10.10 Influence Lines for Statically Indeterminate Beams

In Sec. 6–3 we discussed the use of the Müller-Breslau principle for drawing the influence line for the reaction, shear, and moment at a point in a statically determinate beam. In this section we will extend this method and apply it to statically indeterminate beams.

Recall that, for a beam, *the Müller-Breslau principle states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function.* To draw the deflected shape properly, the capacity of the beam to resist the applied function must be *removed* so the beam can deflect when the function is applied. For *statically determinate beams,* the deflected shapes (or the influence lines) will be a series of *straight line segments.* For *statically indeterminate beams, curves* will result. Construction of each of the three types of influence lines (reaction, shear, and moment) will now be discussed for a statically indeterminate beam. In each case we will illustrate the validity of the Müller-Breslau principle using Maxwell's theorem of reciprocal displacements.

Reaction at A.

To determine the influence line for the reaction at A in Fig. 10–20*a*, a unit load is placed on the beam at successive points, and at each point the reaction at A must be determined. A plot of these results yields the influence line. For example, when the load is at point D, Fig. 10–20*a*, the reaction at A, which represents the ordinate of the influence line at D, can be determined by the force method. To do this, the principle of superposition is applied, as shown in Figs. 10–20*a* through 10–20*c*. The compatibility equation for point A is thus $0 = f_{AD} + A_y f_{AA}$ or $A_y = -f_{AD}/f_{AA}$; however, by Maxwell's theorem of reciprocal displacements $f_{AD} = -f_{DA}$, Fig. 10–20*d*, so that we can also compute A_y (or the ordinate of the influence line at D) using the equation

$$A_y = \left(\frac{1}{f_{AA}}\right) f_{DA}$$

By comparison, the Müller-Breslau principle requires removal of the support at A and application of a vertical unit load. The resulting deflection curve, Fig. 10–20*d*, is to some scale the shape of the influence line for A_y. From the equation above, however, it is seen that the scale factor is $1/f_{AA}$.

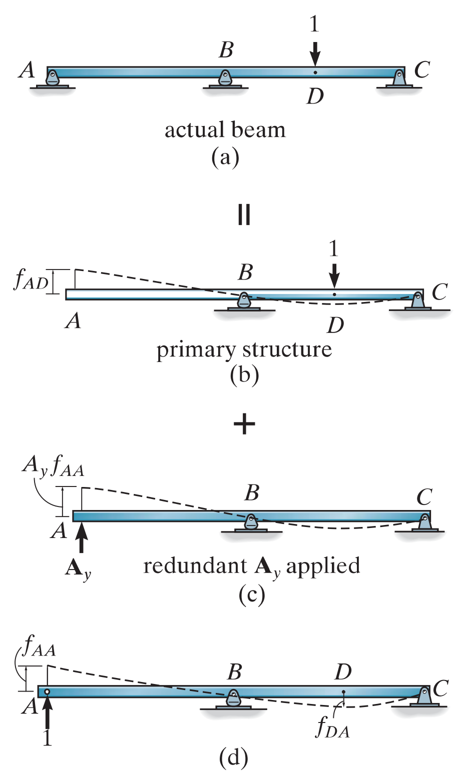


Fig. 10–20

Shear at *E*. If the influence line for the shear at point E of the beam in Fig. 10–21*a* is to be determined, then by the Müller-Breslau principle the beam is imagined cut open at this point and a *sliding device* is inserted at E, Fig. 10–21*b*. This device will transmit a moment and normal force but no shear. When the beam deflects due to positive unit shear loads acting at E, the slope on each side of the guide remains the same, and the deflection curve represents to some scale the influence line for the shear at E, Fig. 10–21*c*. Had the basic method for establishing the influence line for the shear at E been applied, it would then be necessary to apply a unit load at each point D and compute the internal shear at E, Fig. 10–21*a*. This value, V_E, would represent the ordinate of the influence line at D. Using the force method and Maxwell's theorem of reciprocal displacements, as in the previous case, it can be shown that

$$V_E = \left(\frac{1}{f_{EE}}\right) f_{DE}$$

This again establishes the validity of the Müller-Breslau principle, namely, a positive unit shear load applied to the beam at E, Fig. 10–21*c*, will cause the beam to deflect into the *shape* of the influence line for the shear at E. Here the scale factor is $(1/f_{EE})$.

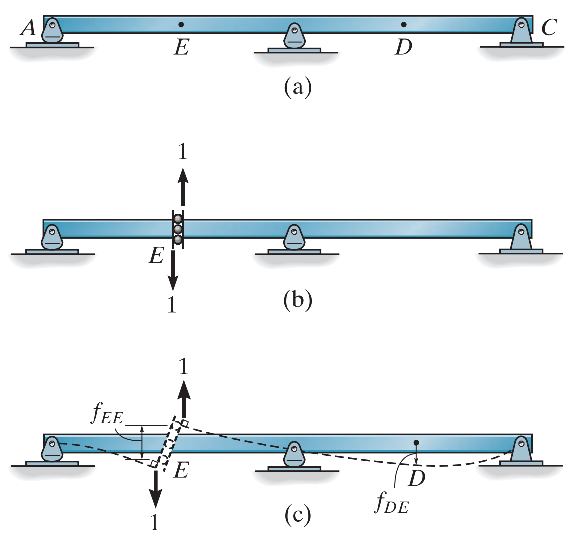


Fig. 10–21

Moment at *E*. The influence line for the moment at E in Fig. 10–22*a* can be determined by placing a *pin* or *hinge* at E, since this connection transmits normal and shear forces but cannot resist a moment, Fig. 10–22*b*. Applying a positive unit couple moment, the beam then deflects to the dashed position in Fig. 10–22*c*, which yields to some scale the influence line, again a consequence of the Müller-Breslau principle. Using the force method and Maxwell's reciprocal theorem, we can show that

$$M_E = \left(\frac{1}{\alpha_{EE}}\right) f_{DE}$$

The scale factor here is $(1/\alpha_{EE})$.

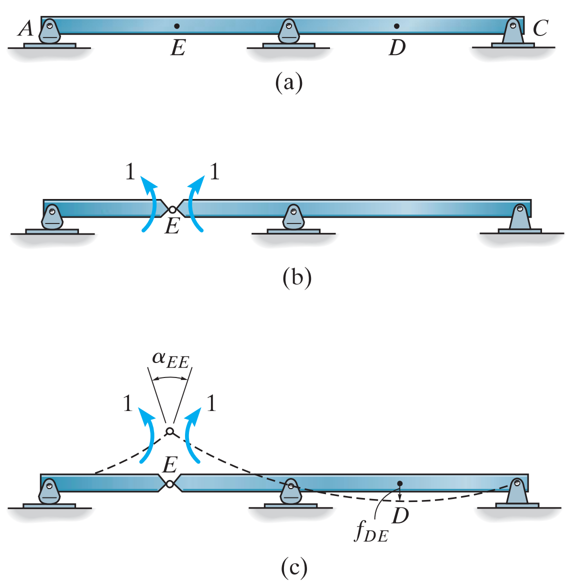


Fig. 10–22

Procedure for Analysis

The following procedure provides a method for establishing the influence line for the reaction, shear, or moment at a point on a beam using the Müller-Breslau technique.

Qualitative Influence Line

At the point on the beam for which the influence line is to be determined, place a connection that will remove the capacity of the beam to support the function of the influence line. If the function is a vertical *reaction,* use a vertical *roller guide;* if the function is *shear,* use a *sliding device;* or if the function is *moment,* use a *pin* or *hinge.* Place a unit load at the connection acting on the beam in the "positive direction" of the function. Draw the deflection curve for the beam. This curve represents to some scale the shape of the influence line for the beam.

Quantitative Influence Line

If numerical values of the influence line are to be determined, compute the *displacement* of successive points along the beam when the beam is subjected to the unit load placed at the connection mentioned above. Divide each value of displacement by the displacement determined at the point where the unit load acts. By applying this scalar factor, the resulting values are the ordinates of the influence line.

Influence lines for the continuous girder of this trestle were constructed in order to properly design the girder.

10.11 Qualitative Influence Lines for Frames

The Müller-Breslau principle provides a quick method and is of great value for establishing the general shape of the influence line for building frames. Once the influence-line *shape* is known, one can immediately specify the *location* of the live loads so as to create the greatest influence of the function (reaction, shear, or moment) in the frame. For example, the shape of the influence line for the *positive* moment at the center *I* of girder *FG* of the frame in Fig. 10–23*a* is shown by the dashed lines. Thus, uniform loads would be placed only on girders *AB*, *CD*, and *FG* in order to create the largest positive moment at *I*. With the frame loaded in this manner, Fig. 10–23*b*, an indeterminate analysis of the frame could then be performed to determine the critical moment at *I*.

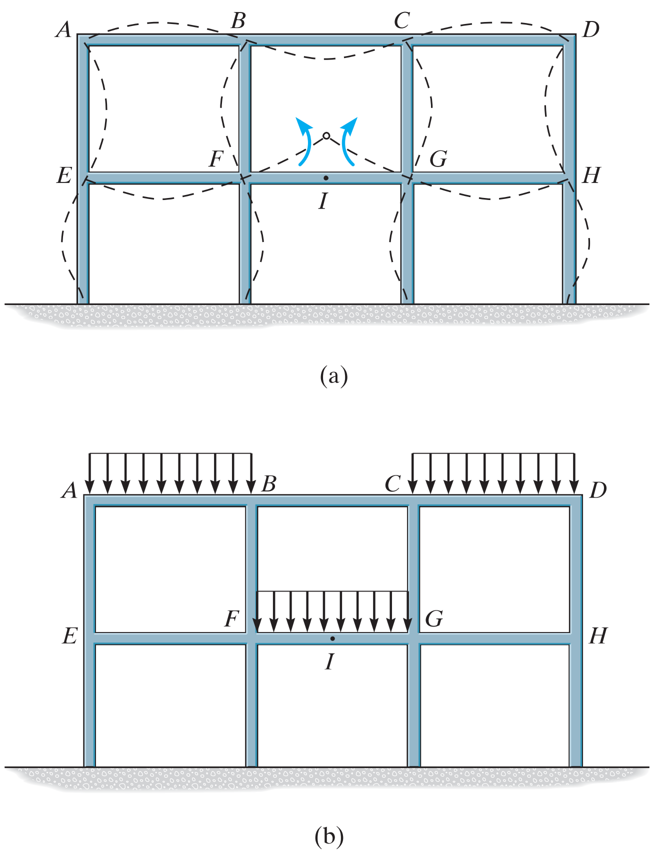
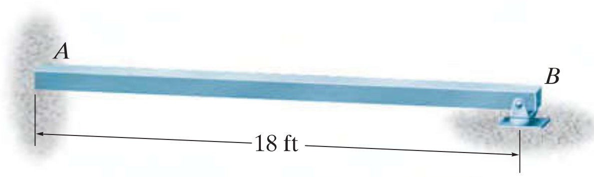


Fig. 10–23

EXAMPLE 10.10

Draw the influence line for the vertical reaction at *A* for the beam in Fig. 10–24*a*. *EI* is constant. Plot numerical values every 6 ft.



(a)

SOLUTION

The capacity of the beam to resist the reaction $\mathbf{A}_y$ is removed. This is done using a vertical roller device shown in Fig. 10–24*b*. Applying a vertical unit load at *A* yields the shape of the influence line shown in Fig. 10–24*c*.

In order to determine ordinates of the influence line we will use the conjugate-beam method. The reactions at *A* and *B* on the "real beam," when subjected to the unit load at *A*, are shown in Fig. 10–24*b*. The corresponding conjugate beam is shown in Fig. 10–24*d*. Notice that the support at A' remains the *same* as that for *A* in Fig. 10–24*b*. This is because a vertical roller device on the conjugate beam supports a moment but no shear, corresponding to a displacement but no slope at *A* on the real beam, Fig. 10–24*c*. The reactions at the supports of the conjugate beam have been computed and are shown in Fig. 10–24*d*. The displacements of points on the real beam, Fig. 10–24*b*, will now be computed.

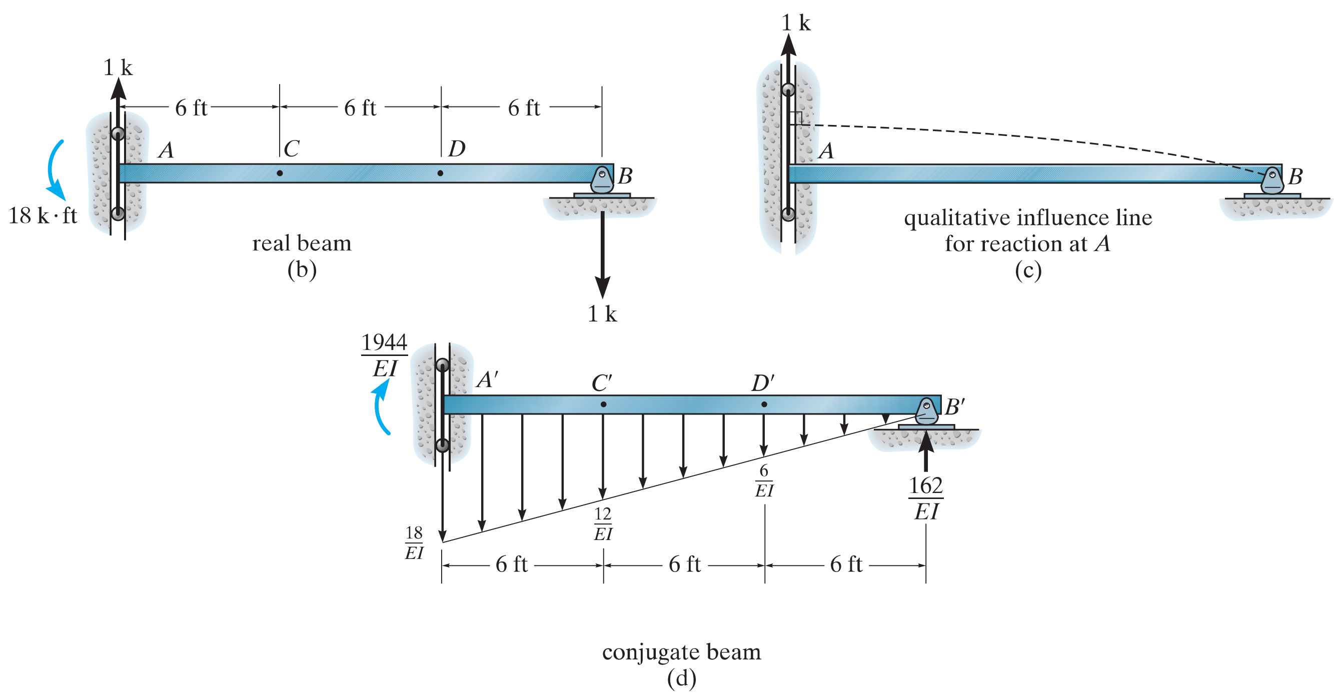
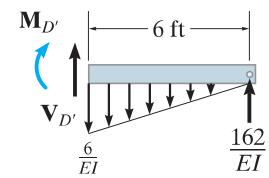


Fig. 10–24

For B', since no moment exists on the conjugate beam at B', Fig. 10–24*d*, then

$$\Delta_B = M_{B'} = 0$$



(e)

For D', Fig. 10–24*e*:

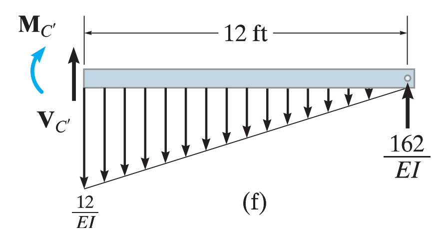
$$\Sigma M_{D'} = 0; \qquad \Delta_D = M_{D'} = \frac{162}{EI}(6) - \frac{1}{2}\left(\frac{6}{EI}\right)(6)(2) = \frac{936}{EI}$$

For C', Fig. 10–24*f*:

$$\Sigma M_{C'} = 0; \qquad \Delta_C = M_{C'} = \frac{162}{EI}(12) - \frac{1}{2}\left(\frac{12}{EI}\right)(12)(4) = \frac{1656}{EI}$$

For A', Fig. 10–24*d*:

$$\Delta_A = M_{A'} = \frac{1944}{EI}$$

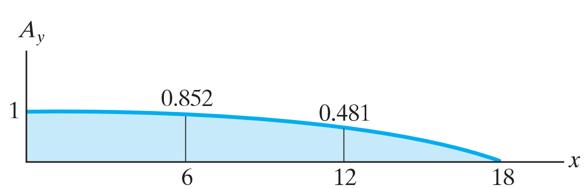


(f)

Since a vertical 1-k load acting at A on the beam in Fig. 10–24*a* will cause a vertical reaction at A of 1 k, the displacement at A, $\Delta_A = 1944/EI$, should correspond to a numerical value of 1 for the influence-line ordinate at A. Thus, dividing the other computed displacements by this factor, we obtain

x	A_y
A	1
C	0.852
D	0.481
B	0

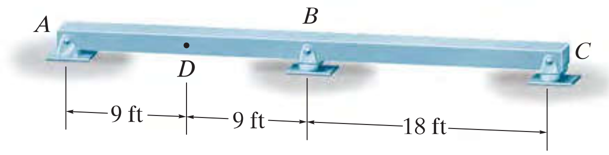
A plot of these values yields the influence line shown in Fig. 10–24*g*.



quantitative influence line
for reaction at A
(g)

EXAMPLE 10.11

Draw the influence line for the shear at D for the beam in Fig. 10–25*a*. EI is constant. Plot numerical values every 9 ft.



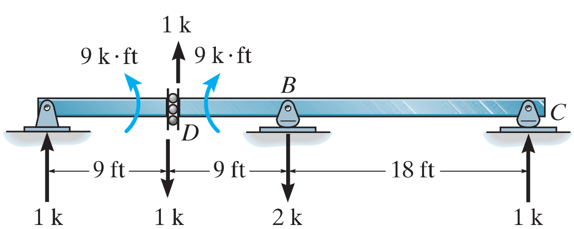
(a)

Fig. 10–25

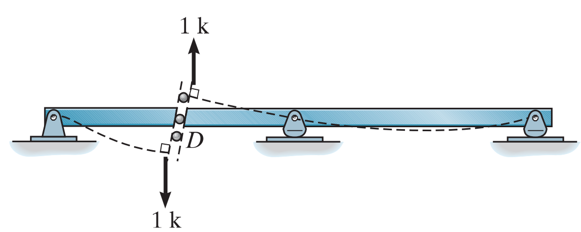
SOLUTION

The capacity of the beam to resist shear at D is removed. This is done using the roller device shown in Fig. 10–25*b*. Applying a positive unit shear at D yields the shape of the influence line shown in Fig. 10–25*c*.

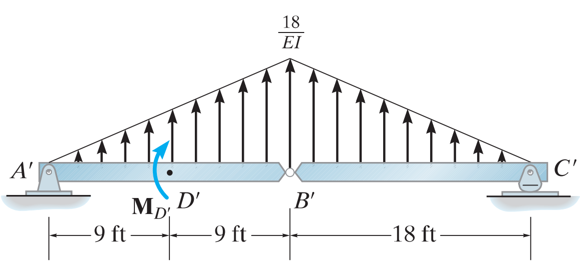
The support reactions at A, B, and C on the "real beam" when subjected to the unit shear at D are shown in Fig. 10–25*b*. The corresponding conjugate beam is shown in Fig. 10–25*d*. Here an external couple moment $\mathbf{M}_{D'}$ must be applied at D' in order to cause a different *internal moment* just to the left and just to the right of D'. These internal moments correspond to the displacements just to the left and just to the right of D on the real beam, Fig. 10–25*c*. The reactions at the supports A', B', C' and the external moment $\mathbf{M}_{D'}$ on the conjugate beam have been computed and are shown in Fig. 10–25*e*. As an exercise verify the calculations.



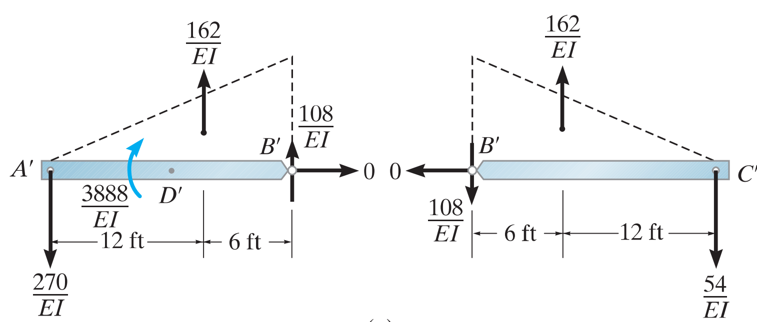
real beam
(b)



qualitative influence line
for shear at D
(c)



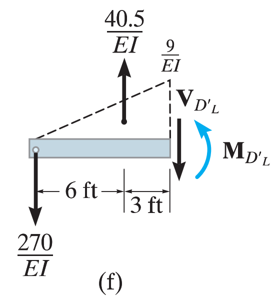
conjugate beam
(d)



(e)

Since there is a *discontinuity* of moment at D', the internal moment just to the left and right of D' will be computed. Just to the left of D', Fig. 10–25*f*, we have

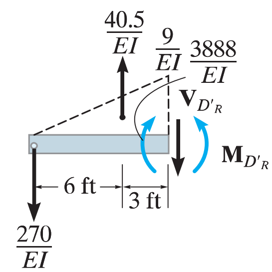
$$\Sigma M_{D'_L} = 0; \qquad \Delta_{D_L} = M_{D'_L} = \frac{40.5}{EI}(3) - \frac{270}{EI}(9) = -\frac{2308.5}{EI}$$



(f)

Just to the right of D', Fig. 10–25*g*, we have

$$\Sigma M_{D'_R} = 0; \qquad \Delta_{D_R} = M_{D'_R} = \frac{40.5}{EI}(3) - \frac{270}{EI}(9) + \frac{3888}{EI} = \frac{1579.5}{EI}$$



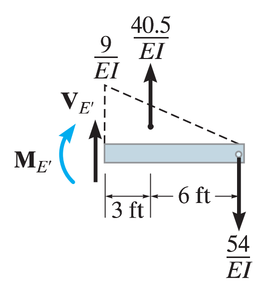
(g)

From Fig. 10–25*e*,

$$\Delta_A = M_{A'} = 0 \qquad \Delta_B = M_{B'} = 0 \qquad \Delta_C = M_{C'} = 0$$

For point E, Fig. 10–25*b*, using the method of sections at the corresponding point E' on the conjugate beam, Fig. 10–25*h*, we have

$$\Sigma M_{E'} = 0; \qquad \Delta_E = M_{E'} = \frac{40.5}{EI}(3) - \frac{54}{EI}(9) = -\frac{364.5}{EI}$$

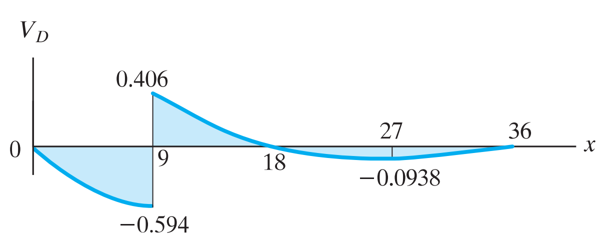


(h)

The ordinates of the influence line are obtained by dividing each of the above values by the scale factor $M_{D'} = 3888/EI$. We have

x	V_D
A	0
D_L	−0.594
D_R	0.406
B	0
E	−0.0938
C	0

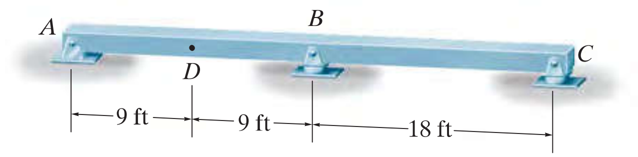
A plot of these values yields the influence line shown in Fig. 10–25*i*.



quantitative influence line
for shear at D
(i)

EXAMPLE 10.12

Draw the influence line for the moment at D for the beam in Fig. 10–26*a*. EI is constant. Plot numerical values every 9 ft.



(a)

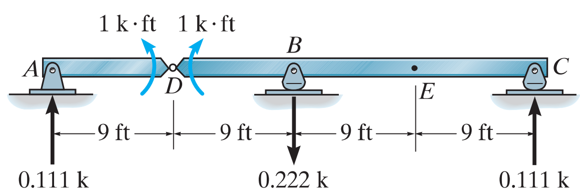
Fig. 10–26

SOLUTION

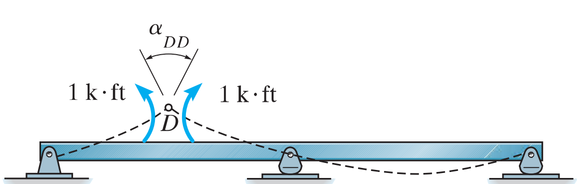
A hinge is inserted at D in order to remove the capacity of the beam to resist moment at this point, Fig. 10–26*b*. Applying positive unit couple moments at D yields the influence line shown in Fig. 10–26*c*.

The reactions at A, B, and C on the "real beam" when subjected to the unit couple moments at D are shown in Fig. 10–26*b*. The corresponding conjugate beam and its reactions are shown in Fig. 10–26*d*. It is suggested that the reactions be verified in both cases. From Fig. 10–26*d*, note that

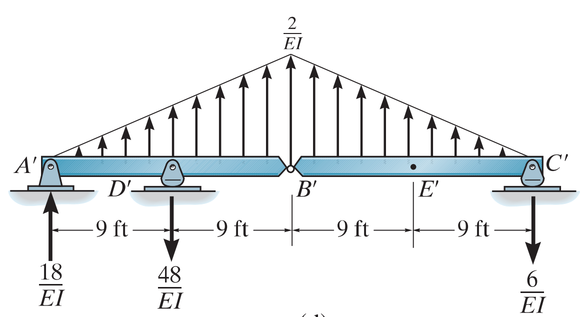
$$\Delta_A = M_{A'} = 0 \qquad \Delta_B = M_{B'} = 0 \qquad \Delta_C = M_{C'} = 0$$



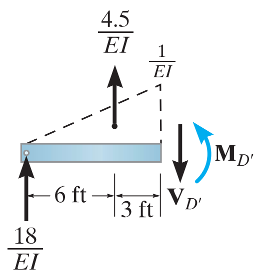
real beam
(b)



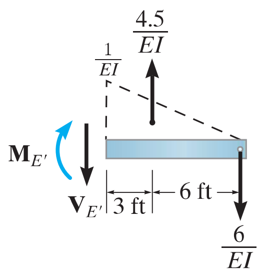
qualitative influence line for moment at D
(c)



(d)



(e)



(f)

For point D', Fig. 10–26e:

$$\Sigma M_{D'} = 0; \qquad \Delta_D = M_{D'} = \frac{4.5}{EI}(3) + \frac{18}{EI}(9) = \frac{175.5}{EI}$$

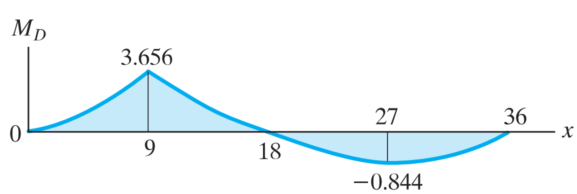
For point E', Fig. 10–26f:

$$\Sigma M_{E'} = 0; \qquad \Delta_E = M_{E'} = \frac{4.5}{EI}(3) - \frac{6}{EI}(9) = -\frac{40.5}{EI}$$

The angular displacement α_{DD} at D of the "real beam" in Fig. 10–26c is defined by the reaction at D' on the conjugate beam. This factor, $D'_y = 48/EI$, is divided into the above values to give the ordinates of the influence line, that is,

x	M_D
A	0
D	3.656
B	0
E	−0.844
C	0

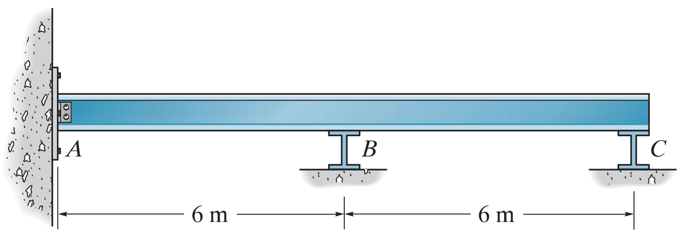
A plot of these values yields the influence line shown in Fig. 10–26g.



quantitative influence line
for moment at D
(g)

PROBLEMS

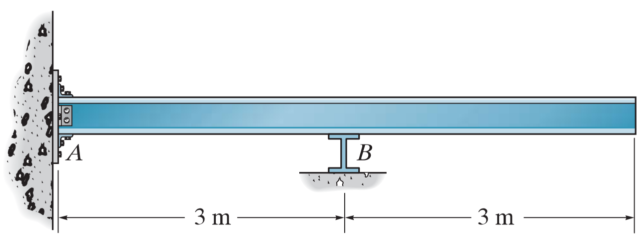
10–41. Draw the influence line for the reaction at *C*. Plot numerical values at the peaks. Assume *A* is a pin and *B* and *C* are rollers. *EI* is constant.



Prob. 10–41

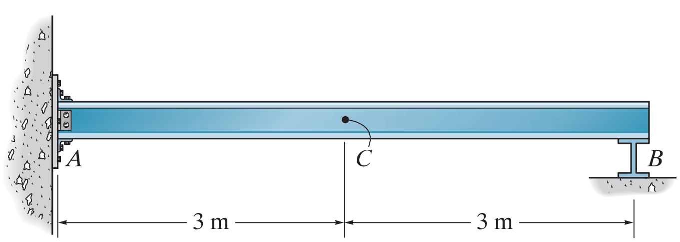
10–42. Draw the influence line for the moment at *A*. Plot numerical values at the peaks. Assume *A* is fixed and the support at *B* is a roller. *EI* is constant.

10–43. Draw the influence line for the vertical reaction at *B*. Plot numerical values at the peaks. Assume *A* is fixed and the support at *B* is a roller. *EI* is constant.



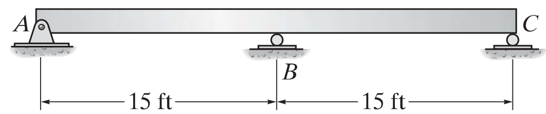
Probs. 10–42/10–43

***10–44.** Draw the influence line for the shear at *C*. Plot numerical values every 1.5 m. Assume *A* is fixed and the support at *B* is a roller. *EI* is constant.



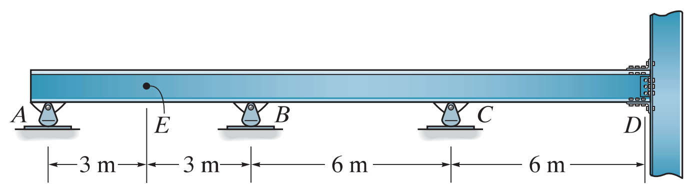
Prob. 10–44

10–45. Draw the influence line for the reaction at *C*. Plot the numerical values every 5 ft. *EI* is constant.



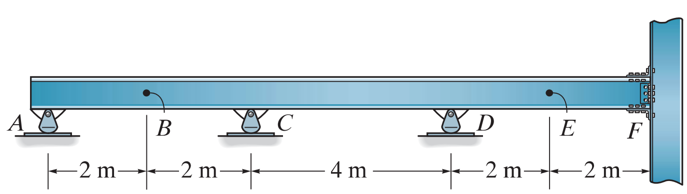
Prob. 10–45

10–46. Sketch the influence line for (a) the moment at *E*, (b) the reaction at *C*, and (c) the shear at *E*. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at *D*.



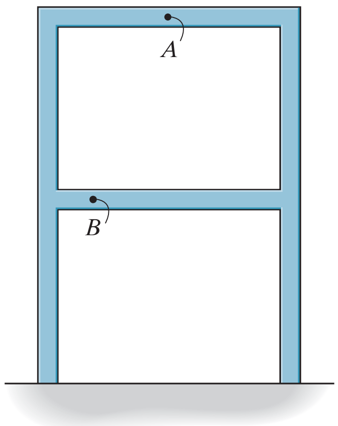
Prob. 10–46

10–47. Sketch the influence line for (a) the vertical reaction at *C*, (b) the moment at *B*, and (c) the shear at *E*. In each case, indicate on a sketch of the beam where a uniform distributed live load should be placed so as to cause a maximum positive value of these functions. Assume the beam is fixed at *F*.



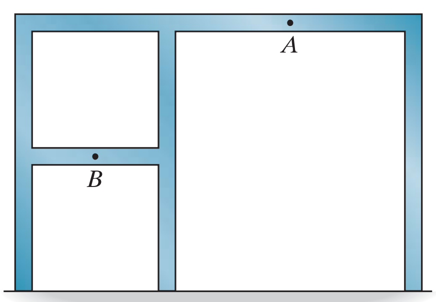
Prob. 10–47

***10–48.** Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



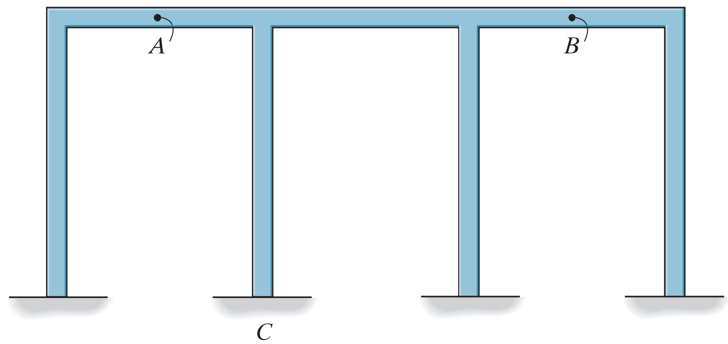
Prob. 10–48

10–49. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



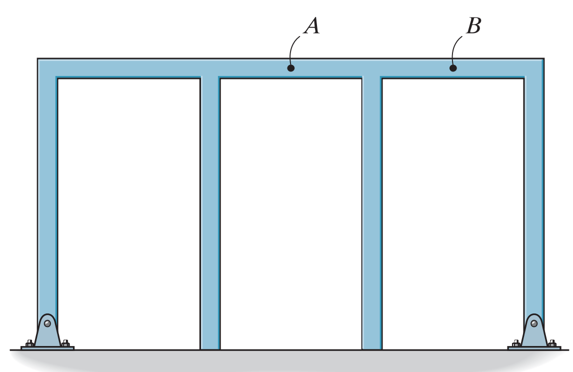
Prob. 10–49

10–50. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



Prob. 10–50

10–51. Use the Müller-Breslau principle to sketch the general shape of the influence line for (a) the moment at *A* and (b) the shear at *B*.



Prob. 10–51