is unacceptable if degree of utilization is > 100%

Verification of drained bearing resistance

$$\text{Degree of utilization } \Lambda'_{\text{GEO},1} = \frac{q'_{\text{Ed}}}{q'_{\text{Rd}}} = \left(\frac{58}{77} \right) \% \quad \text{⑤}$$

Design is unacceptable if degree of utilization is > 100%

**Drained (long-term)
situation is slightly
more critical in DA1-2**

Worked Examples

Design Approach 2

Actions and effects

Partial factors from set A1: $\gamma_G = 1.35$ and $\gamma_Q = 1.5$ **6**

Design action is $V_d = \gamma_G \times (W_{Gk} + V_{Gk}) + \gamma_Q \times V_{Qk} = 629.1 \frac{\text{kN}}{\text{m}}$

Design bearing pressure (total stress) is $q_{Ed} = \frac{V_d}{B} = 251.6 \text{ kPa}$

Design upthrust (favourable): $u_d = \gamma_{G,fav} \times u_{k,b} = 14.7 \text{ kPa}$

Design bearing pressure (effective stress) is $q'_{Ed} = q_{Ed} - u_d = 236.9 \text{ kPa}$

Worked Examples

Material properties and resistance

Partial factors from set M1: $\gamma_{cu} = 1.0$, $\gamma_{\omega} = 1.0$, and $\gamma_c = 1.0$ **6**

Design undrained strength is $c_{ud} = \frac{c_{uk}}{\gamma_c} = 45 \text{ kPa}$

Design angle of shearing resistance is $\varphi_d = \tan^{-1} \left(\frac{\tan(\varphi_k)}{\gamma_\varphi} \right) = 25 \text{ deg}$

Design cohesion is $c'_d = \frac{c'_k}{\gamma_c} = 5 \text{ kPa}$

Worked Examples

Drained bearing capacity factors

$$\text{For overburden: } N_q = e^{(\pi \times \tan(\varphi_d))} \left(\tan\left(45^\circ + \frac{\varphi_d}{2}\right) \right)^2 = 10.7$$

$$\text{For cohesion: } N_c = (N_q - 1) \times \cot(\varphi_d) = 20.7$$

$$\text{For self-weight: } N_\gamma = 2(N_q - 1) \times \tan(\varphi_d) = 9$$

Depth and shape factors

Are the same as for Design Approach 1

Undrained bearing resistance

Total overburden at foundation base is $\sigma_{vk,b} = \gamma_k \times d = 31.5 \text{ kPa}$

Partial factors from set R2: $\gamma_{Rv} = 1.4$ **7**

Worked Examples

Ultimate resistance is $q_{ult} = (\pi + 2) \times c_{ud} \times d_c \times s_c + \sigma_{vk,b} = 348.1 \text{ kPa}$

Design resistance is $q_{Rd} = \frac{q_{ult}}{\gamma_{Rv}} = 248.6 \text{ kPa}$

Drained bearing resistance

Effective overburden at foundation base is $\sigma'_{vk,b} = \sigma_{vk,b} - u_{k,b} = 16.8 \text{ kPa}$

From overburden $q'_{ult_1} = \overrightarrow{(N_q \times \sigma'_{vk,b})} = 179 \text{ kPa}$

From cohesion $q'_{ult_2} = \overrightarrow{(N_c \times c'_d)} = 103.6 \text{ kPa}$

From self-weight $q'_{ult_3} = \overrightarrow{\left[N_\gamma \times (\gamma_k - \gamma_w) \times \frac{B}{2} \right]} = 126.1 \text{ kPa}$

Total resistance $q'_{ult} = \sum q'_{ult} = 408.7 \text{ kPa}$

Design resistance is $q'_{Rd} = \frac{q'_{ult}}{\gamma_{Rv}} = 291.9 \text{ kPa}$

Worked Examples

Verification of undrained bearing resistance

Degree of utilization $\Lambda_{\text{GEO},2} = \frac{q_{\text{Ed}}}{q_{\text{Rd}}} = 101\%$ **8**

Design is unacceptable if degree of utilization factor is > 100%

Verification of drained bearing resistance

Degree of utilization $\Lambda'_{\text{GEO},2} = \frac{q'_{\text{Ed}}}{q'_{\text{Rd}}} = 81\%$ **8**

Design is unacceptable if degree of utilization is > 100%

Worked Examples

Design Approach 3

Actions and effects

Partial factors on actions from set A1: $\gamma_G = 1.35$ and $\gamma_Q = 1.5$ ⑨

Design vertical action $V_d = \gamma_G \times (W_{Gk} + V_{Gk}) + \gamma_Q \times V_{Qk} = 629.1 \frac{\text{kN}}{\text{m}}$

Design bearing pressure (total stress) $q_{Ed} = \frac{V_d}{B} = 251.6 \text{ kPa}$

Design upthrust (favourable): $u_d = \gamma_{G,fav} \times u_{k,b} = 14.7 \text{ kPa}$

Design bearing pressure (effective stress) $q'_{Ed} = q_{Ed} - u_d = 236.9 \text{ kPa}$

Worked Examples

Material properties and resistance

Partial factors from set M2: $\gamma_{cu} = 1.4$, $\gamma_{\phi} = 1.25$, and $\gamma_c = 1.25$ ⑨

Design undrained strength is $c_{ud} = \frac{c_{uk}}{\gamma_{cu}} = 32.1 \text{ kPa}$

Design angle of shearing resistance is $\phi_d = \tan^{-1} \left(\frac{\tan(\phi_k)}{\gamma_{\phi}} \right) = 20.5^\circ$

Design cohesion is $c'_d = \frac{c'_k}{\gamma_c} = 4 \text{ kPa}$

Worked Examples

Drained bearing capacity factors

$$\text{For overburden: } N_q = e^{(\pi \times \tan(\varphi_d))} \times \left(\tan\left(45^\circ + \frac{\varphi_d}{2}\right) \right)^2 = 6.7$$

$$\text{For cohesion: } N_c = (N_q - 1) \times \cot(\varphi_d) = 15.3$$

$$\text{For self-weight: } N_\gamma = 2(N_q - 1) \times \tan(\varphi_d) = 4.3$$

Depth and shape factors

Are the same as for Design Approach 1

Worked Examples

Undrained bearing resistance

Total overburden at foundation base is $\sigma_{vk,b} = \gamma_k \times d = 31.5 \text{ kPa}$

Partial factors from set R3: $\gamma_{Rv} = 1.0$

Ultimate resistance is $q_{ult} = (\pi + 2) \times c_{ud} \times d_c \times s_c + \sigma_{vk,b} = 257.6 \text{ kPa}$

Design resistance is $q_{Rd} = \frac{q_{ult}}{\gamma_{Rv}} = 257.6 \text{ kPa}$

Worked Examples

Drained bearing resistance

Effective overburden at foundation base is $\sigma'_{vk,b} = \sigma_{vk,b} - u_{k,b} = 16.8 \text{ kPa}$

$$\text{From overburden } q'_{ult_1} = \overrightarrow{(N_q \times \sigma'_{vk,b})} = 112.5 \text{ kPa}$$

$$\text{From cohesion } q'_{ult_2} = \overrightarrow{(N_c \times c'_d)} = 61.1 \text{ kPa}$$

$$\text{From self-weight } q'_{ult_3} = \overrightarrow{\left[N_\gamma \times (\gamma_k - \gamma_w) \times \frac{B}{2} \right]} = 59.5 \text{ kPa}$$

$$\text{Total resistance } q'_{ult} = \sum q'_{ult} = 233 \text{ kPa}$$

$$\text{Design resistance is } q'_{Rd} = \frac{q'_{ult}}{\gamma_{Rv}} = 233 \text{ kPa}$$

Worked Examples

Verification of undrained bearing resistance

Degree of utilization $\Lambda_{GEO,3} = \frac{q_{Ed}}{q_{Rd}} = 98\%$ ⑩

Design is unacceptable if degree of utilization is > 100%

Verification of drained bearing resistance

Degree of utilization $\Lambda'_{GEO,3} = \frac{q'_{Ed}}{q'_{Rd}} = 102\%$ ⑩

Design is unacceptable if degree of utilization is > 100%

Worked Examples

Settlement of strip footing on clay Verification of serviceability

Design situation

Consider the infinitely long strip footing from the previous example. There is a rigid layer underlying the footing at a depth of $d_R = 4.5\text{m}$. The clay's

undrained Young's modulus is assumed to be $E_{uk} = 600c_{uk} = 27\text{ MPa}$ and its

characteristic coefficient of compressibility $m_{vk} = 0.12 \frac{\text{m}^2}{\text{MN}}$

Worked Examples

Implicit verification of serviceability (based on ULS check)

Geometrical parameters

Design depth of water table $d_{w,d} = d_w = 1\text{ m}$ **1**

Actions and effects

From previous calculation, characteristic actions are:

imposed permanent $V_{Gk} = 250\text{ kN/m}$

imposed variable action $V_{Qk} = 110\text{ kN/m}$

self-weight of footing $W_{Gk} = 93.8\text{ kN/m}$

Characteristic pore pressure under base $u_{k,b} = \gamma_w \times (d - d_{w,d}) = 4.9\text{ kPa}$ **2**

Worked Examples

Partial load factors for SLS: $\gamma_G = 1$, $\gamma_{G.fav} = 1$, and $\gamma_Q = 1$ ③

Design vertical action: $V_d = \gamma_G \times (W_{Gk} + V_{Gk}) + \gamma_Q \times V_{Qk} = 453.8 \text{ kN/m}$

Design bearing pressure (total stress): $q_{Ed} = \frac{V_d}{B} = 181.5 \text{ kPa}$

Design upthrust (favourable): $u_d = \gamma_{G.fav} \times u_{k.b} = 4.9 \text{ kPa}$ ④

Design bearing pressure (effective stress): $q'_{Ed} = q_{Ed} - u_d = 176.6 \text{ kPa}$

Worked Examples

Material properties and resistance

From previous calculation, characteristic material properties are: undrained strength $c_{uk} = 45$ kPa, shearing resistance $\varphi_k = 25^\circ$, cohesion $c'_k = 5$ kPa

Partial material factors for SLS: $\gamma_{cu} = 1$, $\gamma_\varphi = 1$ and $\gamma_c = 1$ **3**

Design undrained strength $c_{ud} = c_{uk} \div \gamma_{cu} = 45$ kPa

Design shearing resistance $\varphi_d = \tan^{-1} \left(\tan(\varphi_k) \div \gamma_\varphi \right) = 25^\circ$

Design cohesion $c'_d = c'_k \div \gamma_c = 5$ kPa

Drained bearing capacity factors

$$\text{For overburden: } N_q = e^{(\pi \times \tan(\varphi_d))} \times \left(\tan\left(45^\circ + \frac{\varphi_d}{2}\right) \right)^2 = 10.7$$

$$\text{For cohesion: } N_c = (N_q - 1) \times \cot(\varphi_d) = 20.7$$

$$\text{For self-weight: } N_\gamma = 2(N_q - 1) \times \tan(\varphi_d) = 9$$

Depth and shape factors

$$\text{Previous calculation: } d_c = 1 + 0.27 \sqrt{\frac{d}{B}} = 1.21 \quad \& \quad s_c = 1 + 0.17 \sqrt{\frac{d}{B}} = 1.13$$

Undrained bearing resistance

From previous calculation, total overburden under base is $\sigma_{vk,b} = 31.5 \text{ kPa}$

Partial resistance factor for SLS: $\gamma_{Rv,SLS} = 3.0$ **5**

Ultimate resistance is $q_{ult} = (\pi + 2) \times c_{ud} \times d_c \times s_c + \sigma_{vk,b} = 348.1 \text{ kPa}$

Design resistance is $q_{Rd} = \frac{q_{ult}}{\gamma_{Rv,SLS}} = 116 \text{ kPa}$

Worked Examples

Drained bearing resistance

Effective overburden under base is $\sigma'_{vk,b} = \sigma_{vk,b} - u_d = 26.6 \text{ kPa}$

From overburden $q'_{ult_1} = N_q \times \sigma'_{vk,b} = 283.6 \text{ kPa}$

From cohesion $q'_{ult_2} = N_c \times c'_d = 103.6 \text{ kPa}$

From self-weight $q'_{ult_3} = N_\gamma \times (\gamma_k - \gamma_w) \times \frac{B}{2} = 126.1 \text{ kPa}$

Total resistance $q'_{ult} = \sum q'_{ult} = 513.3 \text{ kPa}$

Design resistance is $q'_{Rd} = \frac{q'_{ult}}{\gamma_{Rv,SLS}} = 171.1 \text{ kPa}$

Worked Examples

Verification of undrained bearing resistance

Degree of utilization $\Lambda_{SLS} = \frac{q_{Ed}}{q_{Rd}} = 156\%$ ⑥

Design is unacceptable if the degree of utilization is > 100%

Verification of drained bearing resistance

Degree of utilization $\Lambda'_{SLS} = \frac{q'_{Ed}}{q'_{Rd}} = 103\%$ ⑥

Design is unacceptable if the degree of utilization is > 100%

Worked Examples

Explicit verification of serviceability

Actions and effects

Increase in bearing pressure is $\Delta q_d = q_{Ed} - \sigma_{vk,b} = 150 \text{ kPa}$

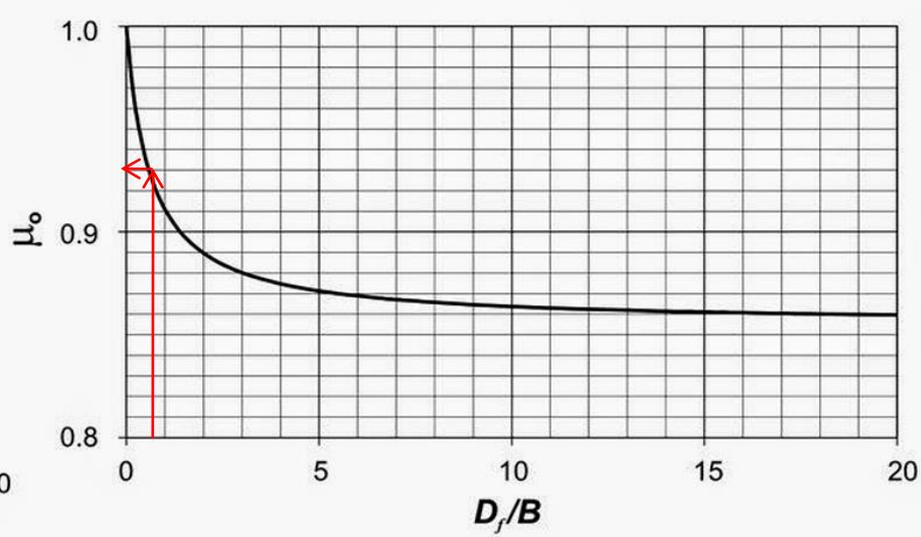
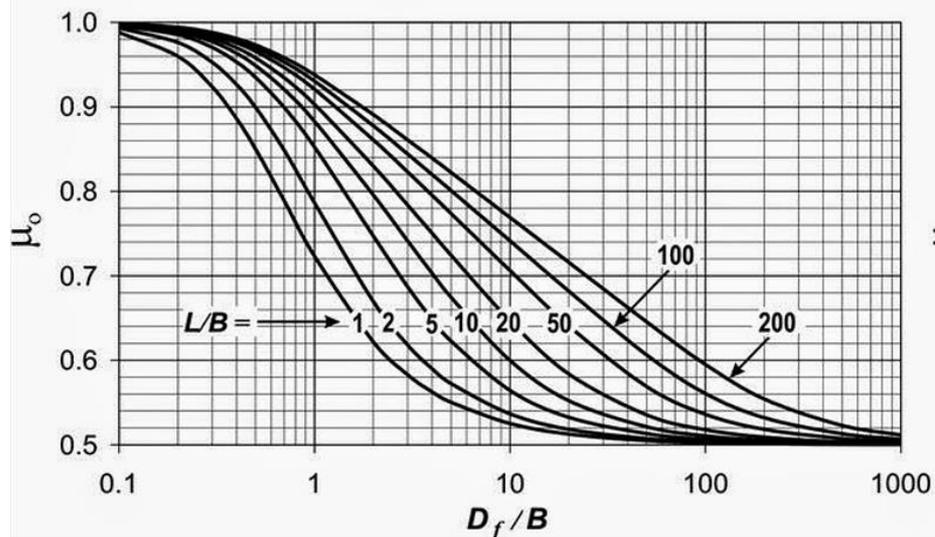
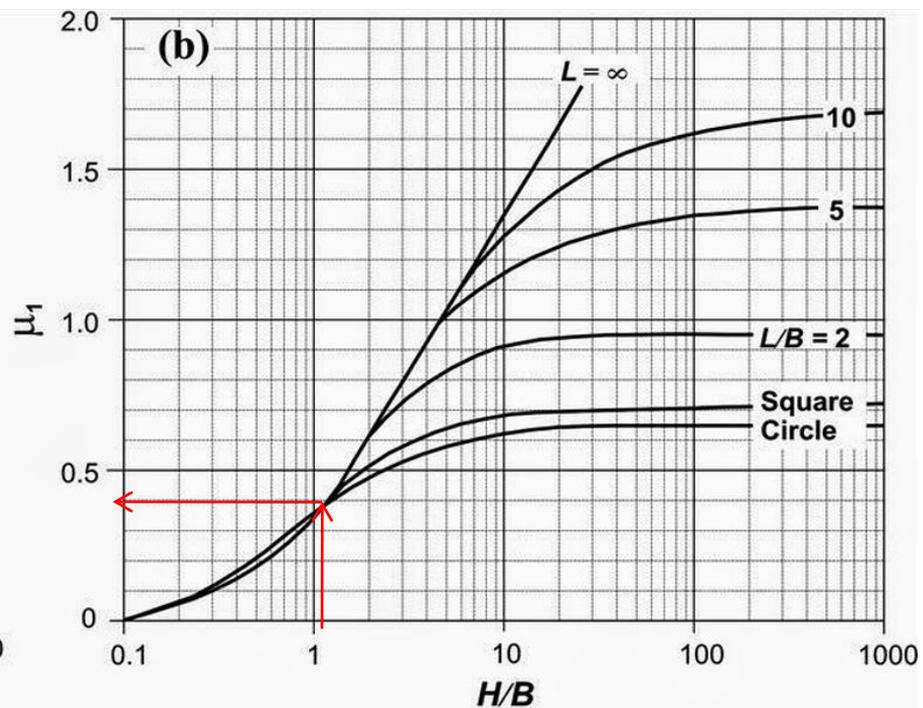
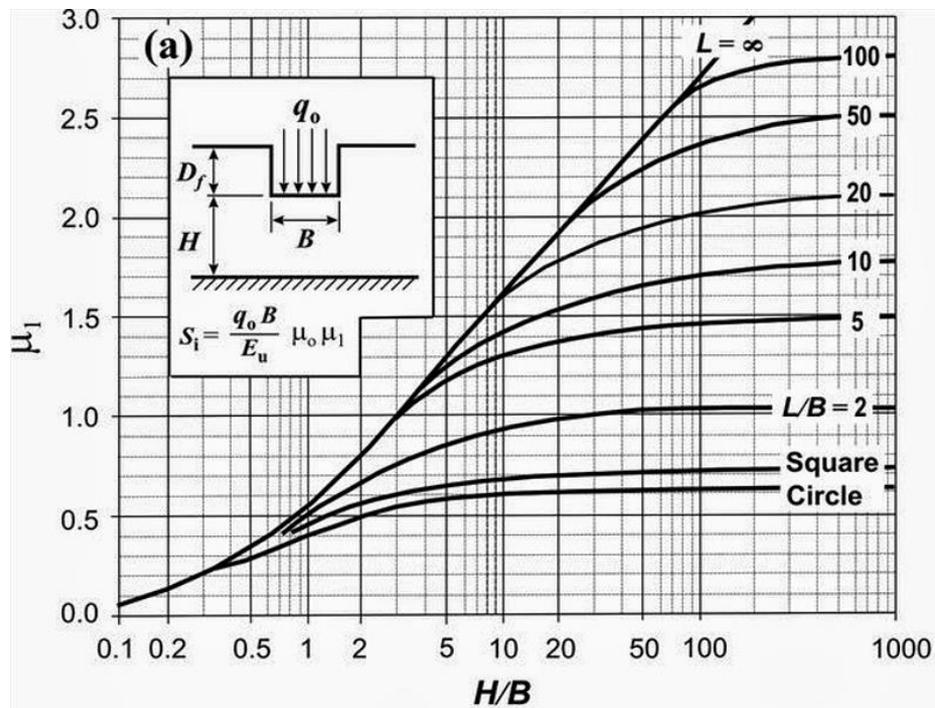
Immediate settlement (Christian & Carrier)

Settlement factor for D/B is $\frac{d}{B} = 0.6$, giving $\mu_0 = 0.93$ (from chart) ⑦

Settlement factor for H/B is $\frac{d_R - d}{B} = 1.2$, giving $\mu_1 = 0.4$ (from chart)

Immediate settlement $s_0 = \frac{\Delta q_d B \mu_0 \mu_1}{E_{uk}} = 5.2 \text{ mm}$

*Janbu, Bjerrum and
Kjaernsli's Method (1956)*



Worked Examples

Consolidation settlement

Divide clay layer into $N = 5$ sub-layers of thickness $\Delta t = \frac{(d_R - d)}{N} = 0.6 \text{ m}$

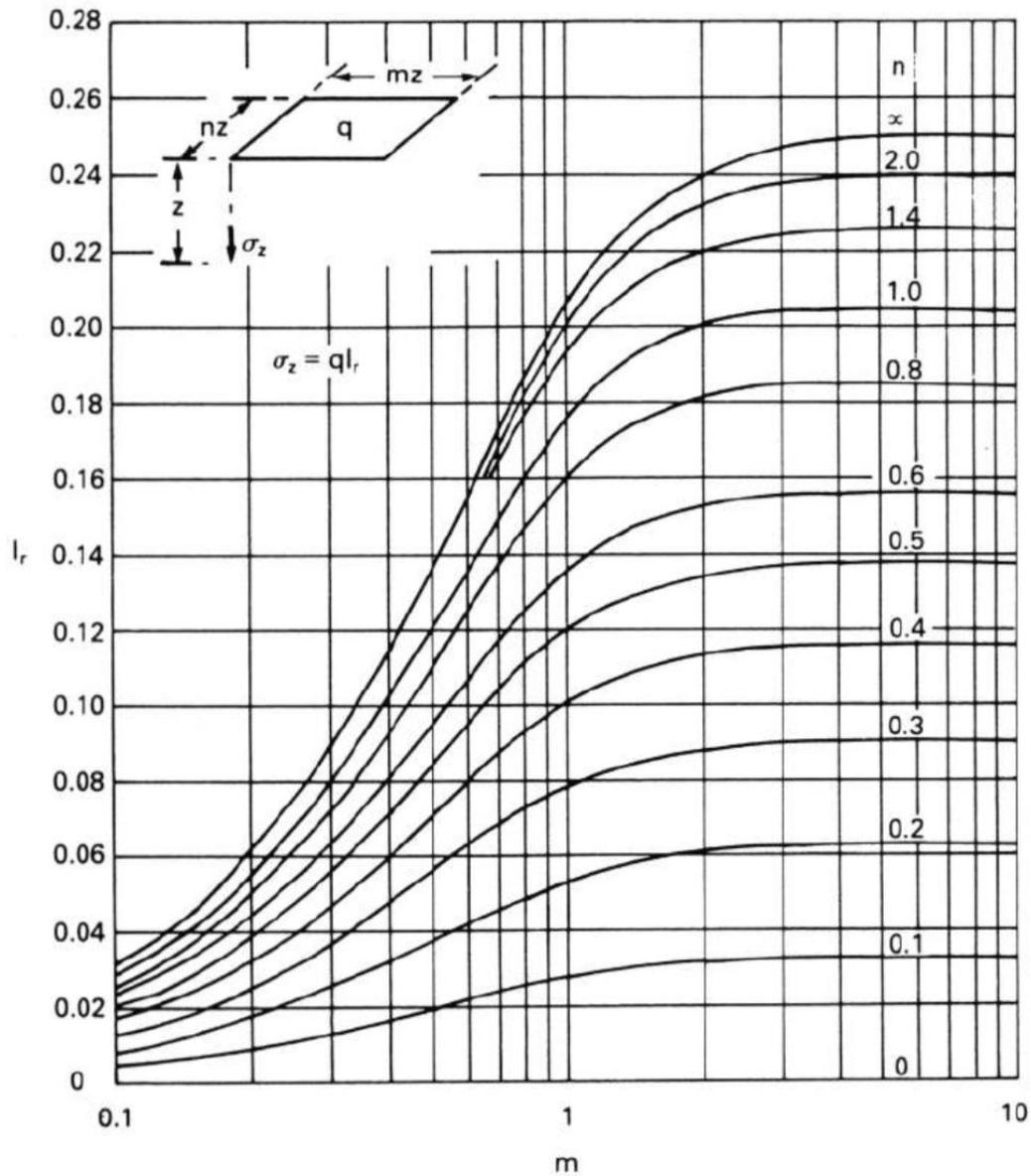
For each layer $i = 1..N$, the depth below base to the centre of each layer is

given by $z_i = (\Delta t \times i) - \frac{\Delta t}{2}$ and the normalized foundation half breadth by

$m_i = \frac{B}{2z_i}$. The influence factor $I_{q_i} = I_{q,\infty}(m_i)$ can be found from Fadum's

chart. The change in vertical stress in each layer is $\Delta\sigma_{v_i} = 4I_{q_i}\Delta q_d$ and the

settlement in each layer $\rho_{c_i} = m_{v_k}\Delta\sigma_{v_i}\Delta t$ ③



Fadum's Chart

Worked Examples

Substituting values into the previous expressions gives:

$$z = \begin{pmatrix} 0.3 \\ 0.9 \\ 1.5 \\ 2.1 \\ 2.7 \end{pmatrix} \text{ m} \quad m = \begin{pmatrix} 4.17 \\ 1.39 \\ 0.83 \\ 0.6 \\ 0.46 \end{pmatrix} \quad I_q = \begin{pmatrix} 0.25 \\ 0.23 \\ 0.19 \\ 0.16 \\ 0.13 \end{pmatrix} \quad \Delta\sigma_v = \begin{pmatrix} 149.2 \\ 135.7 \\ 113.3 \\ 93.2 \\ 77.8 \end{pmatrix} \text{ kPa} \quad \rho_c = \begin{pmatrix} 10.7 \\ 9.8 \\ 8.2 \\ 6.7 \\ 5.6 \end{pmatrix} \text{ mm}$$

The total consolidation settlement is $s_1 = \sum_{i=1}^N \rho_{c_i} = 41 \text{ mm}$

Worked Examples

Total settlement

Sum of settlements is $s = s_0 + s_1 = 46 \text{ mm}$ ⑨

Design effect of actions is $s_{Ed} = s = 46 \text{ mm}$

Verification of settlement

Limiting values of foundation movement for isolated foundation is

$s_{Cd} = 50 \text{ mm}$ ⑩

Degree of utilization is $\Lambda_{SLS} = \frac{s_{Ed}}{s_{Cd}} = 92 \%$

Design is unacceptable if the degree of utilization is $> 100\%$

Design of Isolated Footing

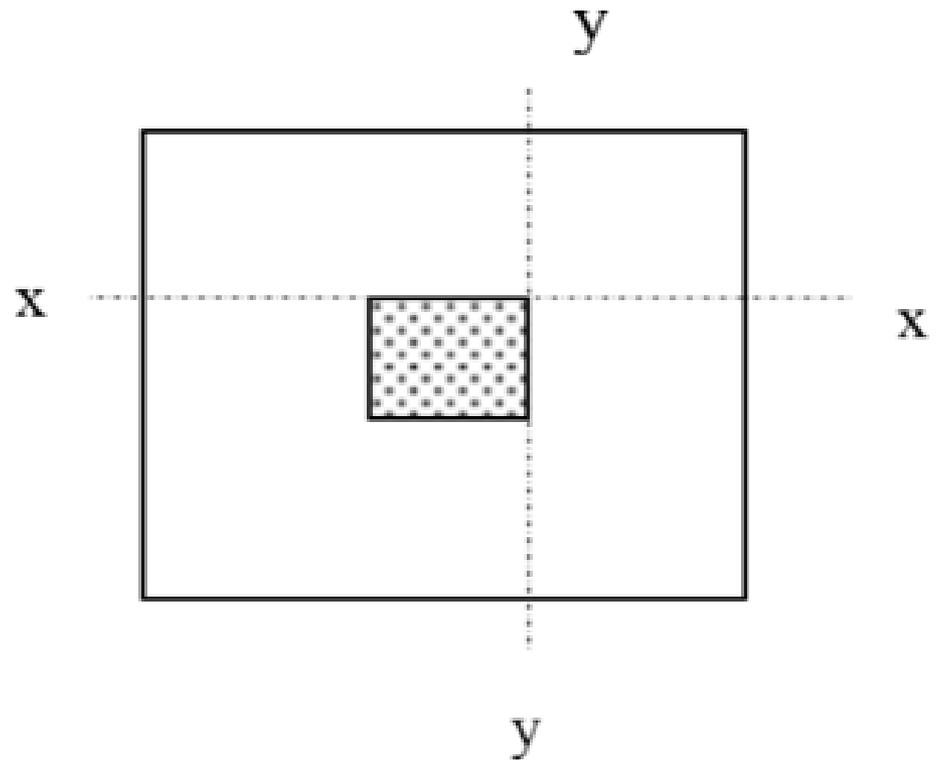
Axially Loaded Pad Bases

- To prevent footing failure $\text{Load} < \text{Capacity}$
 - G_k = Characteristic dead load from the Column(KN)
 - Q_k = Characteristic imposed load from the Column (KN)
 - W = Weight of the base
 - L, B = Base Length and Breadth (m)
 - P_b = Safe Bearing pressure (KN/m² or KPa)- a serviceability value as it is used to control settlement of the foundation
- These loads cause possible failure mechanisms.
- $\text{Load} < \text{Capacity}$ must be fulfilled and the depth and reinforcement of the footing determined.

Design of Isolated Footing

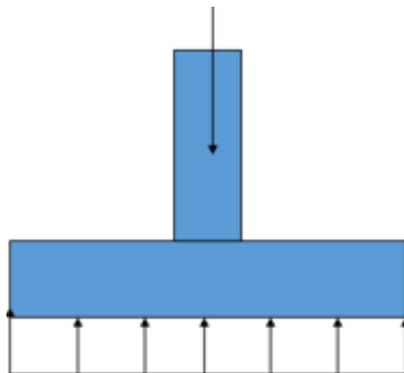
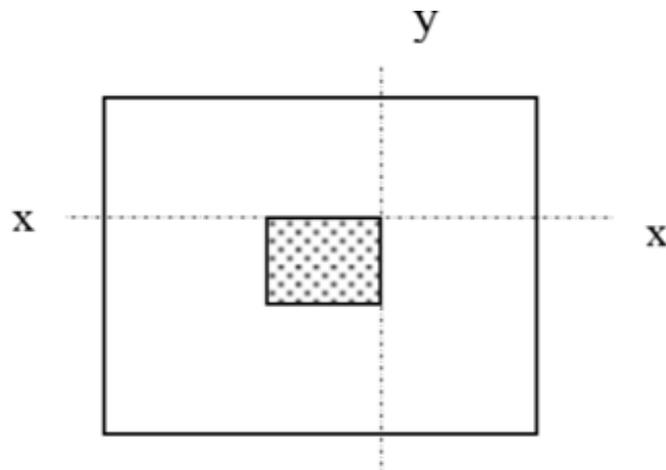
1. Bending moment

- The Critical section for bending is at the face of the Column



Design of Isolated Footing

1. Bending moment



$$M_{Ed} = w l^2 B / 2$$

$$A_s = M / \{0.87 f_{yk} z\}$$

$$\frac{z}{d} = 0.5 \left\{ 1.0 + \sqrt{1 - 3 \frac{k}{\eta}} \right\} \quad k = \frac{M}{b d^2 f_{ck}}$$

$$A_{s,min} = 0.26 \frac{f_{ctm}}{f_{yk}} b_t d \quad \text{but not less than } 0.0013 b_t d$$

$$f_{ctm} = 0.3 \times f_{ck}^{0.67}$$

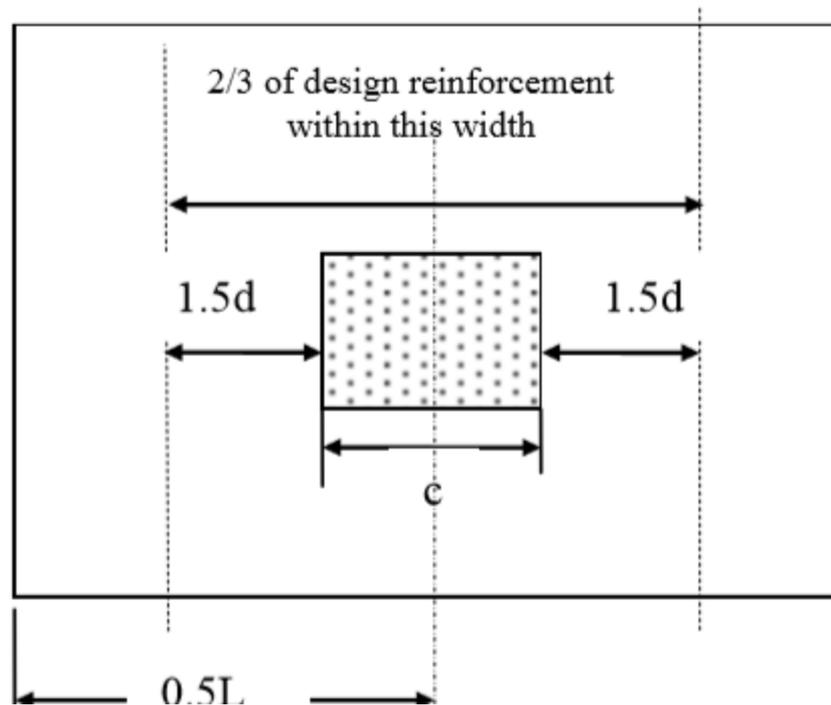
Design of Isolated Footing

1. Bending moment

- Because of the greater concentration of bending moment near the column than towards the edges, traditionally the practice has been to concentrate the reinforcement in a narrow width near the center.
 - Rule- If the distance from the center line of the column to the edge of the pad exceeds $0.75(c+3d)$, $2/3$ of the required reinforcement for the given direction should be concentrated within a zone from the center line of the column to a distance $1.5d$ from the face of the column
 - c- Column width
 - d- the effective depth of the base slab

Design of Isolated Footing

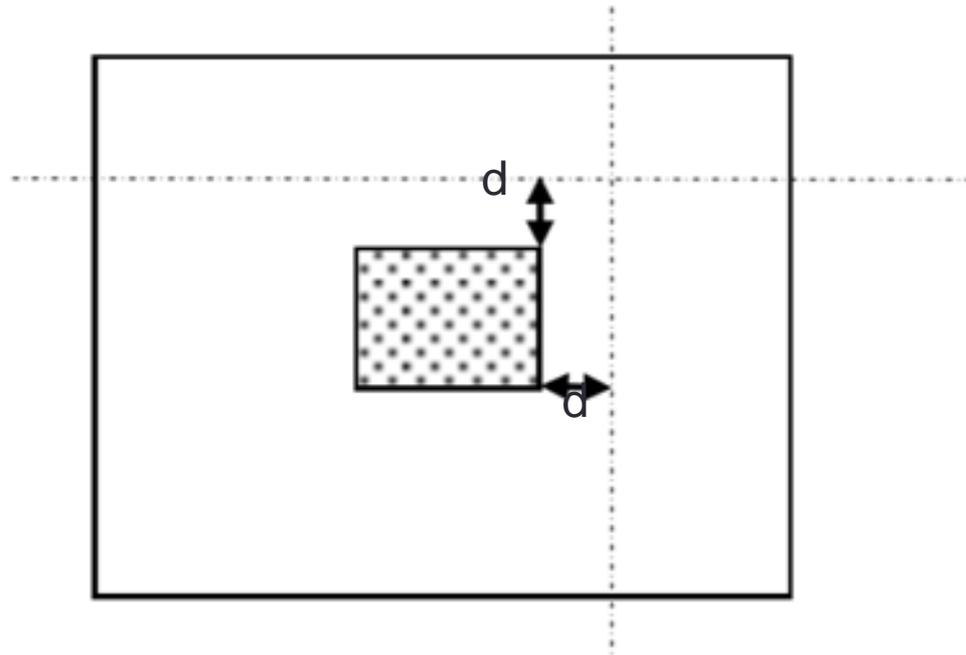
1. Bending Failure



Design of Isolated Footing

2. Vertical Shear

- Shear stress is checked at distance d from the face of column.
- d is effective width (depth to reinforcement)



Design of Isolated Footing

2. Vertical Shear

- Capacity (v_{RD}, f_c):

$$V_{Rd,c} = [C_{Rd,c} k \{100 \rho_1 f_{ck}\}^{1/3}] \quad v_{min} = 0.035 k^{1.5} \sqrt{f_{ck}}$$

$$C_{Rd,c} = \frac{0.18}{(\gamma_c = 1.5)} = 0.12$$

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \quad \rho_1 = \frac{A_{s1}}{b_w d} \leq 0.02$$

- Load (v_{ED}):

- $v_{ED} = \frac{V_{ED}}{A_{sur}}$

- Consider the section with more load to determine V_{ED} .

- $A_{sur} = B.d$

V_{Ed} = Design value of applied shear force

$V_{Rd,c}$ = Design shear resistance of a member without shear reinforcement

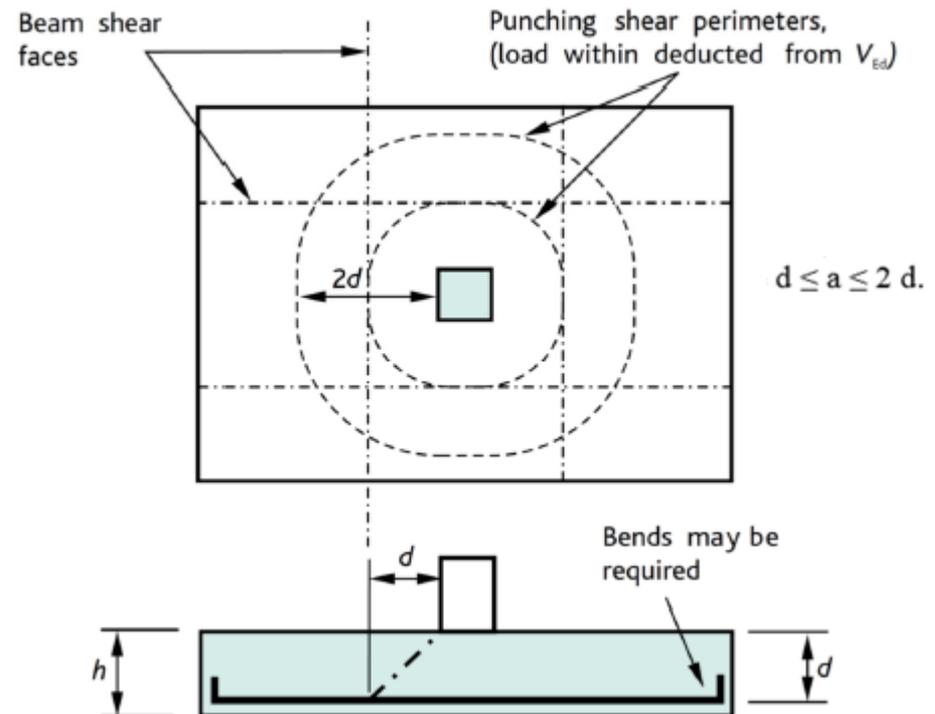
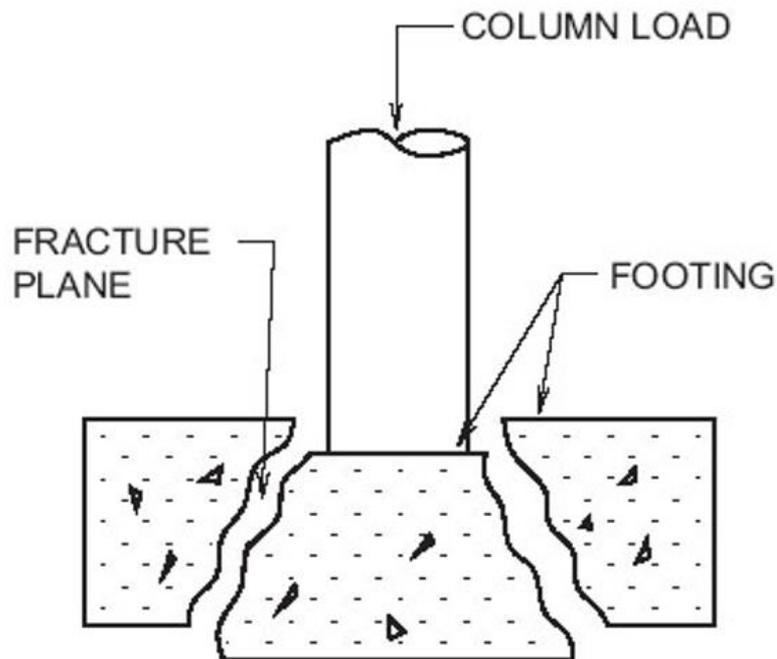
A_{s1} = reinforcement area along direction considered

b = length of side considered

Design of Isolated Footing

3. Punching Shear

- The punching shear resistance is checked at the face of the column and at a distance of **d** to **2d** from the face of the column.



Design of Isolated Footing

3. Punching Shear

- At the column perimeter or perimeter of the loaded area, u_o

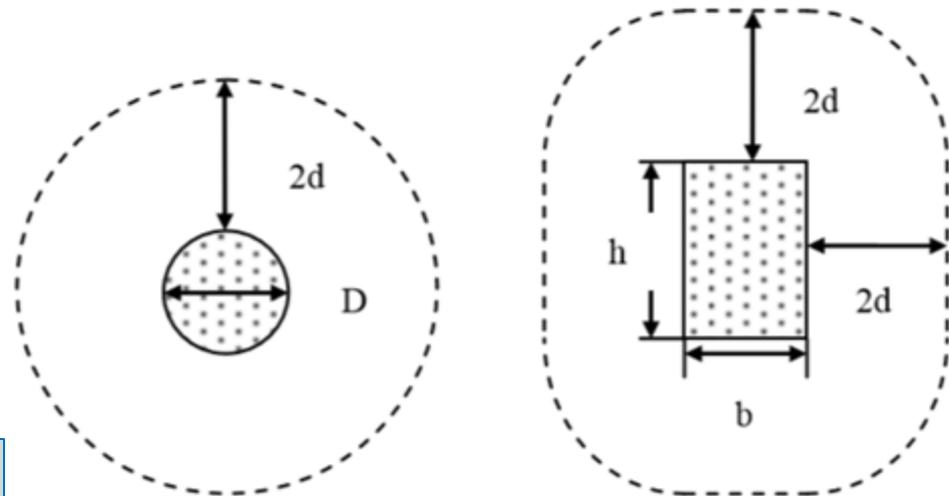
$$\frac{V_{Ed}}{u_o d} < [v_{Rd,max} = 0.5 v f_{cd}] \text{ where } v = 0.6 (1 - f_{ck}/250), f_{cd} = f_{ck} / (\gamma_c = 1.5)$$

$$V_{Ed} = V_{Ed,red} / ud$$

$$V_{Ed,red} = V_{Ed} - \Delta V_{Ed}$$

$$V_{Ed} \leq V_{Rd,c}$$

ΔV_{Ed} = Design value of applied shear force



(a) $\pi (D + 4d)$

(b) $2(b+h) + 4\pi d$

Design of Isolated Footing

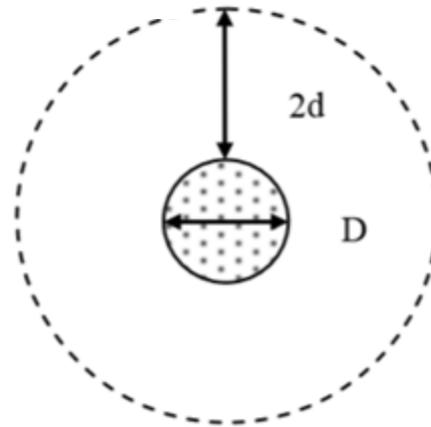
3. Punching Shear

- At the basic control perimeter, u_1

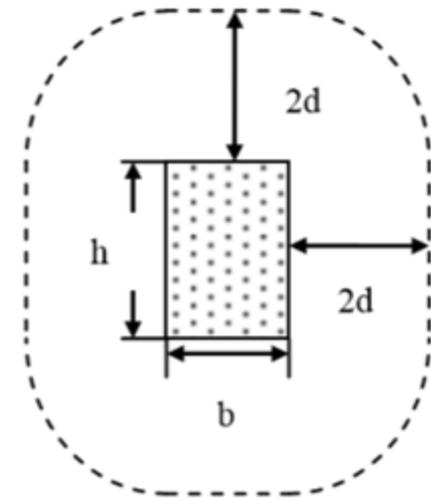
$$V_{Ed} = V_{Ed, red} / ud$$

$$V_{Rd} = C_{Rd,c} k (100 \rho_1 f_{ck})^{1/3} \times 2d/a \geq v_{min} \times 2d/a$$

$$V_{Ed} \leq V_{Rd,c}$$



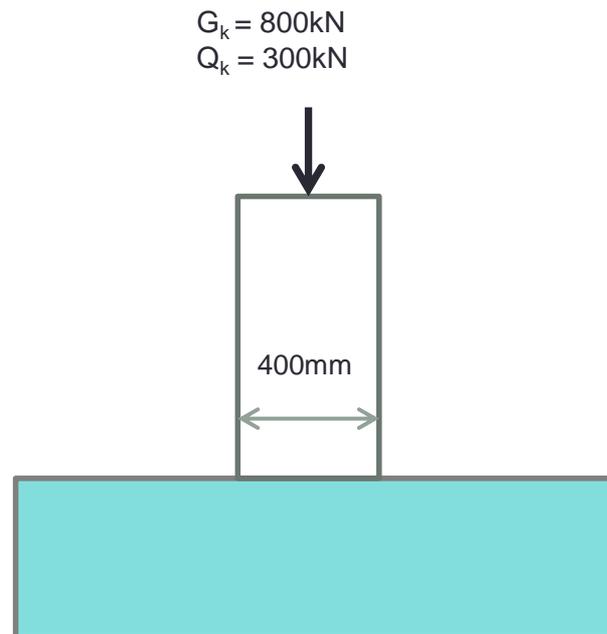
(a) $\pi (D + 4d)$



(b) $2(b+h) + 4\pi d$

Worked Examples

1. A column 400 mm x 400 mm carries a dead load of 800 kN and an imposed load of 300 kN. The safe bearing pressure is 200kN/m². Design a square base to resist the loads. $f_{ck} = 30$ Mpa and $f_{yk} = 500$ MPa.



Worked Examples

a. Size of base

Assume the weight is 100 kN.

$$\text{Service load} = 800 + 300 + 100 = 1200 \text{ kN}$$

$$\text{Area of base} = 1200/200 = 6.0 \text{ m}^2. \text{ Make the base } 2.5 \text{ m} \times 2.5 \text{ m.}$$

$$\text{Ultimate load} = (1.35 \times 800) + (1.5 \times 300) = 1530 \text{ kN}$$

$$\text{Ultimate base pressure} = 1530/6.25 = 245 \text{ kN/m}^2$$

b. Moment Steel

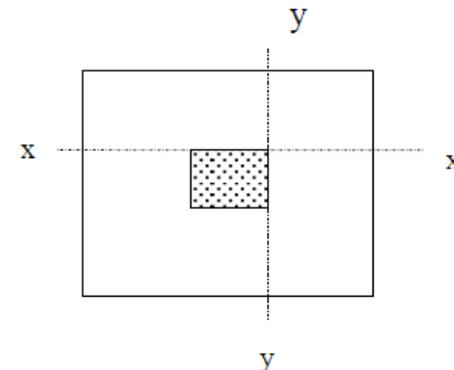
Critical section at the face of column.

$$M_{yy} = 245 \times 1.05 \times 2.5 \times 1.05/2 = 337.6 \text{ kNm}$$

Try an overall depth of 650 mm with 16 mm bars both ways.

$$\text{The weight of the footing} = 2.5 \times 2.5 \times 0.65 \times 25 = 102 \text{ kN}$$

$$102 \text{ kN} \approx 100 \text{ kN assumed in design.}$$



The effective depth of the top layer of steel is

$$d = 650 - 40 - 16 - 16/2 = 586 \text{ mm}$$

Worked Examples

$$k = M / (bd^2 f_{ck}) = 337.6 \times 10^6 / (2500 \times 586^2 \times 30) = 0.013 < 0.196$$

$$\frac{x}{d} = 0.5 \left[1.0 + \sqrt{\left(1 - 3 \frac{k}{\eta} \right)} \right]$$

$$z/d = 0.99$$

$$f_{yk} = 500, f_{yd} = 500/1.15 = 435 \text{ MPa}$$

$$A_s = 337.6 \times 10^6 / (435 \times 0.99 \times 586) = 1338 \text{ mm}^2$$

Check minimum steel:

$$A_{s, \min} = 0.26 \times (f_{ctm}/f_{yk}) \times bd \geq 0.0013 bd$$

$$f_{ctm} = 0.3 \times f_{ck}^{0.67} = 0.3 \times 30^{0.67} = 2.9 \text{ MPa}, f_{yk} = 500 \text{ MPa},$$

$$b = 2500 \text{ mm}, d = 586 \text{ mm}$$

$$A_{s, \min} = 0.26 \times (2.9/500) \times 2500 \times 586 \geq 0.0013 \times 2500 \times 586$$

$$A_{s, \min} = 2209 \text{ mm}^2$$

Area of H16 bar = 201 mm². Number of 16 mm bars = 2209/201 ≈ say, 11
Provide 11H16 bars, A_s = 2212 mm².

Worked Examples

The distribution of the reinforcement is determined to satisfy the rule.

$$3/4(c + 3d) = 0.75 (400 + 3 \times 586) = 1619 \text{ mm}$$

$$0.5 L = 2500/2 = 1250 \text{ mm} < 1619 \text{ mm}$$

The bars can be spaced equally at 240 mm centres.

The full anchorage length required past the face of the column. From Table 5.5, Chapter 5, for $f_{ck} = 30 \text{ MPa}$, the anchorage length required is 36 bar diameters. Anchorage length = $36 \times 16 = 576 \text{ mm}$. Adequate anchorage is available.

Worked Examples

c. Vertical shear

$$V_{Ed} = 245 \times 2.5 \times (1050 - 586) \times 10^{-3} = 284.2 \text{ kN}$$

$$v_{Ed} = 284.2 \times 10^3 / (2500 \times 586) = 0.19 \text{ MPa}$$

$$C_{Rd,c} = 0.18 / (\gamma_c = 1.5) = 0.12, k = 1 + \sqrt{(200/586)} = 1.58 \leq 2.0,$$

The bars extend 565 mm, i.e., more than d , beyond the critical section and so all the steel can be taken into account when calculating A_{sl} .

$$A_{sl} = 11H16 = 2212 \text{ mm}^2, \rho_1 = A_{sl} / (b_w d) = 2212 / (2500 \times 586) = 0.0015 \leq 0.02$$

$$C_{Rd,c} \times k \times (100 \times \rho_1 \times f_{ck})^{0.33} = 0.12 \times 1.58 \times (100 \times 0.0015 \times 30)^{0.33} = 0.31$$

$$v_{min} = 0.035 \times k^{1.5} \times \sqrt{f_{ck}} = 0.035 \times 1.58^{1.5} \times \sqrt{30} = 0.38 > 0.31$$

$$v_{Rd,c} = 0.38 \text{ MPa}$$

$$(v_{Ed} = 0.19) < (v_{Rd,c} = 0.38)$$

The shear stress is satisfactory and no shear reinforcement is required.

Worked Examples

d. Punching shear

Check shear stress at column perimeter:

Column load = 1530 kN, $u_0 = 2 \times (400 + 400) = 1600$ mm, $d = 586$ mm

Upward load = Base pressure \times column area = $245 \times 0.4 \times 0.4 = 39.2$ kN

$$v_{Ed} = (1530 - 39.2) \times 10^3 / (1600 \times 586) = 1.59 \text{ MPa}$$

$$v_{Rd, \max} = 0.3 \times (1 - f_{ck}/250) \times f_{cd} = 0.3 \times (1 - 30/250) \times 30/1.5 = 5.28 \text{ MPa}$$

$$(v_{Ed} = 1.59) < (v_{Rd, \max} = 5.28)$$

Slab depth is adequate.

Check punching shear on a perimeter at d to $2d$ from the column face.

The critical perimeter is shown in Fig. 11.2(d).

Let a = distance of the perimeter from the column face. $d \leq a \leq 2d$.

$$u = 2 \times [(c_1 = 400) + (c_2 = 400)] + 2 \times \pi \times a.$$

Let A = Area inside the perimeter.

$$A = \pi a^2 + 2 \times [(c_1 = 400) + (c_2 = 400)] \times a + [(c_1 = 400) \times (c_2 = 400)] \text{ mm}^2.$$

p = base pressure at ULS = 245 kN/m^2 or kPa.

Worked Examples

e. Cracking

The required and provided areas of reinforcement are respectively 1338 mm² and 2209 mm². The loads at SLS and ULS are 1100 kN and 1300 kN respectively. The stress in steel at serviceability limit state is

$$f_s = \left[\frac{1338}{2209} \right] \times \left[\frac{1100}{1530} \right] \times 435 = 189 \text{ MPa}$$

From Table 7.2N of the code, the maximum bar diameter for 0.3 mm wide crack is 25 mm. From Table 7.3N of the code the maximum spacing of bars is 250 mm. Both the criteria are satisfied. No further checks are required.

Worked Examples

Column load at ULS = 1530 kN.

$$V_{Ed, red} = \text{Column load} - A \times p = 1530 - 245 \times A \times 10^{-6} \text{ kN.}$$

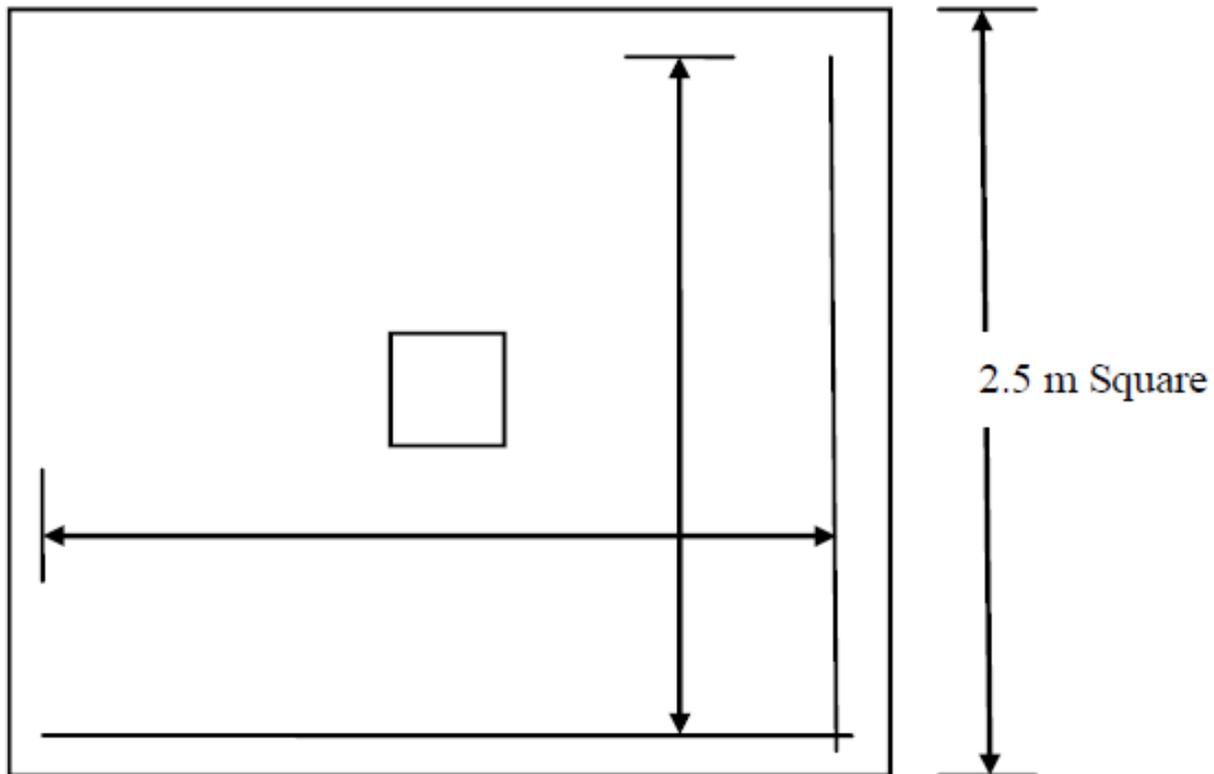
$$v_{Ed} = V_{Ed, red} / (u \times d).$$

Table 11.4 shows the calculation of punching shear stress v_{Ed} . The maximum value at $a = d$ is 0.32 MPa which is less than (see code equation (6.50)

$v_{Rd} = (v_{rd, c} \times 2d/a) = 0.76 \text{ MPa}$. The slab does not require shear reinforcement.

a	A, m ²	u, mm	$V_{Ed, red}$, kN	v_{Ed} , MPa
586	2.18	5281.96	996.78	0.32
644.6	2.50	5650.15	918.30	0.28
703.2	2.84	6018.35	834.54	0.24
761.8	3.20	6386.54	745.49	0.20
820.4	3.59	6754.74	651.16	0.16
879	3.99	7122.93	551.54	0.13
937.6	4.42	7491.13	446.63	0.10
996.2	4.87	7859.32	336.44	0.07
1054.8	5.34	8227.52	220.96	0.05
1113.4	5.84	8595.71	100.19	0.02
1172	6.35	8963.91	-25.86	0.00

Worked Examples



11H16 at 240 mm both ways