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ENGINEERING HYDROLOGY

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ENGINEERING HYDROLOGY

Students as well as teachers have for long felt a strong need for an undergraduate text on hydrology that covers the syllabi requirements of Indian institutes and is also written in a simple manner. This book has been developed to meet this need and is an ideal text for an easy and thorough understanding of the subject. It is in SI units and contains a large number of worked examples which effectively illustrate the concepts. Each chapter has a set of problems and objective type questions which form a useful revision tool and a [illegible] comprehension. An attractive feature is the inclusion of Indian data and examples. The matter is presented systematically and mathematics is kept to the minimum necessary.

The book is a completely sufficient text for civil engineering students. It will also be useful to agricultural engineering students and students taking the examinations of the Institution of Engineers (India). Students appearing in competitive examinations will find it a particularly valuable source of information.

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PREFACE

Water is vital to life and development in all parts of the world. In Third World countries where the agricultural sector plays a key role in their economic growth, the management of water resources is an item of high priority in their developmental activities. The basic inputs in the evaluation of water resources are from hydrological parameters and the subject of hydrology forms the core in the evaluation and development of water resources. In the civil engineering curriculum, this subject occupies an important position.

During my long teaching experience, I have felt a strong need for a text-book oriented to the Indian environment and written in simple and lucid style. The present book is a response to the same. This book is intended to serve as a text for a first course in engineering hydrology at the under-graduate level in the civil engineering discipline. Students specializing in various aspects of water-resources engineering, such as water-power engineering and agricultural engineering will find this book useful. This book also serves as a source of useful information to professional engineers working in the area of water-resources evaluation and development.

Engineering hydrology encompasses a wide spectrum of topics and a book like the present one meant for the first course must necessarily main-tain a balance in the blend of topics. The subject matter has been developed in a logical and coherent manner and covers the prescribed syllabi of various Indian universities. The mathematical part is kept to the minimum and emphasis is placed on the applicability to field situations relevant to Indian conditions. SI units are used throughout the book.

Designed essentially for a one-semester course, the material in the book is presented in nine chapters. The hydrologic cycle and world-water balance are covered in Chap. 1. Aspects of precipitation, essentially rainfall, are dealt in sufficient detail in Chap. 2. Hydrologic abstractions including evapotranspiration and infiltration are presented in Chap. 3. Streamflow-measurement techniques and assessment of surface-flow yield of a catch-ment form the subject matter of Chaps. 4 and 5 respectively. The charac-teristics of flood hydrographs and the unit hydrograph theory together with an introduction to instantaneous unit hydrograph are covered in sufficient detail with numerous worked examples in Chap. 6. Floods, a topic of considerable importance, constitute the subject matter of Chaps. 7 and 8. While in Chap. 7 the flood-peak estimation and frequency studies are described in detail, Chap. 8 deals with the aspects of flood routing, flood control and forecasting. Basic information on the hydrological aspects of groundwater has been covered in Chap. 9.

Numerous worked examples, a set of problems and a set of objective-type multiple-choice questions are provided at the end of each chapter to enable the student to gain good comprehension of the subject. Questions and problems included in the book are largely original and are designed to enhance the capabilities of comprehension, analysis and application of the student.

I am grateful to: UNESCO for permission to reproduce several figures from their publication, *Natural Resources of Humid Tropical Asia—Natural Resources Research XII*, © UNESCO, 1974; the Director-General of Meteorology, India Meteorological Department, Government of India for permission to reproduce several maps; M/s Leupold and Stevens, Inc., Beaverton, Oregon, USA, for photographs of hydrometeorological instruments; M/s Alsthom-Atlantique, Neyrtec, Grenoble France, for photographs of several Neyrtec Instruments; M/s Lawrence and Mayo (India) Pvt. Ltd., New Delhi for the photograph of a current meter.

Thanks are due to Professor KVGK Gokhale for his valuable suggestions and to Sri Suresh Kumar for his help in the production of the manuscript. I wish to thank my student-friends who helped in this endeavour in many ways. The financial support received under the Quality Improvement Programme (QIP), Government of India, through the Indian Institute of Technology, Kanpur for the preparation of the manuscript is gratefully acknowledged.

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CONTENTS

gaseous, and in various degrees of motion. Evaporation of water from water bodies such as oceans and lakes, formation and movement of clouds, rain and snowfall, streamflow and groundwater movement are some examples of the dynamic aspects of water. The various aspects of water related to the earth can be explained in terms of a cycle known as the *hydrologic cycle.*

Figure 1.1 is a schematic representation of the hydrologic cycle. A convenient starting point to describe the cycle is in the oceans. Water in the oceans evaporate due to the heat energy provided by solar radiation. The water vapour moves upwards and form clouds. While much of the clouds condense and fall back to the oceans as rain, a part of the clouds is driven to the land areas by winds. There they condense and *precipitate* onto the land mass as rain, snow, hail, sleet, etc. A part of the precipitation may *evaporate* back to the atmosphere even while falling. Another part may be *intercepted* by vegetation, structures and other such surface modifications from which it may be either evaporated back to atmosphere or move down to the ground surface.

A portion of the water that reaches the ground enters the earth's surface through *infiltration*, enhance the moisture content of the soil and reach the

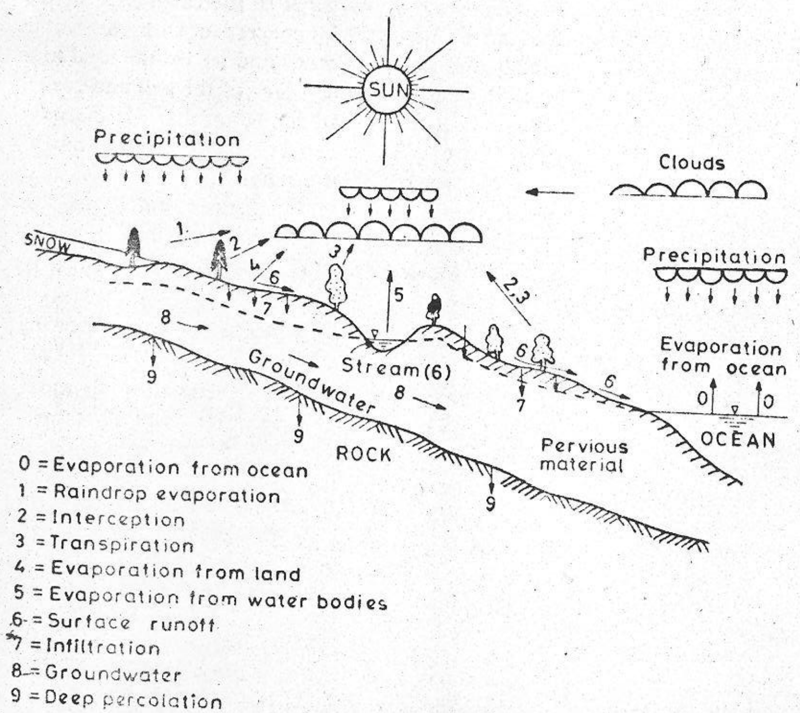

Fig. 1.1 The hydrologic cycle

groundwater body. Vegetation sends a portion of the water from under the ground surface back to the atmosphere through the process of *transpiration.* The precipitation reaching the ground surface after meeting the needs of infiltration and evaporation moves down the natural slope over the surface and through a network of gullies, streams and rivers to reach the ocean. The groundwater may come to the surface through springs and other outlets after spending a considerably longer time than the surface flow. The portion of the precipitation which by a variety of paths above and below the surface of the earth reaches the stream channel is called *runoff.* Once it enters a stream channel, runoff becomes *stream flow.*

The sequence of events as above is a simplistic picture of a very complex cycle that has been taking place since the formation of the earth. It is seen that the hydrologic cycle is a very vast and complicated cycle in which there are a large number of paths of varying time scales. Further, it is a continuous recirculating cycle in the sense that there is neither a beginning nor an end or a pause. Each path of the hydrologic cycle involves one or more of the following aspects: (i) transportation of water, (ii) temporary storage and (iii) change of state. For example, (a) the process of rainfall has the change of state and transportation and (b) the groundwater path has storage and transportation aspects. The quantities of water going through various individual paths of the hydrological cycle can be described by the continuity equation known as *water-budget equation* or *hydrologic equation.*

For a given problem area, say a catchment, in an interval of time Δt,

Mass inflow − mass outflow = change in mass storage

If the density of the inflow, outflow and storage volumes are same.

$$V_i - V_o = \Delta S \qquad (1.1)$$

where V_i = inflow volume of water into the problem area during the time period, V_o = outflow volume of water from the problem area during the time period, and ΔS = change in the storage of the water volume over and under the given area during the given period. In applying this continuity equation [Eq. (1.1)] to the paths of the hydrologic cycle involving change of state, the volumes considered are the equivalent volumes of water at a reference temperature. In hydrologic calculations, the volumes are often expressed as average depths over the catchment area. Thus, for example, if the annual stream flow from a 10 km² catchment is 10^7 m³, it corresponds to a depth of $\left(\frac{10^7}{10 \times 10^6}\right) = 1$ m = 100 cm. Rainfall, evaporation and often runoff volumes are expressed in units of depth over the catchment.

It is important to note that the total water resources of the earth is constant and the sun is the source of energy for the hydrologic cycle. A recognition of the various processes such as evaporation, precipitation and groundwater flow helps one to study the science of hydrology in a systematic way. Also, one realises that man can interfere with virtually any part of the hydrologic cycle, e.g. through artificial rain, evaporation suppression, change of vegetal cover and land use, extraction of ground-water, etc. Interference at one stage can cause serious repurcussions at some other stage of the cycle.

The hydrological cycle has important influences in a variety of fields including agriculture, forestry, geography, economics, sociology and political science. Engineering applications of the knowledge of the hydrologic cycle, and hence of the subjects of hydrology, are found in the design and operation of projects dealing with water supply, irrigation and drainage, water power, flood control, navigation, coastal works, salinity control and recreational uses of water.

1.3 WORLD WATER INVENTORY[2,3]

The total quantity of water in the world is roughly 1357.5 million cubic kilometres (M km^3). About 97% of this water is contained in the oceans as saline water. Thus only about 37.5 M km^3 is fresh water. Out of this about 8.5 M km^3 is both liquid and fresh and the remaining is contained in the frozen state as ice in the polar regions and on mountain tops and glaciers. A rough distribution of the fresh water on the earth is as follows:

1. 29.00 M km^3 (77%) in polar ice caps and in glaciers
2. 4.150 M km^3 (11%) as ground water in depths up to 800 m
3. 4.150 M km^3 (11%) as ground water in depths below 800 m
4. 0.120 M km^3 in lakes, rivers and streams
5. 0.067 M km^3 as soil moisture and seepage
6. 0.013 M km^3 in atmosphere as water vapour

Total 37.50 M km^3

The total amount of rain and snow falling on the earth each year is about 0.42 M km^3 (0.32 M km^3 on the ocean and 0.10 M km^3 on the land). Over the oceans 9% more water evaporates than falls back as rain, to be balanced by excess precipitation over the land mass. Thus the volume of water carried by rivers and springs to the sea each year is about 0.038 M km^3. It is interesting to know that less than 4% of this total river flow is used for irrigation and the rest flows to the sea unutilized by man.

Water-balance studies of the subareas in the world indicate interesting facts. Table 1.1 gives the water balance of oceans. It is seen that there is considerable water transfer between the oceans and the evaporation and precipitation values vary from one ocean to another.

The water balance of the continental land mass is shown in Table 1.2. t is interesting to see from this table that Africa, in spite of its equatorial

TABLE 1.1 WATER BALANCE OF OCEANS[2] mm/yr

Ocean	Area (M km^2)	Precipitation	Inflow from adjacent continents	Evaporation	Water Exchange with other oceans
Atlantic	107	780	200	1040	—60
Arctic	12	240	230	120	350
Indian	75	1010	70	1380	—300
Pacific	167	1210	60	1140	130

TABLE 1.2 WATER BALANCE OF CONTINENTS[2] mm/yr

Continent	Area (M km^2)	Precipitation	Total run-off	Runoff as % of precipitation	Evaporation
Africa	30.3	686	139	20	547
Asia	45.0	726	293	40	433
Australia	8.7	736	226	30	510
Europe	9.8	734	319	43	415
N. America	20.7	670	287	43	383
S. America	17.8	1648	583	35	1065

forest zones, is the driest continent in the world with only 20% of precipitation going as runoff. On the other hand, North America and Europe emerge as the continents with the highest runoff rates. Extending the same analysis to a smaller land mass, viz. to the Indian subcontinent, the long-term average runoff for India is found to be 46% (Table 1.3).

Each year the rivers of the world discharge about 38,000 km^3 of water into the sea. This amounts to an annual average flow of 1.2 M m^3/s. The world's largest river, the Amazon, has an average discharge of 200,000 m^3/s; i.e. one-sixth of the world's total discharge rate. India's largest river,

TABLE 1.3 WATER BALANCE OF INDIA, mm/yr

Area (M km^2)	Precipitation	Evaporation	Runoff	Runoff as % of precipitation
3.05	1190	642	548	46

the Brahmaputra, and the second largest the Ganga flow into the Bay of Bengal with mean annual discharges of 16200 m³/s and 15600 m³/s respectively.

1.4 HISTORY OF HYDROLOGY

Water is the prime requirement for the existence of life and thus it has been man's endeavour from time immemorial to utilise the available water resources. History has instances of civilizations that flourished with the availability of dependable water supplies and then collapsed when the water supply failed. Numerous references exist in Vedic literature to groundwater availability and its utility. During 3000 BC groundwater development through wells was known to the people of the Indus Valley civilizations as revealed by archaeological excavations at Mohenjodaro. Quotations in ancient Hindu scriptures indicate the existence of the knowledge of the hydrologic cycle even as far back as the Vedic period. The first description of the raingauge and its use is contained in the *Arthashastra* by Chanakya (300 BC). Varahamihira's (AD 505-587) *Brihatsamhita* contains descriptions of the raingauge, wind vane and prediction procedures for rainfall. Egyptians knew the importance of the stage measurement of rivers and records of the stages of the Nile dating back to 1800 BC have been located. The knowledge of the hydrologic cycle came to be known to Europe much later, around AD 1500.

Chow[1] classifies the history of hydrology into eight periods as:

1. Period of speculation—prior to AD 1400
2. Period of observation—1400-1600
3. Period of measurement—1600-1700
4. Period of experimentation—1700-1800
5. Period of modernization—1800-1900
6. Period of empiricism—1900-1930
7. Period of rationalization—1930-1950
8. Period of theorization—1950-to date

Most of the present-day science of hydrology has been developed since 1930, thus giving hydrology the status of a young science. The worldwide activities in water-resources development since the last few decades by both developed and developing countries aided by rapid advances in instrumentation for data acquisition and in the computer facilities for data analysis have contributed towards the rapid growth rate of this young science.

1.5 APPLICATIONS IN ENGINEERING

Hydrology finds its greatest application in the design and operation of water-resources engineering projects, such as those for (i) irrigation, (ii) water supply, (iii) flood control, (iv) water power and (v) navigation. In all these projects hydrological investigations for the proper assessment of the following factors are necessary:

1. The capacity of storage structures such as reservoirs.
2. The magnitude of flood flows to enable safe disposal of the excess flow.
3. The minimum flow and quantity of flow available at various seasons.
4. The interaction of the flood wave and hydraulic structures, such as levees, reservoirs, barrages and bridges.

The hydrological study of a project should of necessity precede structural and other detailed design studies. It involves the collection of relevant data and analysis of the data by applying the principles and theories of hydrology to seek solutions to practical problems.

Many important projects in the past have failed due to improper assessment of the hydrological factors. Some typical failures of hydraulic structures are: (i) overtopping and consequent failure of an earthen dam due to an inadequate spillway capacity, (ii) failure of bridges and culverts due to excess flood flow and (iii) inability of a large reservoir to fill up with water due to overestimation of the stream flow. Such failure, often called *hydrologic failures* underscore the uncertainty aspect inherent in hydrological studies.

Various phases of the hydrological cycle, such as rainfall, runoff, evaporation and transpiration are all nonuniformly distributed both in time and space. Further, practically all hydrologic phenomena are complex and at the present level of knowledge, they can at best be interpreted with the aid of probability concepts. Hydrological events are treated as random processes and the historical data relating to the event are analysed by statistical methods to obtain information on probabilities of occurrence of various events. The probability analysis of hydrologic data is an important component of present-day hydrological studies and enables the engineer to take suitable design decisions consistent with economic and other criteria to be taken in a given project.

1.6 SOURCES OF DATA

The main components of the hydrological cycle are rainfall (precipitation), evaporation, transpiration, infiltration, runoff and ground water. Schematically the interdependency of these components can be represented as in Fig. 1.2. Depending upon the problem at hand, a hydrologist would require data relating to the various relevant phases. The data normally required are:

1. Weather records—temperature, humidity and wind velocity,
2. precipitation data,

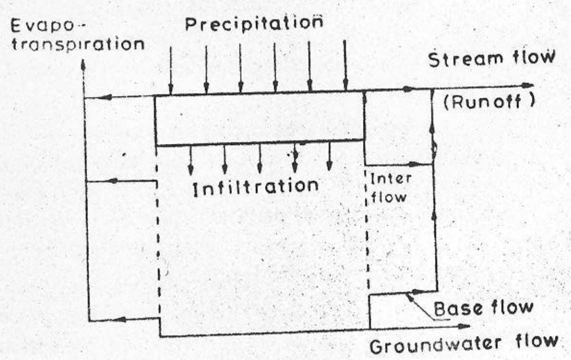

Fig. 1.2 Components of the hydrologic cycle

3. stream-flow records,
4. evaporation and transpiration data,
5. infiltration characteristics of the area,
6. groundwater characteristics, and
7. physical and geological characteristics of the area under consideration.

In India, meteorological data—including weather and rainfall data—are collected by the India Meteorological Department (IMD) and by some state government agencies. Stream flow-data of various rivers and streams are usually available with the state irrigation department. Flow in major rivers are monitored by Central government agencies such as the Central Water Commission (CWC). Groundwater data will normally be available with the Central Groundwater Board and State Government groundwater development organisations. Data on evaporation, transpiration and infiltration will be available with State government organisations, such as the Irrigation Department and Department of Agriculture. The physical data of the basin have to be obtained from a study of topographical maps available with the Survey of India. The geological characteristics of the basin under study are available with the Geological Survey of India and the State Geology Directorate.

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2
PRECIPITATION

2.1 INTRODUCTION

The term "precipitation" denotes all forms of water that reach the earth from the atmosphere. The usual forms are rainfall, snowfall, hail, frost and dew. Of all these, only the first two contribute significant amounts of water. Rainfall being the predominant form of precipitation causing stream flow, especially the flood flow in a majority of rivers in India, unless otherwise stated the term "rainfall" is used in this book synonymously with precipitation. The magnitude of precipitation varies with time and space. Differences in the magnitude of rainfall in various parts of a country at a given time and variations of rainfall at a place in various seasons of the year are obvious and need no elaboration. It is this variation that is responsible for many hydrological problems, such as floods and droughts. The study of precipitation forms a major portion of the subject of hydrometeorology. In this chapter, a brief introduction is given to familiarize the engineer with important aspects of rainfall and, in particular, with the collection and analysis of rainfall data.

For precipitation to form: (i) the atmosphere must have moisture, (ii) there must be sufficient nuclei present to aid condensation, (iii) weather conditions must be good for condensation of water vapour to take place, and (iv) the products of condensation must reach the earth. Under proper weather conditions, the water vapour condenses over nuclei to form tiny water droplets of sizes less than 0.1 mm in diameter. The nuclei are usually salt particles or products of combustion and are normally available in plenty. Wind speed facilitates the movement of clouds while its turbulence retains the water droplets in suspension. Water droplets in a cloud are somewhat similar to the particles in a colloidal suspension. Precipitation results when water droplets come together and coalesce to form larger drops that can drop down. A considerable part of this precipitation gets evaporated back to the atmosphere. The net precipitation at a place and its form depend upon a number of meteorological factors, such as the weather elements like wind, temperature, humidity and pressure in the volume region enclosing the clouds and the ground surface at the given place.

2.2 FORMS OF PRECIPITATION

Some of the common forms of precipitation are: rain, snow, drizzle, glaze, sleet and hail.

Rain

It is the principal form of precipitation in India. The term "rainfall" is used to describe precipitations in the form of water drops of sizes larger than 0.5 mm. The maximum size of a raindrop is about 6 mm. Any drop larger in size than this tends to break up into drops of smaller sizes during its fall from the clouds. On the basis of its intensity, rainfall is classified as:

Type	Intensity
1. Light rain	trace to 2.5 mm/h
2. Moderate rain	2.5 mm/h to 7.5 mm/h
3. Heavy rain	> 7.5 mm/h

Snow

Snow is another important form of precipitation. Snow consists of ice crystals which usually combine to form flakes. When new, snow has an initial density varying from 0.06 to 0.15 g/cm^3 and it is usual to assume an average density of 0.1 g/cm^3. In India, snow occurs only in the Himalayan regions.

Drizzle

A fine sprinkle of numerous water droplets of size less than 0.5 mm and intensity less than 1 mm/h is known as drizzle. In this the drops are so small that they appear to float in the air.

Glaze

When rain or drizzle come in contact with cold ground at around 0°C, the water drops freeze to form an ice coating called *glaze* or *freezing rain*.

Sleet

It is frozen raindrops of transparent grains which form when rain falls through air at subfreezing temperature. In Britain, *sleet* denotes precipitation of snow and rain simultaneously.

Hail

It is a showery precipitation in the form of irregular pellets or lumps of ice of size more than 8 mm. Hails occur in violent thunderstorms in which vertical currents are very strong.

2.3 WEATHER SYSTEMS FOR PRECIPITATION

For the formation of clouds and subsequent precipitation, it is necessary that the moist air masses cool to form condensation. This is normally accomplished by adiabatic cooling of moist air through a process of being lifted to higher altitudes. Some of the terms and processes connected with the weather systems associated with precipitation are given below.

Front

A front is the interface between two distinct air masses. Under certain favourable conditions when a warm air mass and cold air mass meet, the warmer air mass is lifted over the colder one with the formation of a front. The ascending warmer air cools adiabatically with the consequent formation of clouds and precipitation.

Cyclone

A cyclone is a large low pressure region with circular wind motion. Two types of cyclones are recognised: tropical cyclones and extratropical cyclones.

Tropical cyclone: A tropical cyclone, also called cyclone in India, *hurricane* in USA and *typhoon* in South-East Asia, is a wind system with an intensely strong depression with MSL pressures sometimes below 915 mbars. The normal areal extent of a cyclone is about 100-200 km in diameter. The isobars are closely spaced and the winds are anticlockwise in the northern hemisphere. The centre of the storm, called the *eye*, which may extend to about 10-50 km in diameter, will be relatively quiet. However, right outside the eye, very strong winds reaching to as much as 200 kmph exist. The wind speed gradually decreases towards the outer edge. The pressure also increases outwards (Fig. 2.1). The rainfall will normally be heavy in the entire area occupied by the cyclone.

During summer months, tropical cyclones originate in the open ocean at around 5-10° Latitude and move at speeds of about 10-30 kmph to higher

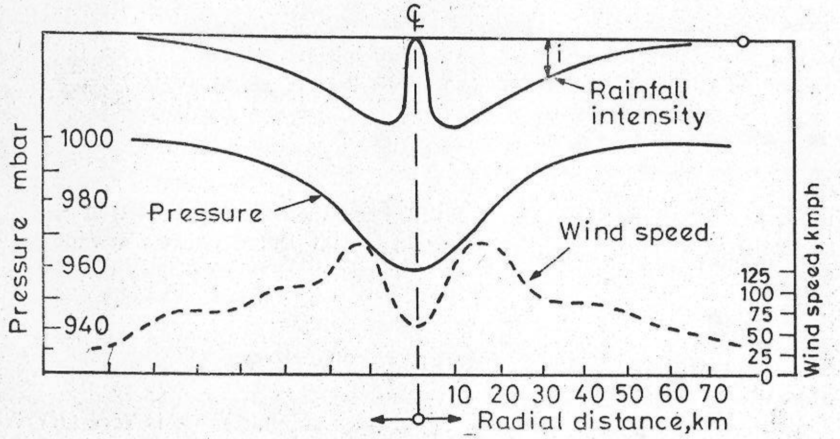

Fig. 2.1 Schematic section of a tropical cyclone

latitudes in an irregular path. They derive their energy from the latent heat of condensation of ocean water vapour and increase in size as they move on oceans. When they move on land the source of energy is cut off and the cyclone dissipates its energy very fast. Hence, the intensity of the storm decreases rapidly. Tropical cyclones cause heavy damage to life and property on their land path and intense rainfall and heavy floods in streams are its usual consequences. Tropical cyclones give moderate to excessive precipitation over very large areas, of the order of 10^3 km^2 for several days.

Extratropical cyclone: These are cyclones formed in locations outside the tropical zone. Associated with a frontal system, they possess a strong counter-clockwise wind circulation in the northern hemisphere. The magnitude of precipitation and wind velocities are relatively lower than those of a tropical cyclone. However, the duration of precipitation is usually longer and the areal extent also is longer.

Anticyclones

These are regions of high pressure, usually of large areal extent. The weather is usually calm at the centre. Anticyclones cause clockwise wind circulations in the northern hemisphere. Winds are of moderate speed, and at the outer edges, cloudy and precipitation conditions exist.

Convective Precipitation

In this type of precipitation a packet of air which is warmer than the surrounding air due to localised heating rises because of its lesser density. Air from cooler surroundings flows to take up its place thus setting up a convective cell. The warm air continues to rise, undergoes cooling and

results in precipitation. Depending upon the moisture, thermal and other conditions light showers to thunderstorms can be expected in convective precipitation. Usually the areal extent of such rains is small, being limited to a diameter of about 10 km.

Orographic Precipitation

The moist air masses may get lifted-up to higher altitudes due to the presence of mountain barriers and consequently undergo cooling, condensation and precipitation. Such a precipitation is known as Orographic precipitation. Thus in mountain ranges, the windward slopes have heavy precipitation and the leeward slopes light rain fall.

2.4 CHARACTERISTICS OF PRECIPITATION ON INDIA

From the point of view of climate the Indian subcontinent can be considered to have two major seasons and two transitional periods as :

1. South-west monsoon (June–September)
2. Transition-I, post-monsoon (October–November)
3. Winter season (December–February)
4. Transition-II, Summer, (March–May)

The chief precipitation characteristics of these seasons are given below.

South-West Monsoon (June-September)

The south-west monsoon (popularly known as the *monsoon)* is the principal rainy season of India when over 75% of the annual rainfall is received over a major portion of the country. Excepting the south-eastern part of the peninsula and Jammu and Kashmir, for the rest of the country the south-west monsoon is the principal source of rain with July as the rainiest month. The monsoon originates in the Indian ocean and heralds its appearance in the southern part of Kerala by the end of May. The onset of monsoon is accompanied by high south-westerly winds at speeds of 20-40 knots and low-pressure regions at the advancing edge. The monsoon winds advance across the country in two branches: (i) the Arabian sea branch and (ii) the Bay of Bengal branch. The former sets in at the extreme southern part of Kerala and the latter at Assam, almost simultaneously in the first week of June. The Bay branch first covers the north-eastern regions of the country and turns westwards to advance into Bihar and UP. The Arabian sea branch moves northwards over Karnataka, Maharashtra and Gujarat. Both the branches reach Delhi around the same time by about the fourth week of June. A low-pressure region known as *monsoon trough* is formed between the two branches. The trough extends from the Bay of Bengal to Rajasthan and the precipitation pattern over the country is generally determined by its position. The monsoon winds increase from June to July and begin to weaken in September. The withdrawal of the monsoon, marked by a substantial rainfall activity starts in September in the northern part of the country. The onset and withdrawal of the monsoon at various parts of the country are shown in Fig. 2.2.

The monsoon is not a period of continuous rainfall. The weather is generally cloudy with frequent spells of rainfall. Heavy rainfall activity in various parts of the country owing to the passage of low pressure regions is common. Depressions formed in the Bay of Bengal at a frequency of 2–3 per month move along the trough causing excessive precipitation of about 100-200 mm per day. Breaks of about a week in which the rainfall activity is the least is another feature of the monsoon. The south-west monsoon rainfall over the country is indicated in Fig. 2.3. As seen from this figure, the heavy rainfall areas are Assam and the north-eastern region with 200-300 cm; West Bengal 200-400 cm; west coast and western ghats with 200-300 cm; West Bengal with 120-160 cm, UP, Haryana and the Punjab with 100-120 cm.

Post-Monsoon (October-November)

As the south-west monsoon retreats, low-pressure areas form in the Bay of Bengal and a north-easterly flow of air that picks up moisture in the Bay of Bengal is formed. This air mass strikes the east coast of the southern peninsula (Tamilnadu) and causes rainfall. Also, in this period, specially in November, severe tropical cyclones form in the Bay of Bengal and the Arabian sea. The cyclones formed in the Bay of Bengal are about twice as many as in the Arabian sea. These cyclones strike the coastal areas and cause intense rainfall and heavy damage to life and property.

Winter Season (December-February)

By about mid-December, disturbances of extra tropical origin travel eastwards across Afghanistan and Pakistan. Known as *western disturbances*, they cause moderate to heavy rain and snowfall (about 25 cm) in the Himalayas and Jammu and Kashmir. Some light rainfall also occurs in the northern plains. Low-pressure areas in the Bay of Bengal formed in these months cause 10-12 cm of rainfall in the southern parts of Tamilnadu.

Summer (Pre-monsoon) (March-May)

There is very little rainfall in India in this season. Convective cells cause some thunderstorms mainly in Kerala, West Bengal and Assam. Some cyclone activity, dominantly on the east coast, also occurs.

Annual Rainfall

The annual rainfall over the country is shown in Fig. 2.4. Considerable areal variation exists for the annual rainfall in India with high rainfall of the magnitude of 200 cm in Assam and north-eastern parts and the western

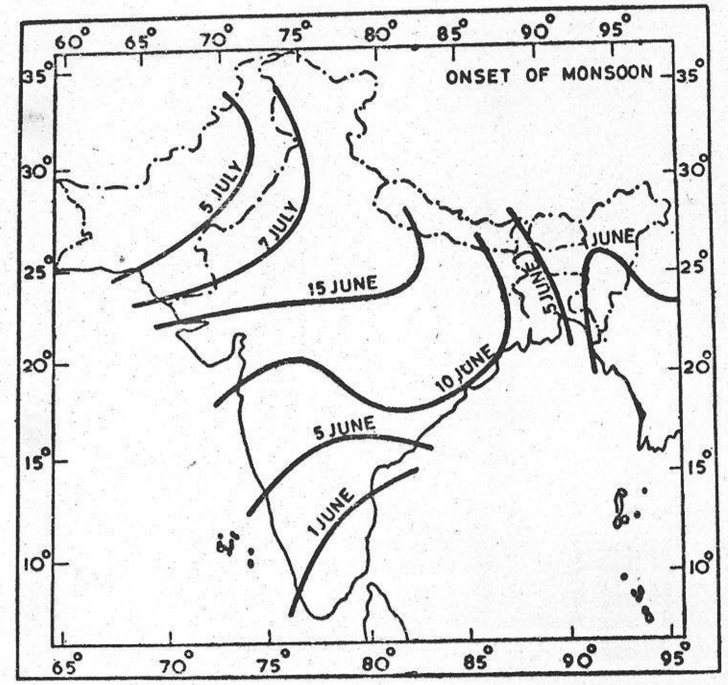

Fig. 2.2 (a) Normal dates of onset of monsoon

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The territorial waters of India extend into the sea to a distance of twelve nautical miles measured from the appropriate baseline

Responsibility for the correctness of internal details on the map rests with the publisher

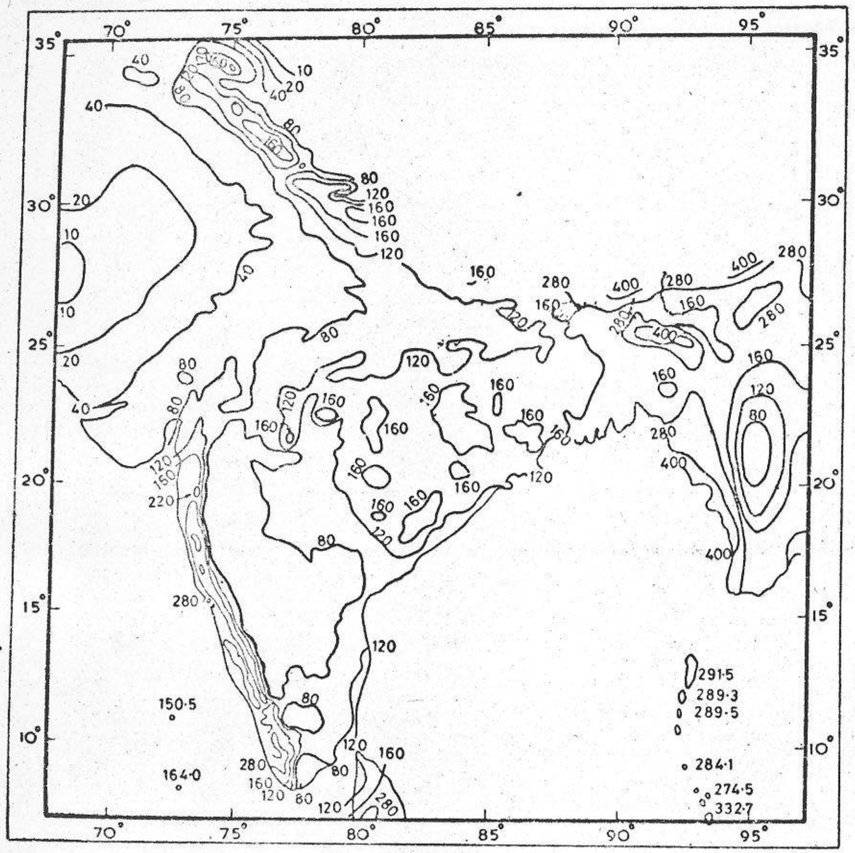


Fig. 2.4 Annual rainfall (cm) over India and neighbourhood

(Reproduced from *Natural Resources of Humid Tropical Asia—Natural Resources Research XII*,

The territorial waters of India extend into the sea to a distance of twelve nautical miles measured from the appropriate baseline

Responsibility for the correctness of internal details on the map rests with the publisher

ghats, and scanty rainfall in eastern Rajasthan and parts of Gujarat, Maharashtra and Karnataka. The average annual rainfall for the entire country is estimated[5] as 119 cm.

It is well-known that there is considerable variation of annual rainfall in time at a place. The coefficient of variation,

$$C_v = \frac{100 \times \text{standard deviation}}{\text{mean}}$$

of the annual rainfall varies between 15 to 70, from place to place with an average value of about 30. Variability is least in regions of high rainfall and largest in regions of scanty rainfall. Gujarat, Haryana, the Punjab and Rajasthan have large variability of rainfall.

2.5 MEASUREMENT

Precipitation is expressed in terms of the depth to which rainfall water would stand on an area if all the rain were collected on it. Thus 1 cm of rainfall over a catchment area of 1 km^2 represents a volume of water equal to 10^4 m^3. In the case of snowfall, an equivalent depth of water is used as the depth of precipitation. The precipitation is collected and measured in a *raingauge*. Terms such as ***pluviometer***, ***ombrometer*** and ***hyetometer*** are also sometimes used to designate a raingauge.

A raingauge essentially consists of a cylindrical-vessel assembly kept in the open to collect rain. The rainfall catch of the raingauge is affected by its exposure conditions. To enable the catch of raingauge to accurately represent the rainfall in the area surrounding the raingauge standard settings are adopted. For siting a raingauge the following considerations are important:

1. The ground must be level and in the open and the instrument must present a horizontal catch surface.
2. The gauge must be set as near the ground as possible to reduce wind effects but it must be sufficiently high to prevent splashing, flooding etc.
3. The instrument must be surrounded by an open fenced area of at least 5.5 m × 5.5 m. No object should be nearer to the instrument than 30 m or twice the height of the obstruction.

Raingauges can be broadly classified into two categories as (i) nonrecording raingauges and (ii) recording gauges.

Nonrecording Gauges

The nonrecording gauge extensively used in India is the *Symons' gauge*. It essentially consists of a circular collecting area of 12.7 cm (5.0 inch) diameter connected to a funnel. The rim of the collector is set in a horizontal plane at a height of 30.5 cm above the ground level. The funnel

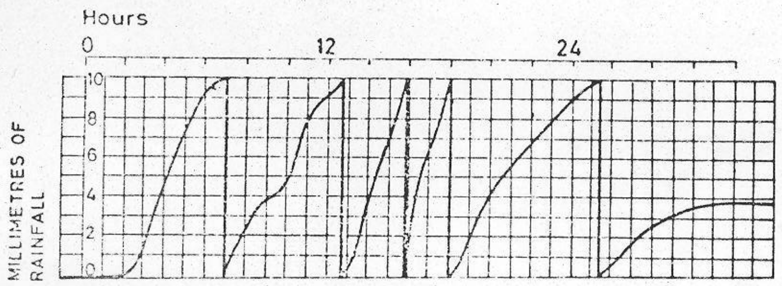


Fig. 2.6 Recording from a natural syphon-type gauge (schematic)

action which resets the pen to zero level. It is obvious that the natural syphon-type recording raingauge gives a plot of the mass curve of rainfall.

Telemetering Raingauges

These raingauges are of the recording type and contain electronic units to transmit the data on rainfall to a base station both at regular intervals and on interrogation. The tipping-bucket type raingauge, being ideally suited, is usually adopted for this purpose. Any of the other types of recording raingauges can also be used equally effectively. Telemetering gauges are of utmost use in gathering rainfall data from mountainous and generally inaccessible places.

Radar Measurement of Rainfall

The meteorological radar is a powerful instrument for measuring the areal extent, location and movement of rain storms. Further, the amounts of rainfall over large areas can be determined through the radar with a good degree of accuracy.

The radar emits a regular succession of pulses of electromagnetic radiation in a narrow beam. When raindrops intercept a radar beam, it has been shown that

$$P_r = \frac{C\,Z}{r^2} \qquad (2.1)$$

where P_r = average echopower, Z = radar-echo factor, r = distance to target volume and C = a constant. Generally the factor Z is related to the intensity of rainfall as

$$Z = a\,I^b \qquad (2.2)$$

where a and b are coefficients and I = intensity or rainfall in mm/h. The values a and b for a given radar station have to be determined by calibration with the help of recording raingauges. A typical equation for Z is

$$Z = 200\,I^{1.60}$$

Meteorological radars operate with wavelengths ranging from 3 to 10 cm the common values being 5 and 10 cm. For observing details of heavy flood-producing rains, a 10-cm radar is used while for light rain and snow a 5-cm radar is used. The hydrological range of the radar is about 200 km. Thus a radar can be considered to be a remote-sensing super gauge covering an areal extent of as much as 100,000 km². Radar measurement is continuous in time and space. Present-day developments in the field include (*i*) On-line processing of radar data on a computer and (*ii*) Doppler-type radars for measuring the velocity and distribution of raindrops.

2.6 RAINGAUGE NETWORK

Since the catching area of a raingauge is very small compared to the areal extent of a storm, it is obvious that to get a representative picture of a storm over a catchment the number of raingauges should be as large as possible, i.e. the catchment area per gauge should be small. On the other hand, economic considerations to a large extent and other considerations, such as topography, accessibility, etc. to some extent restrict the number of gauges to be maintained. Hence one aims at an optimum density of gauges from which reasonably accurate information about the storms can be obtained. Towards this the World Meteorological Organisation (WMO) recommends the following densities.

1. In flat regions of temperate, Mediterranean and tropical zones:
   ideal—1 station for 600-900 km^2,
   acceptable—1 station for 900-3000 km^2;
2. in mountainous regions of temperate, Mediterranean and tropical zones:
   ideal—1 station for 100-250 km^2
   acceptable—1 station for 250-1000 km^2; and
3. in arid and polar zones: 1 station for 1500-10,000 km^2 depending on the feasibility.

Ten per cent of raingauge stations should be equipped with self-recording gauges to know the intensities of rainfall.

From practical considerations of Indian conditions, the Indian Standard (IS : 4987-1968) recommends the following densities as sufficient.

1. In plains: 1 station per 520 km^2;
2. in regions of average elevation 1000 m : 1 station per 260-390 km^2; and
3. in predominently hilly areas with heavy rainfall: 1 station per 130 km^2.

Adequacy of Raingauge Stations

If there are already some raingauge stations in a catchment, the optimal number of stations that should exist to have an assigned percentage of in the estimation of mean rainfall is obtained by statistical analysis as:

$$N = \left(\frac{C_v}{\epsilon}\right)^2 \tag{2.3}$$

where N = optimal number of stations, ϵ = allowable degree of error in the estimate of the mean rainfall and C_v = coefficient of variation of the rainfall values at the existing m stations (in per cent). If there are m stations in the catchment each recording rainfall values $P_1, P_2, \ldots, p_i, \ldots p_m$ in a known time, the coefficient of variation C_v is calculated as:

$$C_v = \frac{100 \times \sigma_{m-1}}{\bar{P}}$$

$$\text{where } \sigma_{m-1} = \sqrt{\left[\sum_1^m (P_i^2) - \frac{\left(\sum_1^m P_i\right)^2}{m}\right] / (m-1)}$$

P_i = precipitation magnitude in the i^{th} station

$$\bar{P} = \frac{1}{m}\left(\sum_1^m P_i\right)$$

In calculating N from Eq. (2.3) it is usual to take $\epsilon = 10\%$. It is seen that if the value of ϵ is small, the number of raingauge stations will be more.

EXAMPLE 2.1 *A catchment has six raingauge stations. In a year, the annual rainfall recorded by the gauges are as follows:*

Station	*A*	*B*	*C*	*D*	*E*	*F*
Rainfall (cm)	*82.6*	*102.9*	*180.3*	*110.3*	*98.8*	*136.7*

For a 10% error in the estimation of the mean rainfall, calculate the optimum number of stations in the catchment.

For this data, $m = 6$

$\bar{P} = 118.6$

$\sigma_{m-1} = 35.04$

$\epsilon = 10$

$$C_v = \frac{100 \times 35.04}{118.6} = 29.54$$

$$N = \left(\frac{29.54}{10}\right)^2 = 8.7, \text{ say 9 stations}$$

The optimal number of stations for the catchment is 9. Hence three more additional stations are needed.

2.7 PREPARATION OF DATA

Before using the rainfall records of a station, it is necessary to first check the data for continuity and consistency. The continuity of a record may be broken with missing data due to many reasons such as damage or fault in a raingauge during a period. The missing data can be estimated by using the data of the neighbouring stations. In these calculations the *normal* **rainfall** is used as a standard of comparison. The normal rainfall is the average value of rainfall at a particular date, month or year over a specified 30-year period. The 30-year normals are recomputed every decade. Thus the term "normal annual precipitation" at station A means the average annual precipitation at A based on a specified 30 years of record.

Estimation of Missing Data

Given the annual precipitation values, $P_1, P_2, P_3, \ldots P_m$ at neighbouring M stations 1, 2, 3,..., M respectively, it is required to find the missing annual precipitation P_x at a station X not included in the above M stations. Further, the normal annual precipitations $N_1, N_2, \ldots, N_i \ldots$ at each of the above $(M + 1)$ stations including station X are known.

If the normal annual precipitations at various stations are within about 10% of the normal annual precipitation at station X, then a simple arithmetic average procedure is followed to estimate P_x. Thus

$$P_x = \frac{1}{M}[P_1 + P_2 + \ldots + P_m] \tag{2.4}$$

If the normal precipitations vary considerably, then P_x is estimated by weighing the precipitation at the various stations by the ratios of normal annual precipitations. This method, known as the *normal ratio method* gives P_x as

$$P_x = \frac{N_x}{M}\left[\frac{P_1}{N_1} + \frac{P_2}{N_2} + \ldots + \frac{P_m}{N_m}\right] \tag{2.5}$$

EXAMPLE 2.2 *The normal annual rainfall at stations* A, B, C, *and* D *in a basin are 80.97, 67.59, 76.28 and 92.01 cms respectively. In the year 1975, the station* D *was inoperative and the stations* A, B *and* C *recorded annual precipitations of 91.11, 72.23 and 79.89 cm respectively. Estimate the rainfall at station D in that year.*

As the normal rainfall values vary more than 10%, the normal ratio method is adopted. Using Eq. (2.5),

$$P_D = \frac{92.01}{3} \times \left(\frac{91.11}{80.97} + \frac{72.23}{67.59} + \frac{79.89}{76.28}\right)$$

$$= 99.41 \text{ cm}$$

Test for Consistency of Record

If the conditions relevant to the recording of a raingauge station have undergone a significant change during the period of record, inconsistency would arise in the rainfall data of that station. This inconsistency would be felt from the time the significant change took place. Some of the common causes for inconsistency of record are: (i) shifting of a raingauge station to a new location, (ii) the neighbourhood of the station undergoing a marked change, (iii) change in the ecosystem due to calamities, such as forest fires, land slides, and (iv) occurrence of observational error from a certain date. The checking for inconsistency of a record is done by the *double-mass curve technique*. This technique is based on the principle that when each recorded data comes from the same parent population, they are consistent.

A group of 5 to 10 base stations in the neighbourhood of the problem station X is selected. The data of the annual (or monthly mean) rainfall of the station X and also the average rainfall of the group of base stations covering a long period is arranged in the reverse chronological order (i.e. the latest record as the first entry and the oldest record as the last entry in the list). The accumulated precipitation of the station X (i.e. ΣP_x) and the accumulated values of the average of the group of base stations (i.e. ΣP_{av}) are calculated starting from the latest record. Values of ΣP_x are plotted against ΣP_{av} for various consecutive time periods (Fig. 2.7). A decided break in the slope of the resulting plot indicates a change in the precipitation regime of station X. The precipitation values at station X beyond the period of change of regime (point 63 in Fig. 2.7) is corrected by using the relation

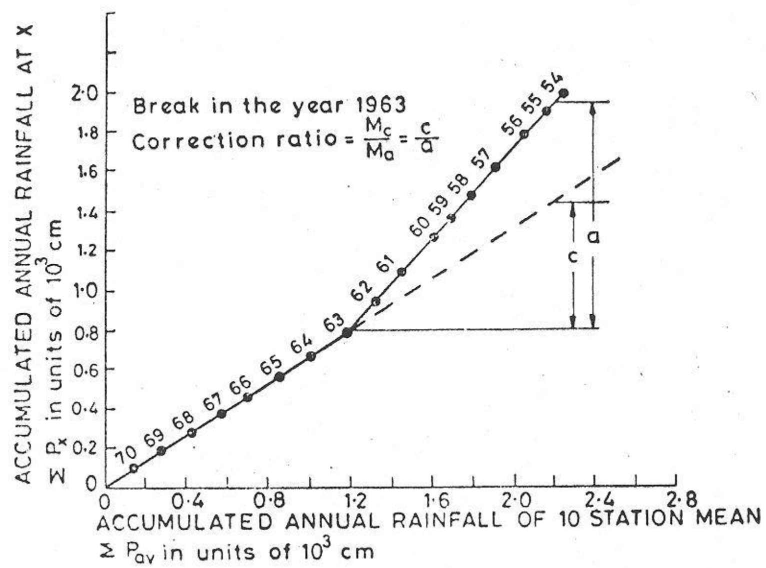


Fig. 2.7 Double-mass curve

$$P_{cx} = P_x \frac{M_c}{M_a} \tag{2.6}$$

where P_{cx} = corrected precipitation at any time period t_1 at station X

P_x = original recorded precipitation at time period t_1 at station X

M_c = corrected slope of the double-mass curve

M_a = original slope of the mass curve

In this way the older records are brought up to the new regime of the station. It is apparent that the more homogeneous the base station records are, the more accurate will be the corrected values at station X. A change in slope is normally taken as significant only where it persists for more than five years. The double-mass curve is also helpful in checking arithmetical errors in transferring rainfall data from one record to another.

2.8 PRESENTATION OF RAINFALL DATA

A few commonly used methods of presentation of rainfall data which have been found to be useful in interpretation and analysis of such data are given below.

Mass Curve of Rainfall

The mass curve of rainfall is a plot of the accumulated precipitation against time, plotted in chronological order. Records of float type and weighing-bucket type gauges are of this form. A typical mass curve of rainfall at a station during a storm is shown in Fig. 2.8. Mass curves of rainfall are very useful in extracting the information on the duration and magnitude of a storm. Also, intensities at various time intervals in a storm can be obtained by the slope of the curve. For nonrecording raingauges, mass curves are prepared from a knowledge of the approximate beginning and end of a

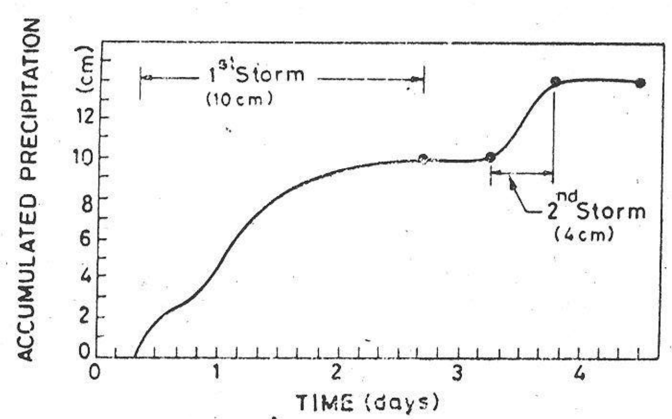


Fig. 2.8 Mass curve of rainfall

storm and by using the mass curves of adjacent recording gauge stations as a guide.

Hyetograph

A hyetograph is a plot of the intensity of rainfall against the time interval. The hyetograph is derived from the mass curve and is usually represented as a bar chart (Fig. 2.9). It is a very convenient way of representing the characteristics of a storm and is particularly important in the development of design storms to predict extreme floods. The area under a hyetograph represents the total precipitation received in that period. The time interval used depends on the purpose; in urban-drainage problems small durations are used while in flood-flow computations in larger catchments the intervals are of about 6 h.

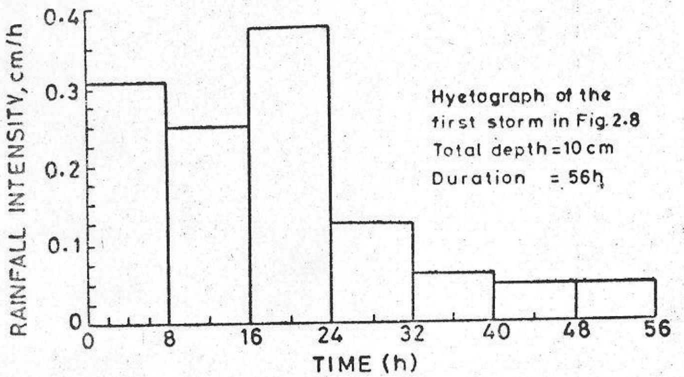


Fig. 2.9 Hyetograph of a storm

Point Rainfall

Point rainfall, also known as station rainfall refers to the rainfall data of a station. Depending upon the need, data can be listed as daily, weekly, monthly, seasonal or annual values for various periods. Graphically these data are represented as plots of magnitude vs chronological time in the form of a bar diagram. Such a plot, however, is not convenient for discerning a trend in the rainfall as there will be considerable variations in the rainfall values leading to rapid changes in the plot. A moving-average plot, in which the average value of precipitation of three or five consecutive time intervals is plotted at the mid-value of the time interval is useful in smoothening out the variations and bringing out the trend (*see* Prob. 2.9).

2.9 MEAN PRECIPITATION OVER AN AREA

As indicated earlier, raingauges represent only point sampling of the areal

distribution of a storm. In practice, however, hydrological analysis requires a knowledge of the rainfall over an area, such as over a catchment. To convert the point rainfall values at various stations into an average value over a catchment the following three methods are in use : (i) arithmetical-mean method, (ii) Thiessen-polygon method and (iii) Isohyetal method.

Arithmetical-Mean Method

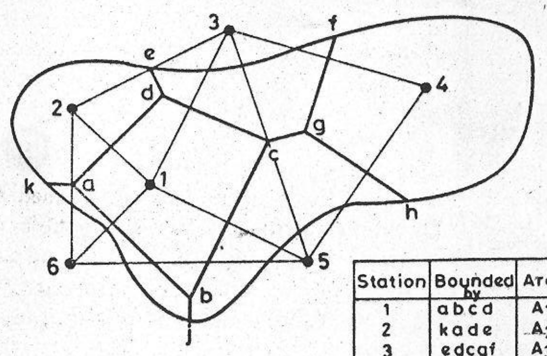
When the rainfall measured at various stations in a catchment show little variation, the average precipitation over the catchment area is taken as the arithmetic mean of the station values. Thus if $P_1, P_2, \ldots, P_i \ldots P_n$ are the rainfall values in a given period in N stations within a catchment, then the value of the mean precipitation $\bar{P}$ over the catchment by the arithmetic-mean method is

$$\bar{P} = \frac{P_1 + P_2 + \ldots + P_i + \ldots + P_n}{N} = \frac{1}{N}\sum_{i=1}^{N} P_i \qquad (2.7)$$

In practice, this method is used very rarely.

Thiessen-Mean Method

In this method the rainfall recorded at each station is given a weightage on the basis of an area closest to the station. The procedure of determining the weighing area is as follows: Consider a catchment area as in Fig. 2.10 containing three raingauge stations. There are three stations outside the catchment but in its neighbourhood. The catchment area is drawn to scale and the positions of the six stations marked on it. Stations 1 to 6 are joined to form a network of triangles. Perpendicular bisectors for each of the sides of the triangle are drawn. These bisectors form a polygon around



Station	Bounded by	Area	Weightage
1	abcd	A_1	A_1/A
2	kade	A_2	A_2/A
3	edcgf	A_3	A_3/A
4	fgh	A_4	A_4/A
5	hgcbj	A_5	A_5/A
6	jbak	A_6	A_6/A

Catchment area = Total = A

Fig. 2.10 Thiessen polygons

each station. The boundary of the catchment, if it cuts the bisectors is taken as the outer limit of the polygon. Thus for station 1, the bounding polygon is *abcd*. For station 2, *kade* is taken as the bounding polygon. These bounding polygons are called *Thiessen polygons*. The areas of these six Thiessen polygons are determined either with a planimeter or by using an overlay grid. If $P_1, P_2, ..., P_6$ are the rainfall magnitudes recorded by the stations 1, 2, ..., 6 respectively, and $A_1, A_2, ..., A_6$ are the respective areas of the Thiessen polygons, then the average rainfall over the catchment $\bar{P}$ is given by

$$\bar{P} = \frac{P_1 A_1 + P_2 A_2 + \dots + P_6 A_6}{(A_1 + A_2 + \dots + A_6)}$$

Thus in general for M stations,

$$\bar{P} = \frac{\sum_{i=1}^{M} P_i A_i}{A} = \sum_{i=1}^{M} P_i \frac{A_i}{A} \qquad (2.8)$$

The ratio A_i/A is called the *weightage factor* for each station.

The Thiessen-polygon method of calculating the average precipitation over an area is superior to the arithmetic-average method as some weightage is given to the various stations on a rational basis. Further, the raingauge stations outside the catchment are also used effectively. Once the weightage factors are determined, the calculation of $\bar{P}$ is relatively easy for a fixed network of stations.

Isohyetal Method

An *isohyet* is a line joining points of equal rainfall magnitude. In the isohyetal method, the catchment area is drawn to scale and the raingauge stations are marked. The recorded values for which areal average $\bar{P}$ is to be determined are then marked on the plot at appropriate stations. Neighbouring stations outside the catchment are also considered. The isohyets of various values are then drawn by considering point rainfalls as guides and interpolating between them by the eye (Figure 2.11). The procedure is similar to the drawing of elevation contours based on spot levels.

The area between two adjacent isohyets are then determined with a planimeter. If the isohyets go out of catchment, the catchment boundary is used as the bounding line. The average value of the rainfall indicated by two isohyets is assumed to be acting over the inter-isohyet area. Thus $P_1, P_2, ..., P_n$ are the values of isohyets and if $a_1, a_2, ..., a_{n-1}$ are the inter-isohyet areas respectively, then the mean precipitation over the catchment of area A is given by

$$\bar{P} = \frac{a_1\left(\frac{P_1 + P_2}{2}\right) + a_2\left(\frac{P_2 + P_3}{2}\right) + \dots + a_{n-1}\left(\frac{P_{n-1} + P_n}{2}\right)}{A} \qquad (2.9)$$

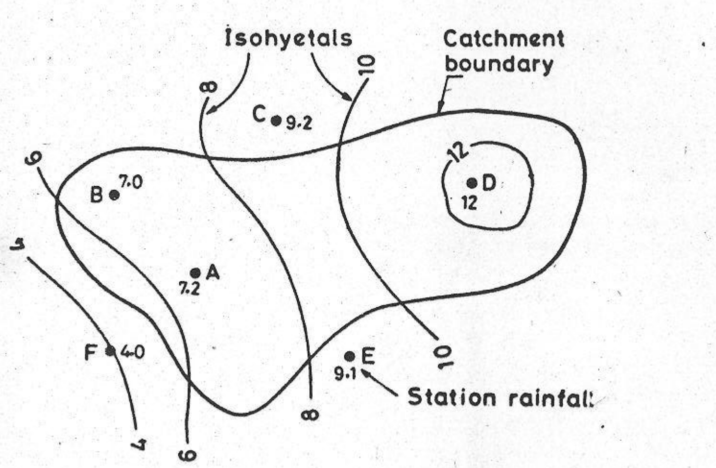


Fig. 2.11 Isohyetals of a storm

The isohyet method is superior to the other two methods especially when the stations are large in number.

EXAMPLE 2.3 *In a catchment area, approximated by a circle of diameter 100 km, four rainfall stations are situated inside the catchment and one station is outside in its neighbourhood. The coordinates of the centre of the catchment and of the five stations are given below. Also given are the annual precipitation recorded by the five stations in 1980. Determine the average annual precipitation by the Thiessen-mean method.*

Centre : (100, 100) Diameter: 100 km.
Distances are in km.

Station	1	2	3	4	5
Coordinates	(30, 80)	(70, 100)	(100, 140)	(130, 100)	(100, 70)
Precipitation (cm)	85.0	135.2	95.3	146.4	102.2

The catchment area is drawn to scale and the stations are marked on it (Figure 2.12). The stations are joined to form a set of triangles and the perpendicular bisector of each side is then drawn. The Thiessen-polygon area enclosing each station is then identified. It may be noted that station 1 in this problem does not have any area of influence in the catchment. The areas of various Thiessen polygons are determined either by a planimeter or by placing an overlay grid.

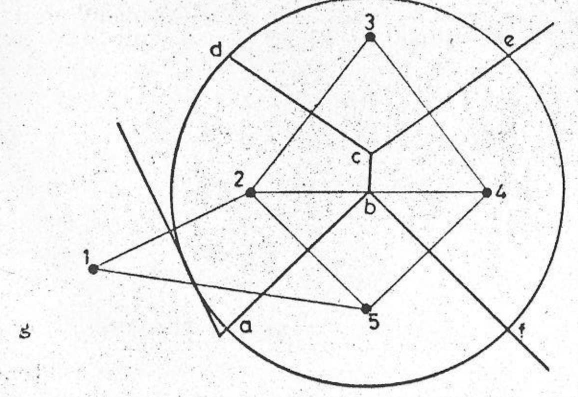


Fig. 2.12 Thiessen polygons—Example 2.12

Station	Boundary of area	Area (km²)	Fraction of total area	Rainfall	Weighted P (cm)
1	—	—	—	85.0	—
2	abcd	2141	0.2726	135.2	36.86
3	dce	1609	0.2049	95.3	19.53
4	ecbf	2141	0.2726	146.4	39.91
5	fba	1963	0.2499	102.2	25.54
Total		7854	1.000		121.84

Mean precipitation = 121.84 cm

EXAMPLE 2.4 *The isohyets due to a storm in a catchment was drawn (Fig. 2.11) and the area of the catchment bounded by isohyets were tabulated as below.*

Isohyets (cm)	Area (km²)
Station–12.0	30
12.0–10.0	140
10.0–8.0	80
8.0–6.0	180
6.0–4.0	20

Estimate the mean precipitation due to the storm.

For the first area consisting of a station surrounded by a closed isohyet, a precipitation value of 12.0 cm is taken. For all other areas, the mean of two bounding isohyets are taken.

Isohyets	Average value	Area (km²)	Fraction of total area	Weighted P (cm)
1	2	3	4	5
12.0	12.0	30	0.0667	0.800
12.0–10.0	11.0	140	0.3111	3.422
10.0–8.0	9.0	80	0.1778	1.600
8.0–6.0	7.0	180	0.4000	2.800
6.0–4.0	5.0	20	0.0444	0.222
Total		450	1.0000	8.844

Mean precipitation $\bar{P}$ = 8.84 cm

2.10 DEPTH-AREA-DURATION RELATIONSHIPS

The areal distribution characteristics of a storm of given duration is reflected in its depth-area relationship. A few aspects of the interdependency of depth, area and duration of storms are discussed below.

Depth-Area Relation

For a rainfall of a given duration, the average depth decreases with the area in an exponential fashion given by

$$\bar{P} = P_0 \exp(-KA^n) \tag{2.10}$$

where $\bar{P}$ = average depth in cms over an area A km², P_0 = highest amount of rainfall in cm at the storm centre and K and n are constants for a given region. On the basis of 42 severemost storms in north India, Dhar and Bhattacharya[3] (1975) have obtained the following values for K and n for storms of different duration.

Duration	K	n
1 day	0.0008526	0.6614
2 days	0.0009877	0.6306
3 days	0.001745	0.5961

Since it is very unlikely that the storm centre coincides over a raingauge station, the exact determination of P_0 is not possible. Hence in the analysis of large area storms the highest station rainfall is taken as the average depth over an area of 25 km^2.

Equation (2.10) is useful in extrapolating an existing storm data over an area.

Maximum Depth-Area-Duration Curves

In many hydrological studies involving estimation of severe floods, it is necessary to have information on the maximum amount of rainfall of various durations occurring over various sizes of areas. The development of relationship, between maximum depth-area-duration for a region is known as DAD analysis and forms an important aspect of hydro-meteorological study. References 1 and 7 can be consulted for details on DAD analysis. A brief description of the analysis is given below.

First, the severemost rainstorms that have occurred in the region under question are considered. Isohyetal maps and mass curves of the storm are compiled. A depth-area curve of a given duration of the storm is prepared. Then from a study of the mass curve of rainfall, various durations and the maximum depth of rainfall in these durations are noted. The maximum depth-area curve for a given duration D is prepared by assuming the area distribution of rainfall for smaller duration to be similar to the total storm. The procedure is then repeated for different storms and the envelope curve of maximum depth-area for duration D is obtained. A similar procedure for various values of D results in a family of envelop curves of maximum depth vs area, with duration as the third parameter (Fig. 2.13). These curves are called DAD curves.

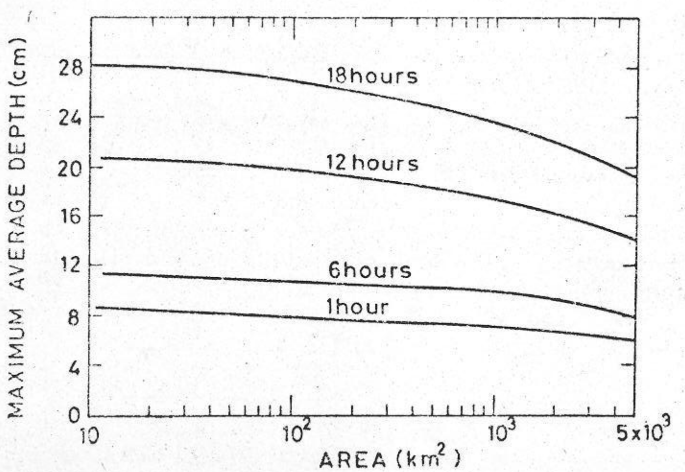


Fig. 2.13 Typical DAD curves

Figure 2.13 shows typical DAD curves for a catchment. In this the average depth denotes the depth averaged over the area under consideration. It may be seen that the maximum depth for a given storm decreases with the area; for a given area the maximum depth increases with the duration.

Preparation of DAD curves involves considerable computational effort and requires meteorological and topographical information of the region. Detailed data on severemost storms in the past are needed. DAD curves are essential to develop design storms for use in computing the design flood in the hydrological design of major structures such as dams.

Maximum rain depths observed over the plains of north India are indicated in Table 2.1. These were due to two storms, which are perhaps the few severemost recorded rainstorms over the world.

TABLE 2.1 MAXIMUM (OBSERVED) RAIN DEPTHS (cm) OVER PLAINS OF NORTH INDIA[4,5]

	Area in km^2 $\times$ 10^4								
Duration	.026	0.13	0.26	1.3	2.6	5.2	7.8	10.5	13.0
1 day	81.0*	76.5*	71.1*	47.2*	37.1*	26.4	20.3†	18.0†	16.0†
2 days	102.9*	97.5*	93.2*	73.4*	58.7*	42.4*	35.6†	31.5†	27.9†
3 days	121.9†	110.7†	103.1†	79.2†	67.1†	54.6†	48.3†	42.7†	38.9†

Note : *—Storm of 17-18 September 1880 over north-west U.P.
†—Storm of 28-30 July 1927 over north Gujarat.

2.11 FREQUENCY OF POINT RAINFALL

In many hydraulic-engineering applications such as those concerned with floods, the probability of occurrence of a particular extreme rainfall, e.g. a 24-h maximum rainfall, will be of importance. Such information is obtained by the frequency analysis of the point-rainfall data. The rainfall at a place is a random hydrologic process and the rainfall data at a place when arranged in chronological order constitute a time series. One of the commonly used data series is the annual series composed of annual values such as annual rainfall. If the extreme values of a specified event occurring in each year is listed, it also constitutes an annual series. Thus for example, one may list the maximum 24-h rainfall occurring in a year at a station to prepare an annual series of 24-h maximum rainfall values. The probability of occurrence of an event in this series is studied by frequency analysis of this annual data series. A brief description of the terminology and a simple method of predicting the frequency of an event is described in this section and for details the reader is referred to standard works on probability and statistics. The analysis of annual series, even though described with

TABLE 2.2 PLOTTING POSITION FORMULAE

Method	P
California	m/N
Hazen	$(m-0.5)/N$
Weibull	$m/(N+1)$
Chegodayev	$(m-0.3)/(N+0.4)$
Blom	$(m-0.44)/(N+0.12)$
Gringorten	$(m-3/8)/(N+1/4)$

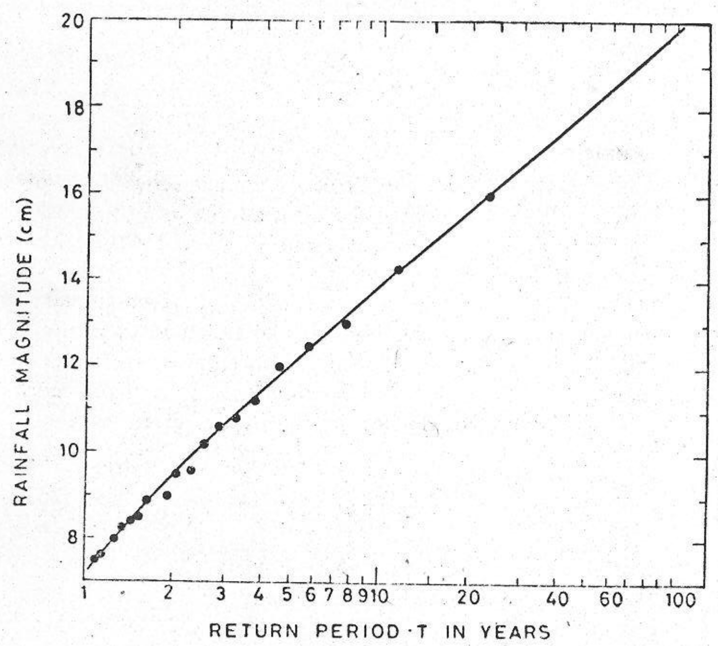


Fig. 2.14 Rainfall frequency curve

accurate work, various analytical calculation procedures using frequency factors are available. Gumbel's extreme value distribution and Log Pearson Type III method are two commonly used analytical methods and are described in Chap. 7 of this book.

EXAMPLE 2.6 *For a station A, the recorded annual 24 h maximum rainfall are given below.* (a) *Estimate the 24 h maximum rainfall with return periods of 13 and 50 years.* (b) *What would be the probability of a rainfall of magnitude equal to or exceeding 10 cm occurring in 24 h at station A.*

TABLE 2.3 ANNUAL MAXIMUM 24 h RAINFALL AT STATION A

Year	1950	'51	'52	'53	'54	'55	'56	'57	'58	'59	'60	'61
Rainfall (cm)	13.0	12.0	7.6	14.3	16.0	9.6	8.0	12.5	11.2	8.9	8.9	7.8

Year	1962	'63	'64	'65	'66	'67	'68	'69	'70	'71
Rainfall (cm)	9.0	10.2	8.5	7.5	6.0	8.4	10.8	10.6	8.3	9.5

The data is arranged in descending order and the probability and recurrence intervals of various events are calculated as indicated in Table 2.4.

TABLE 2.4 CALCULATION OF THE RETURN PERIODS FOR DATA OF TABLE 2.3

N = 22 years.

m	Rainfall (cm)	$P = \frac{m}{N+1}$	$T = 1/P$	m	Rainfall (cm)	$P = \frac{m}{N+1}$	$T = 1/P$
1	16.0	0.043	23.26	12	9.0	0.522	1.92
2	14.3	0.087	11.50	13	8.9	—	—
3	13.0	0.130	7.67	14	8.9	0.609	1.64
4	12.5	0.174	5.75	15	8.5	0.652	1.53
5	12.0	0.217	4.60	16	8.4	0.696	1.44
6	11.2	0.261	3.83	17	8.3	0.739	1.35
7	10.8	0.304	3.29	18	8.0	0.783	1.28
8	10.6	0.348	2.88	19	7.8	0.826	1.21
9	10.2	0.391	2.56	20	7.6	0.870	1.15
10	9.6	0.435	2.30	21	7.5	0.913	1.10
11	9.5	0.478	2.09	22	6.0	0.957	1.05

It may be noted that when two or more magnitudes are same (as m = 13 and 14 in Table 2.4) P is calculated for the largest m value of the set. (Why ?)

A graph is plotted between the rainfall magnitude and the return period

T on a semi-log paper (Fig. 2.14). A smooth curve is drawn through plotted points and the curve extended by judgement. From this curve,

Return period T years	Rainfall magnitude (cm)
13	14.55
50	18.00

For rainfall = 10 cm, T = 2.4 years and P = 0.417.

2.12 INTENSITY-DURATION-FREQUENCY RELATIONSHIP

The intensity of storms decreases with the increase in storm duration. Further, a storm of any given duration will have a larger intensity if its return period is large. In other words, for a storm of given duration, storms of higher intensity in that duration are rarer than storms of smaller intensity. In many design problems related to watershed management, such as runoff disposal and erosion control, it is necessary to know the rainfall intensities of different durations and different return periods. The inter-dependency between the intensity (i cm/h), duration (D h) and return period (T years) is commonly expressed in a general form as

$$i = \frac{K\,T^x}{(D + a)^n} \qquad (2.15)$$

where K, x, a and n are constants for a given catchment. The variation of intensity i with duration D and return period T is shown schematically in Fig. 2.15 (a, b). Typical values of the constants K, x, a and n for a few

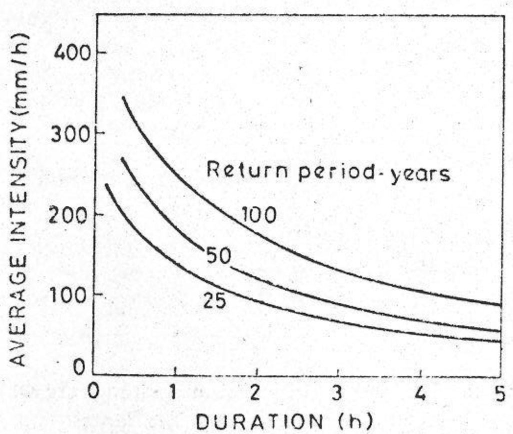


Fig. 2.15 (a) Intensity-duration-frequency curves

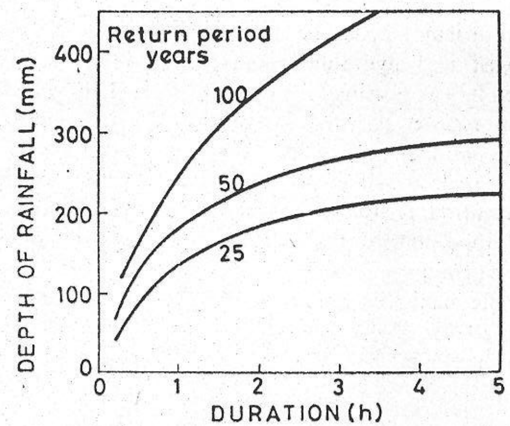


Fig. 2.15 (b) Depth-duration-frequency curves

places in India based on the reported studies of the Central Soil and Water Conservation Research and Training Institute, Dehradun are given in Table 2.5.

TABLE 2.5 TYPICAL VALUES OF CONSTANTS IN EQ. (2.15)

Place	K	x	a	n
Bhopal	6.93	0.189	0.50	0.878
Nagpur	11.45	0.156	1.25	1.032
Chandigarh	5.82	0.160	0.40	0.750
Bellary	6.16	0.694	0.50	0.972
Raipur	4.68	0.139	0.15	0.928

Extreme point rainfall values of different durations and for different return periods have been evaluated by IMD and the *iso-pluvial* (lines connecting equal depths of rainfall) maps covering the entire country have been prepared. These are available for rainfall durations of 15 min, 30 min, 45 min, 1 h, 3 h, 6 h, 9 h, 12 h, 15 h and for return periods of 2, 5, 10, 25, 50 and 100 years. A typical 50 year-12 h maximum rainfall map of the southern peninsula is given in Fig. 2.16(a). The 50 year-1 h maximum rainfall depths over India and the neighbourhood are shown in Figure 2.16(b). Isopluvial maps of the maximum rainfall of various durations and of 50-year return periods covering the entire country are available in Ref. 2.

2.13 PROBABLE MAXIMUM PRECIPITATION (PMP)

In the design of major hydraulic structures such as spillways in large dams, the hydrologist and hydraulic engineer would like to keep the failure probability as low as possible, i.e. virtually zero. This is because the failure of such a major structure will cause very heavy damages to life, property, economy and national morale. In the design and analysis of such structures, the maximum possible precipitation that can reasonably be expected at a given location is used. This stems from the recognition that there is a physical upper limit to the amount of precipitation that can fall over a specified area in a given time.

The probable maximum precipitation (PMP) is defined as the greatest or extreme rainfall for a given duration that is physically possible over a station or basin. From the operational point of view, PMP can be defined as that rainfall over a basin which would produce a flood flow with virtually no risk of being exceeded. The development of PMP for a given region is an involved procedure and requires the knowledge of an experienced hydrometeorologist. Basically two approaches are used: (i) Meteorological methods and (ii) the statistical study of rainfall data. Details of meteorological methods that use storm models are available in published literature.[6]

Statistical studies indicate that PMP can be estimated as

$$PMP = \bar{P} + K\sigma \tag{2.16}$$

where $\bar{P}$ = mean of annual maximum rainfall series, σ = standard deviation of the series and K = a frequency factor which depends upon the statistical distribution of the series, number of years of record and the return period. The value of K is usually in the neighbourhood of 15. Generalised charts for one-day PMP prepared on the basis of the statistical analysis of 60 to 70 years of rainfall data in the North-Indian plain area (Lat. 20°N to 32°N, Long. 68°E to 89°E) are available in Refs 4 and 5. It is found that PMP estimates for North-Indian plains vary from 37 to 100 cm for one-day rainfall.

World's Greatest Observed Rainfall

Based upon the rainfall records available all over the world, a list of world's greatest recorded rainfalls of various durations can be assembled. When this data is plotted on a log-log paper, an enveloping straight line drawn to the plotted points obeys the equation

$$P_m = 42.16\, D^{0.475} \tag{2.17}$$

where P_m = extreme rainfall depth in cm and D = duration in hours. The values obtained from this Eq. (2.17) are of use in PMP estimations.

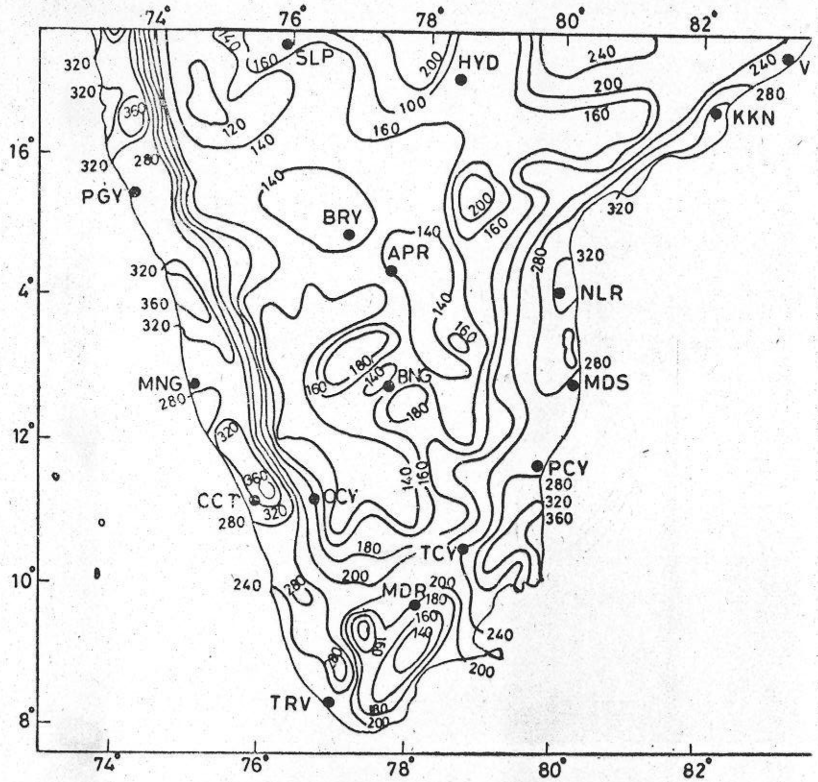


Fig. 2.16 (a) Isopluvial map of 50 yr-24 h maximum rainfall (mm)

(Reproduced with permission from India Meteorological Department)

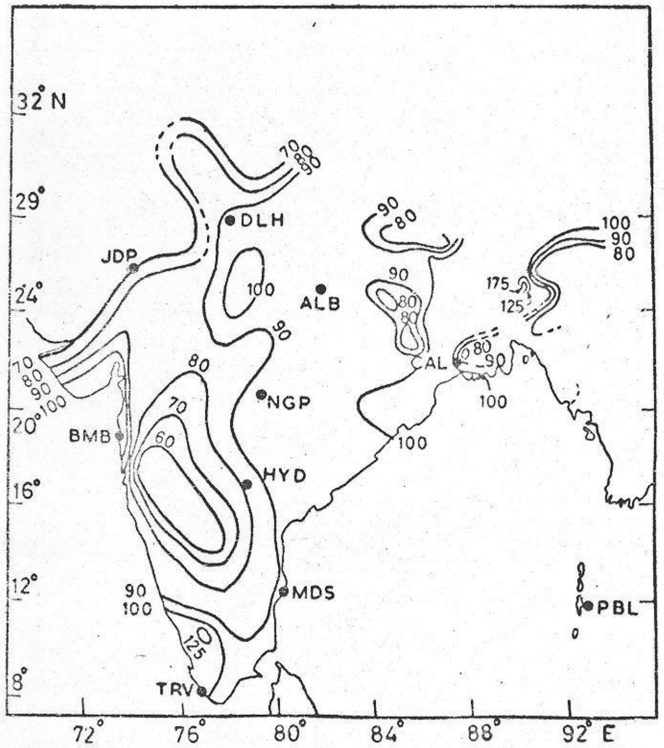


Fig. 2.16 (b) Isopluvial map of 50 yr-1 h maximum rainfall

(Reproduced from *Natural Resources of Humid Tropical Asia—Natural Resources Research XII*, © UNESCO, 1974, with permission of UNESCO)

Based upon Survey of India map with the permission of the Surveyor General of India

The territorial waters of India extend into the sea to a distance of twelve nautical miles measured from the appropriate baseline

Responsibility for the correctness of internal details on the map rests with the publisher

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PROBLEMS

2.1 A catchment area has seven raingauge stations. In a year the annual rainfall recorded by the gauges are as follows:

Station	*P*	*Q*	*R*	*S*	*T*	*U*	*V*
Rainfall (cm)	130.0	142.1	118.2	108.5	165.2	102.1	146.9

For a 5% error in the estimation of the mean rainfall, calculate the minimum number of additional stations required to be established in the catchment.

2.2 The normal annual precipitation of five raingauge stations *P, Q, R, S* and *T* are respectively 125, 102, 76, 113 and 137 cm. During a particular storm the precipitation recorded by stations *P, Q, R*, and *S* are 13.2, 9.2, 6.8 and 10.2 respectively. The instrument at station *T* was inoperative during that storm. Estimate the rainfall at station *T* during that storm.

2.3 Test the consistency of the 22 years of data of the annual precipitation measured at station *A*. Rainfall data for the station *A* as well as the average annual rainfall measured at a group of eight neighbouring stations located in a meteorologically homogeneous region are given below.

Year	1946	'47	'48	'49	'50	'51	'52	'53	'54	'55	'56	'57
Station A (cm)	177	144	178	162	194	168	196	144	160	196	141	158
8 Station average (cm)	143	132	146	147	161	155	152	117	128	193	156	164

Year	1958	'59	'60	'61	'62	'63	'64	'65	'66	'67
Station *A* (cm)	145	132	95	148	142	140	130	137	130	163
8 Station average (cm)	155	143	115	135	163	135	143	130	146	161

(a) In what year is a change in regime indicated ?

(b) Adjust the recorded data at station *A* and determine the mean annual precipitation.

2.4 For a drainage basin of 600 km², isohyetals drawn for a storm gave the following data:

Isohyetals (interval) (cm)	15-12	12-9	9-6	6-3	3-1
Inter-isohyetal area (km²)	92	128	120	175	85

Estimate the average depth of precipitation over the catchment.

2.5 There are 10 raingauge stations available to calculate the rainfall characteristics of a catchment whose shape can be approximately described by straight lines joining the following coordinates (distances in kilometres): (30,0), (80,10), (110,30), (140,90), (130,115), (40,110), (15,60). Coordinates of the raingauge stations and the annual rainfall recorded in them in the year 1981 are given below.

Station	1	2	3	4	5	6	7	8	9	10
Co-ordinates	(0,40)	(50,0)	(140,30)	(140,80)	(90,140)	(0,80)	(40,50)	(90,30)	(90,90)	(40,80)
Annual rainfall (cm)	132	136	93	81	85	124	156	128	102	128

Determine the average annual rainfall over the catchment.

2.6 Figure 2.17 shows a catchment with seven raingauge stations inside it and three stations outside. The rainfall recorded by each of these stations are indicated in the figure. Draw the figure to an enlarged scale and calculate the mean precipitation by (a) Thiessen-mean method, (b) isohyetal method and by (c) arithmetic-mean method.

2.7 Following data are from a self-recording raingauge during a storm.

Time from beginning of storm (minutes)	10	20	30	40	50	60	70	80	90
Accumulated rainfall (mm)	19	41	48	68	91	124	152	160	166

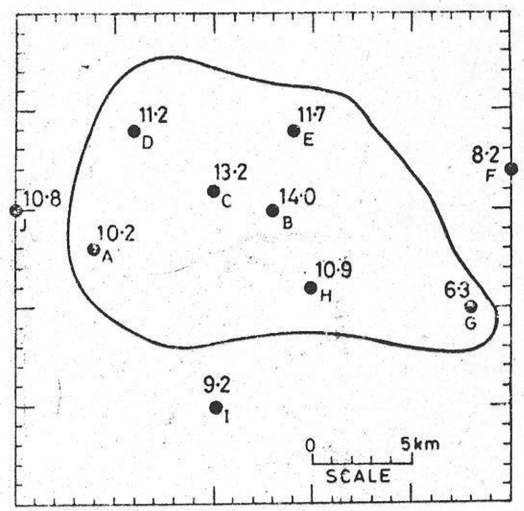

Fig. 2.17

(a) Plot the hyetograph of the storm.

(b) Obtain the values of maximum intensities of this storm for various durations and plot a curve of maximum intensity vs duration.

2.8 The record of annual rainfall at a place is given for 25 years. Estimate the recurrence interval for various magnitudes. By suitable extrapolation, determine the magnitude of annual rainfall at the station corresponding to a recurrence interval of (a) 50 years and (b) 100 years.

Year	1950	'51	'52	'53	'54	'55	'56	'57	'58	'59	'60	'61	'62
Annual rainfall (cm)	113.0	94.5	76.0	87.5	92.7	71.3	77.3	85.1	122.8	69.4	81.0	94.5	86.3

Year	1963	'64	'65	'66	'67	'68	'69	'70	'71	'72	'73	'74
Annual rainfall (cm)	68.6	82.5	90.7	99.8	74.4	66.6	65.0	91.0	106.8	102.2	87.0	84.0

2.9 Plot the three-year moving mean for data of Prob. 2.8. Is there any apparent time trend ? (Hint: Average the annual precipitation value of overlapping three-year periods and plot the average value at the middle year of the period.)

2.10 On the basis of isopluvial maps the 50 year–24 h maximum rainfall at Bangalore is found to be 16.0 cm. Determine the probability of a 24 h rainfall of magnitude equal to or greater than 16.0 cm occurring at Bangalore:

(a) Once in 10 successive years,
(b) two times in 10 successive years, and
(c) at least once in 10 successive years.

2.11 A one-day rainfall of 15.0 cm at a place X was found to have a return period of 100 years. Calculate the probability that a one-day rainfall of this or larger magnitude:

(*a*) Will not occur at X during the next 50 years, and
(*b*) will occur in the next year.

2.12 When long records are not available, records at two or more stations are combined to get one long record for the purposes of recurrence interval calculation. This method is known as ***station-year method.***

The number of times a storm of intensity 4 cm/h was equalled or exceeded in three different raingauge stations were 4, 2, and 5 for periods of records of 36, 25 and 48 years respectively. Find the recurrence interval of the 4 cm/h storm in that area by the station-year method.

2.13 Annual precipitation values at a place having 70 years of rainfall record can be tabulated as follows:

Range (cm)	No. of years
<60.0	6
60–79.9	6
80–99.9	22
100–119.9	25
120–139.9	8
>140	3

Calculate the probability of having:

(a) An annual rainfall equal to or larger than 120 cm,
(b) two successive years in which the annual rainfall is equal to or larger than 140 cm, and
(c) an annual rainfall less than 60 cm.

QUESTIONS

2.1 A tropical cyclone is a

(a) low-pressure zone that occurs in the northern hemisphere only
(b) high-pressure zone with high winds
(c) zone of low pressure with clockwise winds in the northern hemisphere
(d) zone of low pressure with anticlockwise winds in the northern hemisphere.

2.2 Orographic precipitation occurs due to air masses being lifted to higher altitudes by

(a) the density difference of air masses
(b) a frontal action

(c) the presence of mountain barriers
(d) extratropical cyclones.

2.3 The normal onset of monsoon in India is in

(a) early June at Bombay and Madras
(b) early June at Kerala and Assam
(c) early May in Kerala only
(d) November in Tamilnadu and Kerala.

2.4 The average annual rainfall over the whole of India is estimated as

(a) 189 cm (b) 319 cm (c) 89 cm (d) 119 cm.

2.5 Variability of annual rainfall in India is

(a) least in regions of scanty rainfall
(b) largest in regions of high rainfall
(c) least in regions of high rainfall
(d) largest in coastal areas.

2.6 The standard Symons' type raingauge has a collecting area of diameter

(a) 12.7 cm (b) 10 cm (c) 5.08 cm (d) 25.4 cm.

2.7 The standard recording raingauge adopted in India is of

(a) weighing bucket type
(b) natural siphon type
(c) tipping bucket type
(d) telemetry type.

2.8 The following recording raingauges produce the mass curve of precipitation as their record:

(a) Symons' raingauge
(b) tipping-bucket type gauge
(c) weighing-bucket type gauge
(d) natural siphon gauge.

2.9 When specific information about the density of snowfall is not available, the water equivalent of snowfall is taken as

(a) 50% (b) 30% (c) 10% (d) 90%.

2.10 A, B and C are three catchments each having an area of about 10,000 km^2 situated in an arid zone, mountainous region of a tropical zone and flat region of a tropical zone respectively. The desirable number of hydrometeorological stations for each of these three areas, N_A, N_B and N_C respectively will be such that

(a) $N_B > N_B > N_A$ (b) $N_A < N_B < N_C$
(c) $N_A > N_B > N_C$ (d) $N_B = N_C$ and $N_B > N_A$

2.11 The monthly rainfall at a place A during September 1982 was recorded as 55 mm above normal. Here the term "normal" means

(a) the rainfall on the same month on the previous year
(b) the rainfall that was normally expected based on previous month's data
(c) the average rainfall computed from past 12 months' record
(d) The average monthly rainfall for September computed from a specific 30 years of past record.

2.12 The mass curve of rainfall of a storm is a plot of

(a) rainfall depths for various equal durations plotted in decreasing order
(b) rainfall intensity vs time in chronological order
(c) accumulated rainfall intensity vs time
(d) accumulated precipitation vs time in chronological order.

2.13 A hyetograph is a plot of
(a) cumulative rainfall vs time
(b) rainfall intensity vs time
(c) rainfall depth vs duration
(d) discharge vs time.

2.14 The Thiessen polygon is
(a) a polygon obtained by joining adjoining raingauge stations
(b) a representative area used for weighing the observed station precipitation
(c) an area used in the construction of depth-area curves
(d) the descriptive term for the shape of a hydrograph.

2.15 An isohyet is a line joining points having
(a) equal evaporation value
(b) equal barometric pressure
(c) equal height above the MSL
(d) equal rainfall depth in a given duration.

2.16 For a given storm the highest rainfall P_0 and the average rainfall depth $\bar{P}$, are related as $\bar{P}/P_0$ =
(a) $K \exp(A^n)$ (b) $\exp(-KA_n)$ (c) K^{-A} (d) constant.

2.17 By DAD analysis the maximum average depth over an area of 10^4 km² due to one-day storm is found to be 47 cm. For the same area the maximum average depth for a three day storm can be expected to be
(a) < 47 cm (b) > 47 cm (c) = 47 cm (d) inadequate information to conclude.

2.18 A study of the isopluvial maps revealed that at Calcutta a maximum rainfall depth of 200 mm in 12 h has a return period of 50 years. The probability of a 12 h rainfall equal to or greater than 200 mm occurring at Calcutta at least once in 30 years is
(a) 0.45 (b) 0.60 (c) 0.56 (d) 1.0.

2.19 If the maximum depth of a 50 years-15 h-rainfall depth at Bhubaneshwar is 260 mm, the 50 year-3 h-maximum rainfall depth at the same place is
(a) < 260 mm (b) > 260 mm (c) = 260 mm
(d) inadequate date to conclude anything.

2.20 The probable maximum depth of precipitation over a catchment is given by the relation PMP =
(a) $\bar{P} + KA^n$ (b) $\bar{P} + K\sigma$ (c) $\bar{P} \exp(-KA^n)$ (d) $m\bar{P}$.

3

ABSTRACTIONS FROM PRECIPITATION

3.1 LOSSES FROM PRECIPITATION

Evaporation and transpiration form important links in the hydrologic cycle in which water is transferred to the atmosphere as water vapour. In engineering hydrology, runoff is the prime subject of study and evaporation and transpiration phases are treated as "losses" Evaporation from water bodies and soil masses together with the transpiration from vegetation is termed as *evapotranspiration* and is also known variously as *water loss, total loss* or *total evaporation.* The various aspects of evaporation from a water body and evapotranspiration from a basin are discussed in detail in this chapter.

Before the rainfall reaches the outlet of a basin as runoff, certain demands of the catchment such as interception, depression storage and infiltration have to be met. If the precipitation not available for surface runoff is defined as "loss", then these processes are also "losses". In terms of groundwater the infiltration process is a "gain". Aspects of interception, depression storage and infiltration that are important from the point of view of engineering hydrology are dealt briefly in the end of this chapter.

EVAPORATION

3.2 EVAPORATION PROCESS

Evaporation is the process in which a liquid changes to the gaseous state at the free surface, below the boiling point through the transfer of heat

US Weather Bureau and is known as Class A Land Pan. The depth of water is maintained between 18 cm and 20 cm (Fig. 3.1). The pan is normally made of unpainted galvanised iron sheet. Monel metal is used where corrosion is a problem. The pan is placed on a wooden platform of 15 cm height above the ground to allow free circulation of air below the pan. Evaporation measurements are made by measuring the depth of water with a hook gauge in a stilling well.

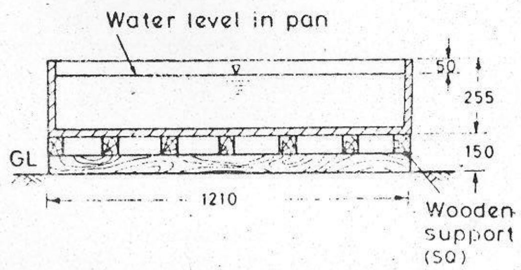

Fig. 3.1 U.S. Class A evaporation pan

ISI Standard Pan

This pan evaporimeter specified by IS : 5973-1970, also known as modified Class A Pan, consists of a pan 1220 mm in diameter with 255 mm of depth. The pan is made of copper sheet of 0.9 mm thickness, tinned inside and painted white outside (Fig. 3.2). A fixed point gauge indicates the level of water. A calibrated cylindrical measure is used to add or remove water maintaining the water level in the pan to a fixed mark. The top of the pan is covered fully with a hexagonal wire netting of galvanized iron to protect the water in the pan from birds. Further, the presence of a wire mesh makes the water temperature more uniform during day and night.

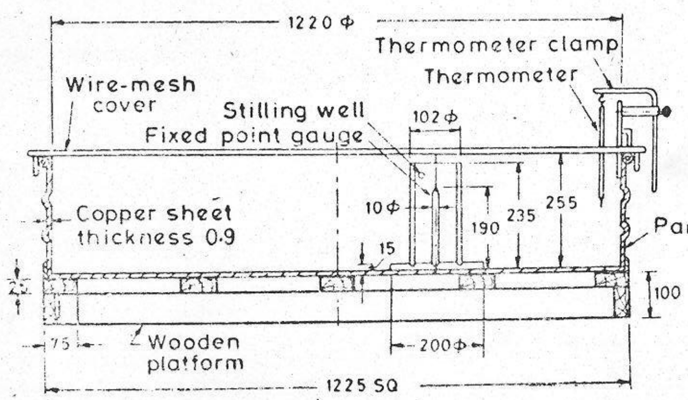

Fig. 3.2 ISI evaporation pan

The evaporation from this pan is found to be less by about 14% compared to that from unscreened pan. The pan is placed over a square wooden platform of 1225 mm width and 100 mm height to enable circulation of air underneath the pan.

Colorado Sunken Pan

This pan, 920 mm square and 460 mm deep is made up of unpainted galvanised iron sheet and buried into the ground within 100 mm of the top (Fig. 3.3). The chief advantage of the sunken pan is that radiation and aerodynamic characteristics are similar to those of a lake. However, it has the following disadvantages: (i) difficult to detect leaks, (ii) extra care is needed to keep the sorrounding area free from tall grass, dust etc. and (iii) expensive to install.

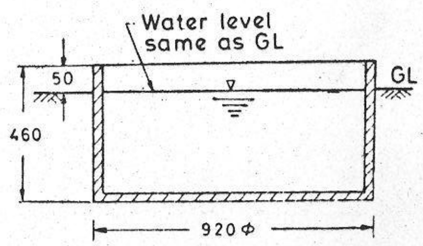

Fig. 3.3 Colorado sunken evaporation pan

US Geological Survey Floating Pan

With a view to simulate the characteristics of a large body of water, this square pan (900 mm side and 450 mm depth) supported by drum floats in the middle of a raft (4.25 m × 4.87 m) is set afloat in a lake. The water level in the pan is kept at the same level as the lake leaving a rim of 75 mm. Diagonal baffles provided in the pan reduce the surging in the pan due to wave action. Its high cost of installation and maintenance together with the difficulty involved in performing measurements are its main disadvantages.

Pan Coefficient, C_p

Evaporation pans are not exact models of large reservoirs and have the following principal drawbacks:

1. They differ in the heat-storing capacity and heat transfer from the sides and bottom. The sunken pan and floating pan aim to reduce this deficiency. As a result of this factor the evaporation from a pan depends to a certain extent on its size. While a pan of 3 m diameter is known to give a value which is about the same as from a neighbouring large lake, a pan of size 1.0 m diameter indicates about 20% excess evaporation than that of the 3 m diameter pan.

2. The height of the rim in an evaporation pan affects the wind action over the surface. Also, it casts a shadow of variable magnitude over the water surface.

3. The heat-transfer characteristics of the pan material is different from that of the reservoir.

In view of the above, the evaporation observed from a pan has to be corrected to get the evaporation from a lake under similar climatic and exposure conditions. Thus a coefficient is introduced as

Lake evaporation = C_p × pan evaporation

in which C_p = pan coefficient. The values of C_p in use for different pans are given in Table 3.1.

TABLE 3.1 VALUES OF PAN COEFFICIENT C_p

S. No.	Type of pan	Average value	Range
1.	Class A Land Pan	0.70	0.60-0.80
2.	ISI Pan (modified Class A)	0.80	0.65-1.10
3.	Colorado Sunken Pan	0.78	0.75-0.86
4.	USGS Floating Pan	0.80	0.70-0.82

Evaporation Stations

It is usual to install evaporation pans in such locations where other meteorological data are also simultaneously collected. The WMO recommends the minimum network of evaporimeter stations as below:

1. Arid zones—One station for every 30,000 km^2,
2. humid temperate climates—one station for every 50,000 km^2, and
3. cold regions—One station for every 100,000 km^2.

Currently India has about 200 pan-evaporimeter stations maintained by the India Meteorological Department.

3.4 EMPIRICAL EVAPORATION EQUATIONS

A large number of empirical equations are available to estimate lake evaporation using commonly available meteorological data. Most formulae are based on the Dalton-type equation and can be expressed in the general form

$$E_L = Kf(u)\,(e_w - e_a) \tag{3.2}$$

where E_L = lake evaporation in mm/day, e_w = saturated vapour pressure at the water-surface temperature in mm of mercury, e_a = actual vapour pressure of overlying air at a specified height in mm of mercury, $f(u)$ = wind-speed correction function and K = a coefficient. The term e_a is measured at the same height at which wind speed is measured. Two commonly used empirical evaporation formulae are:

Meyer's Formula (1915)

$$E_L = K_M\,(e_w - e_a)\left(1 + \frac{u_9}{16}\right) \tag{3.3}$$

in which E_L, e_w, e_a are as defined in Eq. (3.2), u_9 = monthly mean wind velocity in km/h at about 9 m above ground and K_M = coefficient accounting for various other factors with a value of 0.36 for large deep waters and 0.50 for small, shallow waters.

Rohwer's Formula (1931)

Rohwer's formula considers a correction for the effect of pressure in addition to the wind-speed effect and is given by

$$E_L = 0.771\,(1.465 - 0.000732\,p_a)\,(0.44 + 0.0733\,u_0)\,(e_w - e_a) \tag{3.4}$$

in which E_L, e_w and e_a are as defined earlier in Eq. (3.2),

p_a = mean barometric reading in mm of mercury

u_0 = mean wind velocity in km/h at ground level, which can be taken to be the velocity at 0.6 m height above ground .

These empirical formulae are simple to use and permit the use of standard meteorological data. However, in view of the various limitations of the formulae, they can at best be expected to give an approximate magnitude of the evaporation. References 2 and 3 list several other popular empirical formulae.

In using the empirical equations, the saturated vapour pressure at a given temperature (e_w) is found from a table of e_w vs temperature in °C, such as Table 3.3. Often, the wind-velocity data would be available at an elevation other than that needed in the particular equation. However, it is known that in the lower part of the atmosphere, up to a height of about 500 m above the ground level, the wind velocity can be assumed to follow the 1/7 power law as

$$u_h = C\,h^{1/7} \tag{3.5}$$

where u_h = wind velocity at a height h above the ground and C = constant. This equation can be used to determine the velocity at any desired level if u_h is known.

EXAMPLE 3.1 *A reservoir with a surface area of 250 hectares had the following average values of parameters during a week: Water temperature= 20°C, relative humidity = 40%, wind velocity at 1.0 m above ground = 16 km/h. Estimate the average daily evaporation from the lake and the volume of water evaporated from the lake during that one week.*

From Table 3.3,

$e_w = 17.54$ mm of Hg

$e_a = 0.40 \times 17.54 = 7.02$ mm of Hg

u_9 = wind velocity at a height of 9.0 m above ground

$= u_1\,(9)^{1/7} = 16.0\,(9)^{1/7} = 21.9$ km/h

By Meyer's formula [Eq. (3.3)],

$$E_L = 0.36\,(17.54 - 7.02)\left(1 + \frac{21.9}{16}\right) = 8.97 \text{ mm/day}$$

$$\text{Evaporated volume in 7 days} = 7 \times \frac{8.97}{1000} \times 250 \times 10^4 = 157{,}000 \text{ m}^3$$

3.5 ANALYTICAL METHODS OF EVAPORATION ESTIMATION

The analytical methods for the determination of lake evaporation can be broadly classified into three categories as:

1. Water-budget method,
2. energy-balance method, and
3. mass-transfer method.

Water-Budget Method

The water-budget method is the simplest of the three analytical methods and is also the least reliable. It involves writing the hydrological continuity equation for the lake and determining the evaporation from a knowledge or estimation of other variables. Thus considering the daily average values for a lake, the continuity equation is written as

$$P + V_{is} + V_{ig} = V_{os} + V_{og} + E_L + \Delta S + T_L \qquad (3.6)$$

where P = daily precipitation

V_{is} = daily surface inflow into the lake

V_{ig} = daily groundwater inflow

V_{os} = daily surface outflow from the lake

V_{og} = daily seepage outflow

E_L = daily lake evaporation

ΔS = increase in lake storage in a day

T_L = daily transpiration loss

All quantities are in units of volume (m³) or depth (mm) over a reference area. Equation (3.6) can be written as

$$E_L = P + (V_{is} - V_{os}) + (V_{ig} - V_{og}) - T_L - \Delta S \qquad (3.7)$$

n this the terms P, V_{is}, V_{os} and ΔS can be measured. However, it is not

possible to measure V_{ig}, V_{og} and T_L and therefore these quantities can only be estimated. Transpiration losses can be considered to be insignificant in some reservoirs. If the unit of time is kept large, say weeks or months, better accuracy in the estimate of E_L is possible. In view of the various uncertainties in the estimated values and the possibilities of errors in measured variables, the water-budget method cannot be expected to give very accurate results. However, controlled studies such as at Lake Hefner in USA (1952) have given fairly accurate results by this method.

Energy-Budget Method

The energy-budget method is an application of the law of conservation of energy. The energy available for evaporation is determined by considering the incoming energy, outgoing energy and energy stored in the water body over a known time interval.

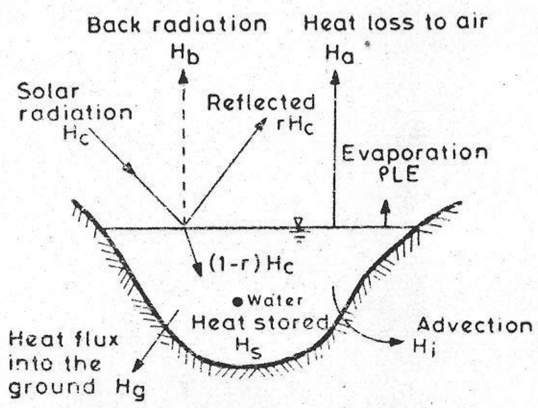

Fig. 3.4 Energy balance in a water body

Considering the water body as in Figure 3.4, the energy balance to the evaporating surface in a period of one day is given by

$$H_n = H_a + H_e + H_g + H_s + H_i \qquad (3.8)$$

where H_n = net heat energy received by the water surface

$= H_c\,(1-r) - H_b$

in which $H_c\,(1-r)$ = incoming solar radiation into a surface of reflection coefficient (albedo) r

H_b = back radiation (long wave) from water body

H_a = sensible heat transfer from water surface to air

H_e = heat energy used up in evaporation

$= \rho L E_L$, where ρ = density of water, L = latent heat of evaporation and E_L = evaporation in mm

H_g = heat flux into the ground

H_s = heat stored in water body

H_i = net heat conducted out of the system by water flow (advected energy)

All the energy terms are in calories per square mm per day. If the time periods are short, the terms H_s and H_i can be neglected as negligibly small. All the terms except H_a can either be measured or evaluated indirectly. The sensible heat term H_a, which cannot be readily measured is estimated using *Bowen's ratio* β given by the expression

$$\beta = \frac{H_a}{\rho L\, E_L} = 6.1 \times 10^{-4} \times p_a \frac{T_w - T_a}{e_w - e_a} \quad (3.9)$$

where p_a = atmospheric pressure in mm of mercury, e_w = saturated vapour pressure in mm of mercury, e_a = actual vapour pressure of air in mm of mercury, T_w = temperature of water surface in °C and T_a = temperature of air in °C. From Eqs (3.8) and (3.9) E_L can be evaluated as

$$E_L = \frac{H_n - H_g - H_s - H_i}{\rho L\,(1 + \beta)} \quad (3.10)$$

Estimation of evaporation in a lake by the energy balance method has been found to give satisfactory results, with errors of the order of 5%, when applied to periods less than a week. Further details of the energy-budget method are available in Refs 2, 3 and 5.

Mass-Transfer Method

This method is based on theories of turbulent mass transfer in boundary layer to calculate the mass water vapour transfer from the surface to the surrounding atmosphere. However, the details of the method are beyond the scope of this book and can be found in published literature[2,5]. With the use of quantities measured by sophisticated (and expensive) instrumentation, this method can give satisfactory results.

3.6 RESERVOIR EVAPORATION AND METHODS FOR ITS REDUCTION

Any of the methods mentioned above may be used for the estimation of reservoir evaporation. Although analytical methods provide better results, they involve parameters that are difficult to asses or expensive to obtain. Empirical equations can at best give approximate values of the correct order of magnitude. Therefore pan measurements find general acceptance for practical application. Mean monthly and annual evaporation data collected at more than 200 stations by IMD will prove to be very valuable

in field estimations. The water volume lost due to evaporation from a reservoir in a month is calculated as

$$V_E = A\, E_{pm}\, C_p \quad (3.11)$$

where V_E = volume of water lost in evaporation in a month (m^3)

A = average reservoir area during the month

E_{pm} = pan evaporation loss in metres in a month

= E_L in mm/day × No. of days in the month × 10^{-3}

C_p = relevant pan coefficient

Evaporation from a water surface is a continuous process. Typically under Indian conditions, evaporation loss from a water body is about 160 cm in a year with enhanced values in arid regions. The quantity of stored water lost by evaporation in a year is indeed considerable as the surface area of many natural and man-made lakes in the country are very large. A small-sized reservoir may have a surface area of about 2000 hectares while large reservoirs such as that at Nagarjuna Sagar have surface area of about 30,000 hectares (300 km^2). The surface areas and capacities of some Indian reservoirs are indicated in Table 3.2.

TABLE 3.2 SURFACE AREAS AND CAPACITIES OF SOME INDIAN RESERVOIRS

S. No.	Reservoir	River	State	Surface area at MRL in (km^2)	Gross capacity of the reservoir (× 10^6 m^3)
1	Hirakud	Mahanadi	Orissa	725	8141
2	Gandhi Sagar	Chambal	Madhya Pradesh	660	8450
3	Tungabhadra	Tungabhadra	Karnataka	378	4040
4	Nagarjuna Sagar	Krishna	Andhra Pradesh	285	11,315
5	Kadana	Mahi	Gujarat	172	1714
6	Bhakra	Sutlej	Punjab	169	9868
7	Panchet	Damodar	Bihar	153	1497
8	Matatila	Betwa	Uttar Pradesh	142	1132
9	Nizam Sagar	Manjira	Andhra Pradesh	130	715
10	Shivaji Sagar	Koyna	Maharashtra	115	2780

As the construction of various reservoirs as a part of the water-resources developmental effort involve considerable inputs of money, which is a scarce resource, evaporation from such bodies signifies an economic loss. In arid zones where water is scarce, the importance of conservation of water through reduction of evaporation is obvious.

The various methods available for reduction of evaporation losses can be considered under three categories:

(i) Reduction of Surface Area

Since the volume of water lost by evaporation is directly proportional to the surface area of the water body, the reduction of surface area wherever feasible reduces evaporation losses. Measures like having deep reservoirs in place of wider ones and elimination of shallow areas can be considered under this category.

(ii) Mechanical Covers

Permanent roofs over the reservoir, temporary roofs and floating roofs such as rafts and light-weight floating particles can be adopted wherever feasible. Obviously these measures are limited to very small water bodies such as ponds, etc.

(iii) Chemical Films

This method consists of applying a thin chemical film on the water surface to reduce evaporation. Currently this is the only feasible method available for reduction of evaporation of reservoirs up to moderate size.

Certain chemicals such as cetyl alcohol (hexadecanol) and stearyl alcohol (octadecanol) form monomolecular layers on a water surface. These layers act as evaporation inhibitors by preventing the water molecules to escape past them. The thin film formed has the following desirable features:

1. The film is strong and flexible and does not break easily due to wave action.
2. If punctured due to the impact of raindrops or by birds, insects, etc., the film closes back soon after.
3. It is pervious to oxygen and carbon dioxide; the water quality is therefore not affected by its presence.
4. It is colourless, odourless and nontoxic.

Cetyl alcohol is found to be the most suitable chemical for use as an evaporation inhibitor. It is a white, waxy, crystalline solid and is available as lumps, flakes or powder. It can be applied to the water surface in the form of powder, emulsion or solution in mineral turpentine. Roughly about 0.35 kg/hectare/day of cetyl alcohol is needed for effective action. The chemical is periodically replenished to make up the losses due to oxidation, wind sweep of the layer to the shore and its removal by birds and insects. Evaporation reduction can be achieved to a maximum if a film pressure of 4×10^{-2} N/m is maintained.

Controlled experiments with evaporation pans have indicated an evaporation reduction of about 60% through use of cetyl alcohol. Under field conditions, the reported values of evaporation reduction range from 20 to 50%. It appears that a reduction of 20-30% can be achieved easily in small size lakes ($\leq$ 1000 hectares) through the use of these monomolecular layers. The adverse effect of heavy wind appears to be the only major impediment affecting the efficiency of these chemical films.

EVAPOTRANSPIRATION

3.7 TRANSPIRATION

Transpiration is the process by which water leaves the body of a living plant and reaches the atmosphere as water vapour. The water is taken up by the plant-root system and escapes through the leaves. The important factors affecting transpiration are: atmospheric vapour pressure, temperature, wind, light intensity and characteristics of the plant, such as the root and leaf systems. For a given plant, factors that affect the free-water evaporation also affect transpiration. However, a major difference exists between transpiration and evaporation. Transpiration is essentially confined to daylight hours and the rate of transpiration depends upon the growth periods of the plant. Evaporation, on the other hand, continues all through the day and night although the rates are different.

3.8 EVAPOTRANSPIRATION

While transpiration takes place, the land area in which plants stand also lose moisture by the evaporation of water from soil and water bodies. In hydrology and irrigation practice, it is found that evaporation and transpiration processes can be considered advantageously under one head as evapotranspiration. The term *consumptive use* is also used to denote this loss by evapotranspiration. For a given set of atmospheric conditions, evapotranspiration obviously depends on the availability of water. If sufficient moisture is always available to completely meet the needs of vegetation fully covering the area, the resulting evapotranspiration is called *potential evapotranspiration* (PET). Potential evapotranspiration no longer critically depends on soil and plant factors but depends essentially on climatic factors. The real evapotranspiration occurring in a specific situation is called *actual evapotranspiration* (AET).

It is necessary to introduce at this stage two terms: *field capacity* and *permanent wilting point*. Field capacity is the maximum quantity of water that the soil can retain against the force of gravity. Any higher moisture input to a soil at field capacity simply drains away. Permanent wilting point is the moisture content of a soil at which the moisture is no longer available in sufficient quantity to sustain the plants. At this stage, even though the soil contains some moisture, it will be so held by the soil grains that the roots of the plants are not able to extract it in sufficient quantities to sustain the plants and consequently the plants wilt. The field capacity and permanent wilting point depend upon the soil characteristics. The difference between these two moisture contents is called *available water*, the moisture available for plant growth.

If the water supply to the plant is adequate, soil moisture will be at the field capacity and AET will be equal to PET. If the water supply is less than PET, the soil dries out and the ratio AET/PET would then be less than unity. The decrease of the ratio AET/PET with available moisture depends upon the type of soil and rate of drying. Generally, for clayey soils, AET/PET$\approx$1.0 for nearly 50% drop in the available moisture. As can be expected, when the soil moisture reaches the permanent wilting point, the AET reduces to zero (Fig. 3.5). For a catchment in a given period of time, the hydrologic budget can be written as

$$P - R_s - G_o - E_{act} = \Delta S \qquad (3.12)$$

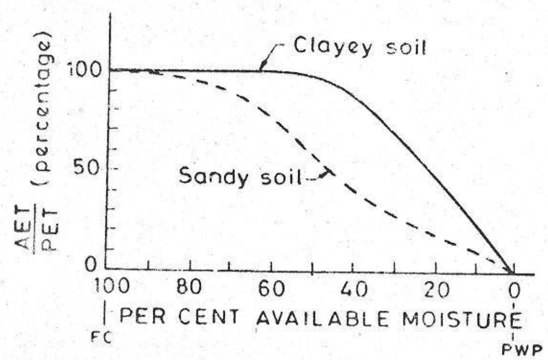

Fig. 3.5 Variation of AET

where P = precipitation, R_s = surface runoff, G_o = subsurface outflow, E_{act} = actual evapotranspiration (AET) and ΔS = change in the moisture storage. This water budgeting can be used to calculate E_{act} by knowing or estimating other elements of Eq. (3.12). The sum of R_s and G_o can be taken as the stream flow R at the basin outlet without much error.

Except in a few specialised studies, all applied studies in hydrology use PET for various estimation purposes. It is generally agreed that PET is a good approximation for lake evaporation.

3.9 MEASUREMENT OF EVAPOTRANSPIRATION

The measurement of evapotranspiration for a given vegetation type can be

carried out in two ways: either by using lysimeters or by the use of field plots.

Lysimeters

A lysimeter is a special watertight tank containing a block of soil and set in a field of growing plants. The plants grown in the lysimeter are the same as in the sorrounding field. Evapotranspiration is estimated in terms of the amount of water required to maintain constant moisture conditions within the tank measured either volumetrically or gravimetrically through an arrangement made in the lysimeter. Lysimeters should be designed to accurately reproduce the soil conditions, moisture content, type and size of the vegetation of the surrounding area. They should be so burried that the soil is at the same level inside and outside the container. Lysimeter studies are time-consuming and expensive.

Field Plots

In special plots all the elements of the water budget in a known interval of time are measured and the evapotranspiration determined as

Evapotranspiration = [precipitation + irrigation input − runoff − increase in soil storage − groundwater loss]

Measurements are usually confined to precipitation, irrigation input, surface runoff and soil moisture. Groundwater loss due to deep percolation is difficult to measure and can be minimised by keeping the moisture condition of the plot at the field capacity. This method provides fairly reliable results.

3.10 EVAPOTRANSPIRATION EQUATIONS

The lack of reliable field data and the difficulties of obtaining reliable evaportranspiration data have given rise to a number of methods to predict PET by using climatological data. Large number of formulae are available; they range from purely empirical ones to those backed by theoretical concepts. Two useful equations are given below.

Penman's Equation

Penman's equation is based on sound theoretical reasoning and is obtained by a combination of the energy-balance and mass-transfer approach. Penman's equation, incorporating some of the modifications suggested by other investigators is

$$\text{PET} = \frac{A H_n + E_a \gamma}{A + \gamma} \qquad (3.13)$$

where PET = daily potential evapotranspiration in mm per day

A = slope of the saturation vapour pressure vs temperature curve at the mean air temperature, in mm of mercury per °C (Table 3.3)

H_n = net radiation in mm of evaporable water per day

E_a = parameter including wind velocity and saturation deficit

γ = psychrometric constant = 0.49 mm of mercury/°C

The net radiation H_n is the same as used in the energy budget [Eq. (3.8)] and is estimated by the following equation:

$$H_n = H_a(1-r)\left(a + b\frac{n}{N}\right) - \sigma T_a^4 (0.56 - 0.092\sqrt{e_a}) \times \left(0.10 + 0.90\frac{n}{N}\right) \quad (3.14)$$

where H_a = incident solar radiation outside the atmosphere on a horizontal surface, expressed in mm of evaporable water per day (it is a function of the latitude and period of the year as indicated in Table 3.4).

r = reflection coefficient (albedo). Usual ranges of values of r are given below.

Surface	range of r values
Close ground crops	0.15–0.25
Bare lands	0.05–0.45
Water surface	0.05
Snow	0.45–0.90

a = a constant depending upon the latitude ϕ and is given by $a = 0.29\cos\phi$

b = a constant with an average value of 0.52

n = actual duration of bright sunshine in hours

N = maximum possible hours of bright sunshine (it is a function of latitude as indicated in Table 3.5)

σ = Stefan-Boltzman constant = 2.01×10^{-9} mm/day

T_a = mean air temperature in degrees kelvin = 273 + °C

e_a = actual mean vapour pressure in the air in mm of mercury

The parameter E_a is estimated as

$$E_a = 0.35\left(1 + \frac{u_2}{160}\right)(e_w - e_a) \quad (3.15)$$

in which

u_2 = mean wind speed at 2 m above ground in km/day

e_w = saturation vapour pressure at mean air temperature in mm of mercury (Table 3.3)

e_a = actual vapour pressure, defined earlier

For the computation of PET, data on n, e_a, u_2, mean air temperature and nature of surface (i.e. value of r) are needed. These can be obtained from actual observations or through available meteorological data of the region. Equations (3.13), (3.14) and (3.15) together with Tables 3.3, 3.4 and 3.5 enable the daily PET to be calculated. It may be noted that Penman's equation can be used to calculate evaporation from a water surface by using r = 0.05. Penman's equation is widely used in India, the UK, Australia and in some parts of USA. Further details about this equation are available elsewhere[2, 4, 5].

TABLE 3.3 SATURATION VAPOUR PRESSURE OF WATER

Temperature (°C)	Saturation vapour pressure e_w (mm of Hg)	A (mm/°C)
0	4.58	0.30
5.0	6.54	0.45
7.5	7.78	0.54
10.0	9.21	0.60
12.5	10.87	0.71
15.0	12.79	0.80
17.5	15.00	0.95
20.0	17.54	1.05
22.5	20.44	1.24
25.0	23.76	1.40
27.5	27.54	1.61
30.0	31.82	1.85
32.5	36.68	2.07
35.0	42.81	2.35
37.5	48.36	2.62
40.0	55.32	2.95
45.0	71.20	3.66

TABLE 3.4 MEAN MONTHLY SOLAR RADIATION AT TOP OF ATMOSPHERE, H_a IN mm OF EVAPORABLE WATER/DAY

North latitude	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0°	14.5	15.0	15.2	14.7	13.9	13.4	13.5	14.2	14.9	15.0	14.6	14.3
10°	12.8	13.9	14.8	15.2	15.0	14.8	14.8	15.0	14.9	14.1	13.1	12.4
20°	10.8	12.3	13.9	15.2	15.7	15.8	15.7	15.3	14.4	12.9	11.2	10.3
30°	8.5	10.5	12.7	14.8	16.0	16.5	16.2	15.3	13.5	11.3	9.1	7.9
40°	6.0	8.3	11.0	13.9	15.9	16.7	16.3	14.8	12.2	9.3	6.7	5.4
50°	3.6	5.9	9.1	12.7	15.4	16.7	16.1	13.9	10.5	7.1	4.3	3.0

TABLE 3.5 MEAN MONTHLY VALUES OF POSSIBLE SUNSHINE HOURS, N

North latitude	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0°	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1	12.1
10°	11.6	11.8	12.1	12.4	12.6	12.7	12.6	12.4	12.9	11.9	11.7	11.5
20°	11.1	11.5	12.0	12.6	13.1	13.3	13.2	12.8	12.3	11.7	11.2	10.9
30°	10.4	11.1	12.0	12.9	13.7	14.1	13.9	13.2	12.4	11.5	10.6	10.2
40°	9.6	10.7	11.9	13.2	14.4	15.0	14.7	13.8	12.5	11.2	10.0	9.4
50°	8.6	10.1	11.8	13.8	15.4	16.4	16.0	14.5	12.7	10.8	9.1	8.1

EXAMPLE 3.2 *Calculate the potential evapotranspiration from an area near New Delhi in the month of November by Penman's formula. The following data are available:*

Latitude	:	*28°4′N*
Elevation	:	*230 maSL*
Mean monthly temperature	:	*19°C*
Mean relative humidity	:	*75%*
Mean observed sunshine hours	:	*9 h*
Wind velocity at 2 m height	:	*85 km/day*
Nature of surface cover	:	*Close-ground green crop*

From Table 3.3,

$$A = 1.00 \text{ mm/°C}$$

$$e_w = 16.50 \text{ mm of Hg}$$

From Table 3.4

$$H_a = 9.506 \text{ mm of water/day}$$

From Table 3.5

$$N = 10.716 \text{ h}$$

$$n/N = 9/10.716 = 0.84$$

From given data:

$$e_a = 16.50 \times 0.75 = 12.38 \text{ mm of Hg}$$

$$a = 0.29 \cos 28° 4' = 0.2559$$

$$b = 0.52$$

$$\sigma = 2.01 \times 10^{-9} \text{ mm/day}$$

$$T_a = 273 + 19 = 292 \text{ K}$$

$$\sigma T_a^4 = 14.613$$

r = albedo for close-ground green crop is taken as 0.25

From Eq. (3.14),

$$H_n = 9.506 \times (1-0.25) \times (0.2559 + (0.52 \times 0.84)) - 14.613 \times (0.56 - 0.092\sqrt{12.38)} \times (0.10 + (0.9 \times 0.84))$$

$$= 4.936 - 2.946$$

$$= 1.990 \text{ mm of water/day}$$

From Eq. (3.15),

$$E_a = 0.35 \times \left(1 + \frac{85}{160}\right) \times (16.50 - 12.38)$$

$$= 2.208 \text{ mm/day}$$

From Eq. (3.13), noting the value of $\gamma = 0.49$

$$\text{PET} = \frac{(1 \times 1.990) + (2.208 \times 0.49)}{(1.00 + 0.49)} = 2.06 \text{ mm/day}$$

EXAMPLE 3.3 *Using the data of Example 3.2, estimate the daily evaporation from a lake situated in that place.*

For estimating the daily evaporatiom from a lake, Penman's equation is used with the albedo $r = 0.05$.

Hence

$$H_n = (4.936) \times \frac{(1.0-0.05)}{(1.0-0.25)} - 2.946$$

$$= 6.252 - 2.946 = 3.306 \text{ mm of water/day}$$

$$E_a = 2.224 \text{ mm/day}$$

From Eq.(3.13),

$$PET = \text{Lake evaporation}$$

$$= \frac{(1.0 \times 3.306) + (2.224 \times 0.49)}{(1.0 + 0.49)}$$

$$= 2.95 \text{ mm/day}$$

Empirical Formulae

A large number of empirical formulae are available for the estimation of PET based on climatological data. These are not universally applicable to all climatic areas. They should be used with caution in areas different from those for which they were derived.

Blaney-Criddle Formula

This is a purely empirical formula based on data from arid western United States. This formula assumes that the PET is related to hours of sunshine and temperature, which are taken as measures of solar radiation at an area. The potential evapotranspiration in a crop-growing season is given by

$$E_T = 2.54\, K\, F$$

and

$$F = \Sigma\, p_h\, \bar{T}_f/100 \qquad (3.16)$$

where E_T = PET in a crop season in cm

K = an empirical coefficient, depends on the type of the crop

F = sum of monthly consumptive use factors for the period

p_h = monthly percent of annual day-time hours, depends on the latitude of the place (Table 3.6)

and $\bar{T}_f$ = mean monthly temperature in °F

Values of K depend on the month and locality. Average value for the season for selected crops is given in Table 3.7. The Blaney-Criddle formula is largely used by irrigation engineers to calculate the water requirements of crops, which is taken as the difference between PET and effective precipitation. Blaney-Morin equation is another empirical formula similar to Eq. (3.16) but with an additional correction for humidity.

EXAMPLE 3.4 *Estimate the PET of an area for the season November to February in which wheat is grown. The area is in North India at a latitude of 30°N with mean monthly temperatures as below:*

Month	Nov	Dec	Jan	Feb
Temp. (°C)	16.5	13.0	11.0	14.5

Use the Blaney-Criddle formula.

TABLE 3.6 MONTHLY DAYTIME HOURS PERCENTAGES, p_h, FOR USE IN BLANEY-CRIDDLE FORMULA (EQ. 3.16)

North latitude (degrees)	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	8.50	7.66	8.49	8.21	8.50	8.22	8.50	8.49	8.21	8.50	8.22	8.50
10	8.13	7.47	8.45	8.37	8.81	8.60	8.86	8.71	8.25	8.34	7.91	8.10
15	7.94	7.36	8.43	8.44	8.98	8.80	9.05	8.83	8.28	8.26	7.75	7.88
20	7.74	7.25	8.41	8.52	9.15	9.00	9.25	8.96	8.30	8.18	7.58	7.66
25	7.53	7.14	8.39	8.61	9.33	9.23	9.45	9.09	8.32	8.09	7.40	7.42
30	7.30	7.03	8.38	8.72	9.53	9.49	9.67	9.22	8.33	7.99	7.19	7.15
35	7.05	6.88	8.35	8.83	9.76	9.77	9.93	9.37	8.36	7.87	6.97	6.86
40	6.76	6.72	8.33	8.95	10.02	10.08	10.22	9.54	8.39	7.75	6.72	6.52

TABLE 3.7 VALUES OF K FOR SELECTED CROPS FOR USE IN EQ. (3.16)

Crop	Value of K	Range of monthly values
Rice	1.10	0.85–1.30
Wheat	0.65	0.50–0.75
Maize	0.65	0.50–0.80
Sugarcane	0.90	0.75–1.00
Cotton	0.65	0.50–0.90
Potatoes	0.70	0.65–0.75
Natural Vegetation:		
(*a*) Very dense	1.30	
(*b*) Dense	1.20	
(*c*) Medium	1.00	
(*d*) Light	0.80	

From Table 3.7, for wheat K = 0.65. Values of p_h for 30°N is read from Table 3.6, the temperatures are converted to Fahrenheight and the calculations are performed in the following table.

Month	$\bar{T}_f$ (°F)	p_h	$p_h\bar{T}_f/100$
Nov.	61.7	7.19	4.44
Dec.	55.4	7.15	3.96
Jan.	51.8	7.30	3.78
Feb.	58.1	7.03	4.08
		$\Sigma\, p_h \bar{T}_f/100 =$	16.26

By Eq. (3.16),

$$E_T = 2.54 \times 16.26 \times 0.65 = 26.85 \text{ cm.}$$

Thornthwaite Formula

This formula was developed from data of eastern USA and uses only the mean monthly temperature together with an adjustment for day-lengths. The PET is given by this formula as

$$ET = 1.6\, L_a \left(\frac{10\,\bar{T}}{I_t}\right)^a \tag{3.17}$$

where ET = monthly PET in cm

L_a = adjustment for the number of hours of daylight and days in the month, related to the latitude of the place (Table 3.8)

$\bar{T}$ = mean monthly air temperature °C

I_t = the total of 12 monthly values of heat index $i = \sum_{1}^{12} i$,

where $i = (\bar{T}/5)^{1.514}$

a = an empirical constant

$= 6.75 \times 10^{-7}\, I_t^3 - 7.71 \times 10^{-5}\, I_t^2 + 1.792 \times 10^{-2}\, I_t + 0.49239$

TABLE 3.8 ADJUSTMENT FACTOR L_a FOR USE IN THORNTHWAITE FORMULA (Eq. 3.17)

North latitude degrees	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
0	1.04	0.94	1.04	1.01	1.04	1.01	1.04	1.04	1.01	1.04	1.01	1.04
10	1.00	0.91	1.03	1.03	1.08	1.06	1.08	1.07	1.02	1.02	0.98	0.99
15	0.97	0.91	1.03	1.04	1.11	1.08	1.12	1.08	1.02	1.01	0.95	0.97
20	0.95	0.90	1.03	1.05	1.13	1.11	1.14	1.11	1.02	1.00	0.93	0.94
25	0.93	0.89	1.03	1.06	1.15	1.14	1.17	1.12	1.02	0.99	0.91	0.91
30	0.90	0.87	1.03	1.08	1.18	1.17	1.20	1.14	1.03	0.98	0.89	0.88
40	0.84	0.83	1.03	1.11	1.24	1.25	1.27	1.18	1.04	0.96	0.83	0.81

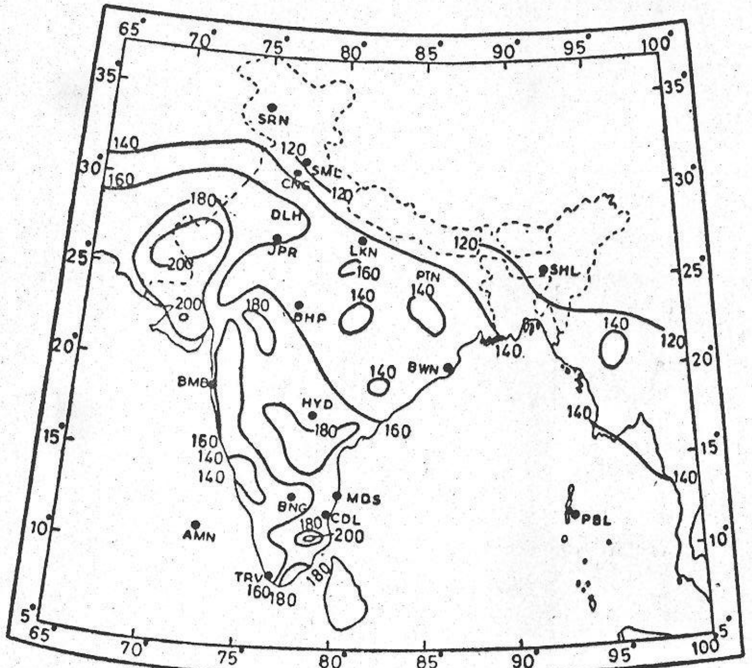

Fig. 3.6 (a) Annual PET (cm) over India

(Source: *Scientific Report No. 136*, IMD, 1971, © Government of India Copyright)

Based upon Survey of India map with the permission of the Surveyor General of India

The territorial waters of India extend into the sea to a distance of twelve nautical miles measured from the appropriate baseline

Responsibility for the correctness of internal details on the map rests with the publisher.

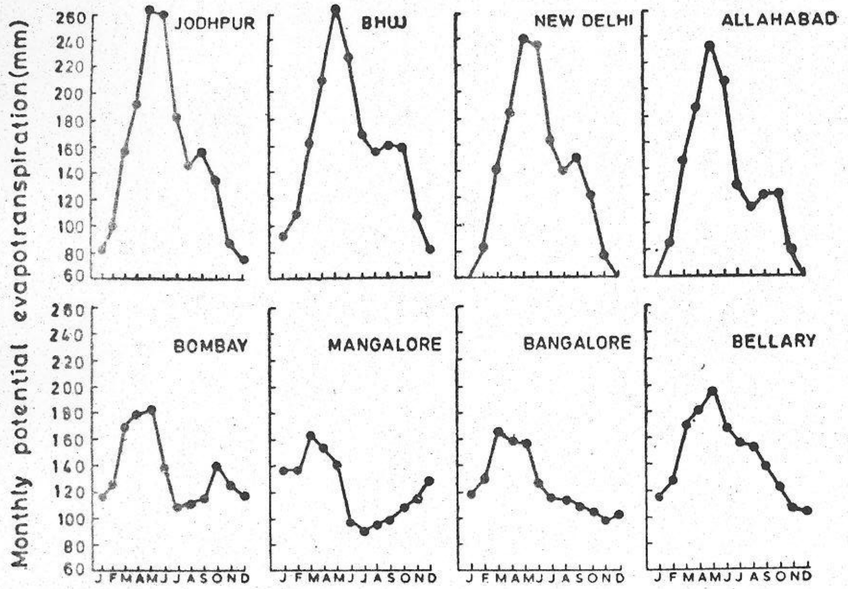

Fig. 3.6 (b) Monthly variation of PET (mm)

(Source: *Scientific Report No. 136*, India Meteorological Department, 1971, © Government of India Copyright)

3.11 POTENTIAL EVAPOTRANSPIRATION OVER INDIA

Using Penman's equation and the available climatalogical data, PET estimate[4] for the country has been made. The mean annual PET (in cm) over various parts of the country is shown in the form of *isopleths*—the lines on a map through places having equal depths of evapotranspiration [Figure 3.6 (a)]. It is seen that the annual PET ranges from 140 to 180 cm over most parts of the country. The annual PET is highest at Rajkot, Gujarat with a value of 214.5 cm. Extreme south-east of Tamil Nadu also show high average values greater than 180 cm. The highest PET for southern peninsula is at Tiruchirapalli, Tamil Nadu with a value of 209 cm. The variation of monthly PET at stations located in various climatic zones in the country is shown in Fig. 3.6 (b). Valuable PET data relevant to various parts of the country are available in Ref. 4.

INITIAL LOSS

In the precipitation reaching the surface of a catchment the major abstraction is from the infiltration process. However, two other processes, though small in magnitude, operate to reduce the water volume available for runoff and thus act as abstractions. These are (i) the interception process and (ii) the depression storage, and together they are called *initial loss*. This abstraction represents the quantity of storage that must be satisfied before overland runoff begins. The following two sections deal with these two processes briefly.

3.12 INTERCEPTION

When it rains over a catchment not all the precipitation falls directly onto the ground. Before it reaches the ground, a part of it may be caught by the vegetation and subsequently evaporated. The volume of water so caught is called interception. The intercepted precipitation may follow one of the three possible routes:

1. It may be retained by the vegetation as surface storage and returned to the atmosphere by evaporation; a process termed *interception loss*;
2. it can drip off the plant leaves to join the ground surface or the surface flow; this is known as *throughfall*; and
3. the rainwater may run along the leaves and branches and down the stem to reach the ground surface. This part is called *stemflow*.

Interception loss is solely due to evaporation and does not include transpiration, throughfall or stemflow.

The amount of water intercepted in a given area is extremely difficult to measure. It depends on the species composition of vegetation, its density and also on the storm characteristics. It is estimated that of the total rainfall in an area during a plant-growing season the interception loss is about 10 to 20%. Interception is satisfied during the first part of a storm and if an area experiences a large number of small storms, the annual interception loss due to forests in such cases will be high, amounting to greater than 25% of the annual precipitation. Quantitatively, the variation of interception loss with the rainfall magnitude per storm for small storms is as shown in Fig. 3.7. It is seen that the interception loss is large for a small rainfall and levels off to a constant value for larger storms. For a given storm, the interception loss is estimated as

$$I_i = S_i + K_i Et \tag{3.18}$$

where I_i = interception loss in mm, S_i = interception storage whose value varies from 0.25 to 1.25 mm depending on the nature of vegetation, K_i = ratio of vegetal surface area to its projected area, E = evaporation rate in mm/h during the precipitation and t = duration of rainfall in hours.

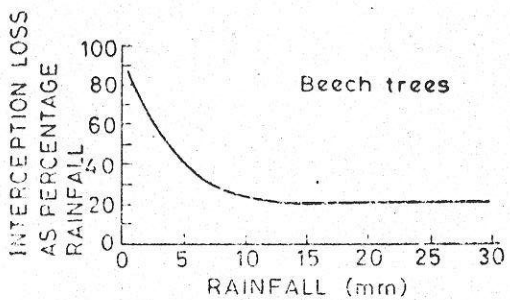

Fig. 3.7 Typical interception loss curve

It is found that coniferous trees have more interception loss than deciduous ones. Also, dense grasses have nearly same interception losses as full-grown trees and can account for nearly 20% of the total rainfall in a season. Agricultural crops in their growing season also contribute high interception losses. In view of these the interception process has a very significant impact on the ecology of the area related to silvicultural aspects and in the water balance of a region. However, in hydrological studies dealing with floods interception loss is rarely significant and is not separately considered. The common practice is to allow a lump sum value as the initial loss to be deducted from the initial period of the storm.

3.13 DEPRESSION STORAGE

When the precipitation of a storm reaches the ground, it must first fill up all depressions before it can flow over the surface. The volume of water

trapped in these depressions is called *depression storage*. This amount is eventually lost to runoff through processes of infiltration and evaporation and thus form a part of the initial loss. Depression storage depends on a vast number of factors the chief of which are : (i) the type of soil, (ii) the condition of the surface reflecting the amount and nature of depression, (iii) the slope of the catchment and (iv) the antecedent precipitation, as a measure of the soil moisture. Obviously, general expressions for quantitative estimation of this loss are not available. Qualitatively, it has been found that antecedent precipitation has a very pronounced effect on decreasing the loss to runoff in a storm due to depression. Values of 0.50 cm in sand, 0.4 cm in loam and 0.25 cm in clay can be taken as representatives for depression-storage loss during intensive storms.

INFILTRATION

3.14 INFILTRATION PROCESS

It is well-known that when water is applied to the surface of a soil, a part of it seeps into the soil. This movement of water through the soil surface is known as *infiltration* and plays a very significant role in the runoff process by affecting the timing, distribution and magnitude of the surface runoff. Further, infiltration is the primary step in the natural groundwater recharge.

Infiltration is the flow of water into the ground through the soil surface and the process can be easily understood through a simple analogy. Consider a small container covered with wire gauze as in Fig. 3.8. If water is poured over the gauze, a part of it will go into the container and a part overflows. Further, the container can hold only a fixed quantity and when it is full no more flow into the container can take place. This analogy, though a highly simplified one, underscores two important aspects, viz., (i) the maximum rate at which the ground can absorb water, the *infiltration capacity* and (ii) the volume of water that it can hold, *the field capacity*.

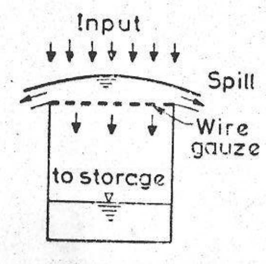

Fig. 3.8 An analogy for infiltration

Since the infiltered water may contribute to groundwater discharge in addition to increasing the soil moisture, the process can be schematically modelled as in Fig. 3.9(a) and (b). This figure considers two situations, viz. low-intensity rainfall and high-intensity rainfall, and is self-explanatory.

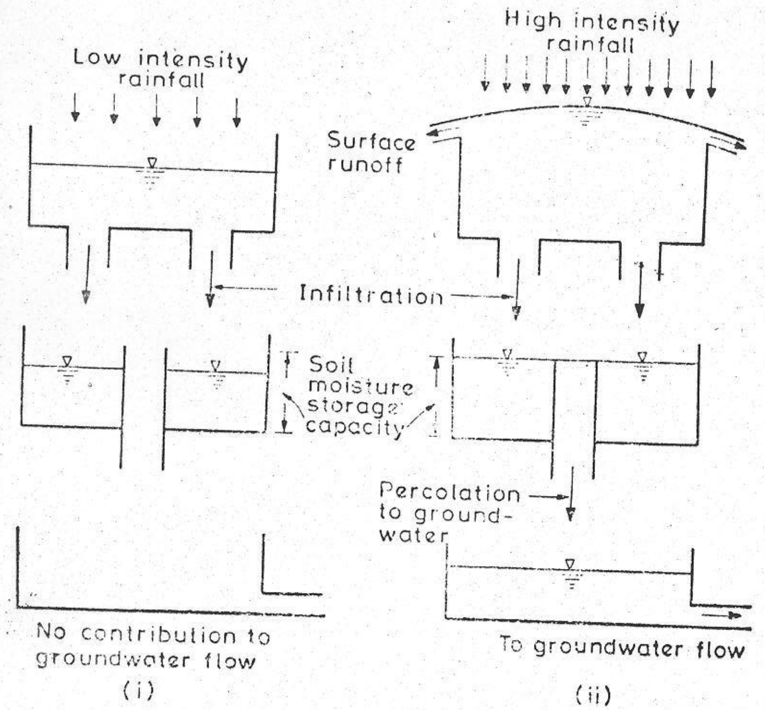

Fig. 3.9 An infiltration model

3.15 INFILTRATION CAPACITY

The maximum rate at which a given soil at a given time can absorb water is defined as the *infiltration capacity*. It is designated as f_c and is expressed in units of cm/h. The actual rate of infiltration f can be expressed as

$$f = f_c \text{ when } i > f_c$$

and

$$f = i \text{ when } i < f_c \qquad (3.19)$$

where i = intensity of rainfall. The infiltration capacity of a soil is high at the beginning of a storm and has an exponential decay as the time elapses. The infiltration process is affected by a large number of factors and a few important ones affecting f_c are described below.

Characteristics of Soil

The type of soil, viz. sand, silt or clay, its texture, structure, permeability and underdrainage are the important characteristics under this category. A loose, permeable, sandy soil will have a larger infiltration capacity than a tight, clayey soil. A soil with good underdrainage, i.e. the facility to transmit the infiltered water downward to a groundwater storage would obviously have a higher infiltration capacity. When the soils occur in layers, the transmission capacity of the layers determine the overall infiltration rate. Also, a dry soil can absorb more water than one whose pores are already full. The landuse has a significant influence on f_c. For example, a forest soil rich in organic matter will have a much higher value of f_c under identical conditions than the same soil in an urban area where it is subjected to compaction.

Surface of Entry

At the soil surface, the impact of raindrops causes the fines in the soils to be displaced and these in turn can clog the pore spaces in the upper layers. This is an important factor affecting the infiltration capacity. Thus a surface covered by grass and other vegetation which can reduce this process has a pronounced influence on the value of f_c.

Fluid Characteristics

Water infiltrating into the soil will have many impurities, both in solution and in suspension. The turbidity of the water, especially the clay and colloid content is an important factor as such suspended particles block the fine pores in the soil and reduce its infiltration capacity. The temperature of the water is a factor in the sense that it affects the viscosity of the water which in turn affects the infiltration rate. Contamination of the water by dissolved salts can affect the soil structure and in turn affect the infiltration rate.

3.16 MEASUREMENT OF INFILTRATION

Information about the infiltration characteristics of the soil at a given location can be obtained by conducting controlled experiments on small areas. The experimental set-up is called an *infiltrometer*. There are two kinds of infiltrometers :

1. Flooding-type infiltrometer, and
2. Rainfall simulator.

These are described below.

Flooding-Type Infiltrometer

This is a simple instrument consisting essentially of a metal cylinder, 30 cm diameter and 60 cm long, open at both ends. This cylinder is driven into the ground to a depth of 50 cm (Fig. 3.10). Water is poured into the top part to a depth of 5 cm and a pointer is set to mark the water level. As

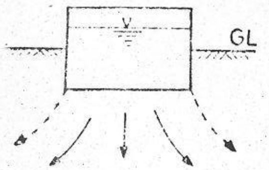

Fig. 3.10 Simple infiltrometer

infiltration proceeds, the volume is made up by adding water from a burette to keep the water level at the tip of the pointer. Knowing the volume of water added at different time intervals, the plot of the infiltration capacity vs time is obtained. The experiments are continued till a uniform rate of infiltration is obtained and this may take 2–3 h. The surface of the soil is usually protected by a perforated disk to prevent formation of turbidity and its settling on the soil surface.

A major objection to the simple infiltrometer as above is that the infiltered water spreads at the outlet from the tube (as shown by dotted lines in Fig. 3.10) and as such the tube area is not representative of the area in which infiltration takes place. To overcome this a ring infiltrometer consisting of a set of two concentric rings (Fig. 3.11) is used. In this two rings are inserted into the ground and water is maintained on the soil surface, in both the rings, to a common fixed level. The outer ring provides a water jacket to the infiltering water of the inner ring and hence prevents the spreading out of the infiltering water of the inner tube. The measurements of water volume is done on the inner ring only.

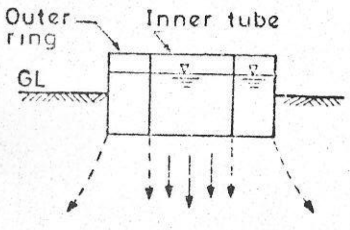

Fig. 3.11 Ring infiltrometer

The main disadvantages of flooding-type infiltrometers are :

1. the raindrop-impact effect is not simulated;
2. the driving of the tube or rings disturbs the soil structure; and
3. the results of the infiltrometer depend to some extent on their size with the larger meters giving less rates than the smaller ones; this is due to the border effect.

Rainfall Simulator

In this a small plot of land, of about 2 m × 4 m size, is provided with a series of nozzles on the longer side with arrangements to collect and measure the surface runoff rate. The specially designed nozzles produce raindrops falling from a height of 2 m and are capable of producing various intensities of rainfall. Experiments are conducted under controlled conditions with various combinations of intensities and durations and the surface runoff is measured in each case. Using the water-budget equation involving the volume of rainfall, infiltration and runoff, the infiltration rate and its variation with time is calculated. If the rainfall intensity is higher than the infiltration rate, infiltration-capacity values are obtained.

Rainfall simulator type infiltrometers given lower values than flooding-type infiltrometers. This is due to the effect of the rainfall impact and turbidity of the surface water present in the former.

3.17 INFILTRATION-CAPACITY VALUES

The typical variation of the infiltration capacity for two soils and for two initial conditions is shown in Fig. 3.12. It is clear from the figure that the infiltration capacity for a given soil decreases with time from the start of rainfall; it decreases with the degree of saturation and depends upon the type of soil. Horton (1930) expressed the decay of the infiltration capacity with time as

$$f_{ct} = f_{cf} + (f_{c0} - f_{cf})\, e^{-K_h t} \quad \text{for } 0 \leqslant t \leqslant t_d \qquad (3.20)$$

where f_{ct} = infiltration capacity at any time t from start of the rainfall

f_{c0} = initial infiltration capacity at $t = 0$

f_{cf} = final steady state value

t_d = duration of the rainfall and

K_h = constant depending upon the soil characteristics and vegetation cover.

The difficulty of finding the variation of the three parameters f_{c0}, f_{cf} and K_h with soil characteristics and antecedent moisture conditions precludes the general use of Eq. (3.20).

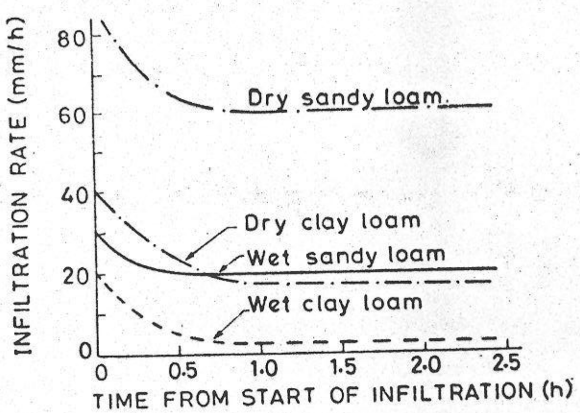

Fig. 3.12 Variation of infiltration capacity

It is apparent that infiltration-capacity values of soils are subjected to wide variations depending upon a large number of factors. Typically, a bare, sandy area will have $f_c \approx 1.2$ cm/h and a bare, clay soil will have

$f_c \approx 0.15$ cm/h. A good grass cover or vegetation cover increases these values by as much as 10 times.

3.18 INFILTRATION INDICES

In hydrological calculations involving floods it is found convenient to use a constant value of infiltration rate for the duration of the storm. The average infiltration rate is called *infiltration index* and two types of indices are in common use.

The ϕ index is the average rainfall above which the rainfall volume is equal to the runoff volume. The ϕ index is derived from the rainfall hyetograph with the knowledge of the resulting runoff volume. The initial loss is also considered as infiltration. The ϕ value is found by treating it as a constant infiltration capacity. If the rainfall intensity is less than ϕ, then the infiltration rate is equal to the rainfall intensity; however, if the rainfall intensity is larger than ϕ the difference between rainfall and infiltration in an interval of time represents the runoff volume (Fig. 3.13). The amount of rainfall in excess of the ϕ index is called *rainfall excess.* The ϕ index thus accounts for the total abstraction and enables runoff magnitudes to be estimated for a given rainfall hyetograph. Example 3.5 illustrates the calculation of the ϕ index.

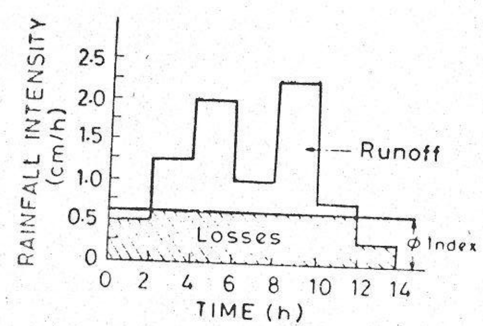

Fig. 3.13 ϕ **Index**

EXAMPLE 3.5: *A storm with 10.0 cm precipitation produced a direct runoff of 5.8 cm. Given the time distribution of the storm as below, estimate the ϕ index of the storm.*

Time from start (h)	1	2	3	4	5	6	7	8
Incremental rainfall in each hour (cm)	0.4	0.9	1.5	2.3	1.8	1.6	1.0	0.5

Total infiltration = 10.0 − 5.8 = 4.2 cm

Assume t_e = time of rainfall excess = 8 h for the first trial.

Then

$$\phi = \frac{4.2}{8} = 0.525 \text{ cm/h}$$

But this value of ϕ makes the rainfalls of the first hour and eighth hour ineffective as their magnitude is less than 0.525 cm/h. The value of t_e is therefore modified.

Assume t_e = 6 h for the second trial

In this period,

Infiltration = (10.0 − 0.4 − 0.5 − 5.8)

= 3.3 cm

$$\phi = \frac{3.3}{6} = 0.55 \text{ cm/h}$$

This value of ϕ is satisfactory as it gives t_e = 6 h and by calculating the rainfall excesses.

Time from start (h)	1	2	3	4	5	6	7	8
Rainfall excess (cm)	0	0.35	0.95	1.75	1.25	1.05	0.45	0

Total rainfall excess = 5.8 cm = total runoff.

In an attempt to refine the ϕ index the initial losses are separated from the total abstractions and an average value of infiltration rate called the W index is defined as

$$W = \frac{P - R - I_a}{t_e} \tag{3.21}$$

where P = total storm precipitation (cm)

R = total storm runoff (cm)

I_a = initial losses (cm)

t_e = duration of the rainfall excess, i.e. the total time in which the rainfall intensity is greater than W (in hours) and

W = average rate of infiltration (cm/h).

Since I_a values are difficult to obtain, the accurate estimation of the W index is rather difficult. The minimum value of the W index obtained under very wet soil conditions, representing the constant minimum rate of infiltration of the catchment, is known as W_{min}. Both the W index and ϕ index vary from storm to storm.

The ϕ index during a storm for a catchment depends in general upon the soil type, vegetal cover, initial moisture condition, storm duration and intensity. To obtain complete information on the interrelationship between these factors, a detailed and expensive study of a catchment is necessary. For practical use in the estimation of flood magnitudes due to critical storms, a simplified relationship for ϕ is adopted. As the maximum flood peaks are invariably produced due to long storms and usually in the wet season, the initial losses are assumed to be negligibly small. Further, only the soil type and rainfall are found to be critical in the estimate of the ϕ index for maximum flood-producing storms.

On the basis of rainfall and runoff correlations, CWC[1] has found the following relationship for the estimation of the ϕ index for flood producing storms and soil conditions prevalent in India:

$$R = \alpha\, I^{1.2} \qquad (3.22)$$

$$\text{and } \phi = \frac{I-R}{24} \qquad (3.22\text{-a})$$

where R = runoff in cm from a 24-h rainfall of intensity I cm/day and α = a coefficient which depends upon the soil type as below.

S. No.	Type of soil	Coefficient α
1	Sandy soils and sandy loam	0.17 to 2.25
2	Coastal alluvium and silty loam	0.25 to 0.34
3	Red soils, clayey loam, grey and brown alluvium	0.42
4	Black-cotton and clayey soils	0.42 to 0.46
5	Hilly soils	0.46 to 0.50

In estimating the maximum floods for design purposes, in the absence of any other data, a ϕ-index value of 0.10 cm/h can be assumed.

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PROBLEMS

3.1 A class A pan was set up adjacent to a lake. The depth of water in the pan at the beginning of a certain week was 195 mm. In that week there was a rainfall of 45 mm and 15 mm of water was removed from the pan to keep the water level within the specified depth range. If the depth of the water in the pan at the end of the week was 190 mm calculate the pan evaporation. Using a suitable pan coefficient estimate the lake evaporation in that week.

3.2 A reservoir has an average area of 50 km² over an year. The normal annual rainfall at the place is 120 cm and the class A pan evaporation is 240 cm. Assuming the land flooded by the reservoir has a runoff coefficient of 0.4, estimate the net annual increase or decrease in the streamflow as a result of the reservoir.

3.3 At a reservoir in the neighbourhood of Delhi the following climatic data were observed. Estimate the mean monthly and annual evaporation from the reservoir using Meyer's formula.

Month	Temp. (°C)	Relative humidity (%)	Wind velocity at 2 m above GL (km/h)
Jan	12.5	85	4.0
Feb	15.8	82	5.0
Mar	20.7	71	5.0
Apr	27.0	48	5.0
May	31.0	41	7.8
June	33.5	52	10.0
July	30.6	78	8.0
Aug	29.0	86	5.5
Sept	28.2	82	5.0
Oct	28.8	75	4.0
Nov	18.9	77	3.6
Dec	13.7	73	4.0

3.4 For the lake in Prob. 3.3, estimate the evaporation in the month of June by (a) Penman formula and (b) Thornthwaite equation by assuming that the lake evaporation is the same as PET, given latitude = 28°N and elevation = 230 m above MSL. Mean observed sunshine 9 h/day.

3.5 A reservoir had an average surface area of 20 km² during June 1982. In that month the mean rate of inflow = 10 m³/s, mean outflow = 15 m³/s, monthly rainfall = 10 cm and change in storage = 16 million m³. Assuming the seepage losses to be 1.8 cm, estimate the evaporation in that month.

3.6 For an area in South India (latitude = 12°N), the mean monthly temperatures are given

Month	June	July	Aug	Sept	Oct
Temp. (°C)	31.5	31.0	30.0	29.0	28.0

Calculate the seasonal consumptive use of water for the rice crop in the season June 16 to October 15, by using the Blaney-Criddle formula.

3.7 A catchment area near Mysore is at latitude 12°18 N and at an elevation of 770 m.a.s.l. The mean monthly temperature is given below.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mean monthly Temp. (°C)	22.5	24.5	27.0	28.0	27.0	25.0	23.5	24.0	24.0	24.5	23.0	22.5

Calculate the monthly and annual PET for this catchment using the Thronthwaite formula.

3.8 The rainfall on five successive days on a catchment were 2, 6, 9, 5 and 3 cm. If the ϕ index for the storm can be assumed as 3 cm/day, find the total surface runoff.

3.9 The mass curve of a rainfall of duration 100 min. is given below. If the catchment had an initial loss of 0.6 cm and a ϕ index of 0.6 cm/h, calculate the total surface runoff from the catchment.

Time from start of rainfall (min)	0	20	40	60	80	100
Cumulative rainfall (cm)	0	0.5	1.2	2.6	3.3	3.5

3.10 An isolated 3-h storm occurred over a basin in the following fashion.

% of catchment area	ϕ Index (cm/h)	Rainfall (cm) 1st hour	2nd hour	3rd hour
20	1.00	0.8	2.3	1.5
30	0.75	0.7	2.1	1.0
50	0.50	1.0	2.5	0.8

Estimate the runoff from the catchment due to this storm.

3.11 An isolated storm in a catchment produced a runoff of 3.5 cm. The mass curve of the average rainfall depth over the catchment was as below:

Time from beginning of storm (h)	0	1	2	3	4	5	6
Accumulated av. rainfall (cm)	0	0.50	1.65	3.55	5.65	6.80	7.75

Calculate the ϕ index for the storm.

3.12 In a 140-min storm the following rates of rainfall were observed in successive 20-min intervals: 3.0, 3.0, 9.0, 6.6, 1.2, 1.2 and 6.0 cm/h. Assuming the ϕ index value as 3.0 cm/h and an initial loss of 0.8 cm, determine the total rainfall, net runoff and W-Index for the storm.

QUESTIONS

3.1 If e_w and e_a are the saturated vapour pressures of the water surface and air respectively, the Dalton's law for evaporation E_L in unit time is given by E_L =

(a) $(e_w - e_a)$ (b) $K e_w e_a$ (c) $K(e_w - e_a)$

(d) $K(e_w + e_a)$

3.2 The average pan coefficient for the standard US Weather Bureau class A pan is

(a) 0.85 (b) 0.70 (c) 0.90 (d) 0.20

3.3 A canal is 80 km long and has an average surface width of 15 m. If the evaporation measured in a class A pan is 0.5 cm/day, the volume of water evaporated in a month of 30 days is (in m³)

(a) 12600 (b) 18000 (c) 180000 (d) 126000

3.4 The chemical that is found to be most suitable as water evaporation inhibitor is

(a) ethyl alcohol (b) methyl alcohol (c) cetyl alcohol
(d) butyl alcohol.

3.5 Evapotranspiration is confined

(a) to daylight hours (b) night-time only
(c) land surfaces only (d) none of these.

3.6 Interception losses

(a) include evaporation, through flow and stemflow
(b) consists of only evaporation loss
(c) includes evaporation and transpiration losses
(d) consists of only stemflow.

3.7 If the wind velocity at a height of 2 m above ground is 5.0 kmph, its value at a height of 9 m above ground can be expected to be in km/h about

(a) 9.0 (b) 6.2 (c) 2.3 (d) 10.6.

3.8 The highest value of annual evapotranspiration in India is at Rajkot, Gujarat. Here the annual PET is about

(a) 150 cm (b) 150 mm (c) 210 cm. (d) 310 cm.

3.9 The infiltration capacity of a soil was measured under fairly identical general conditions by a flooding type infiltrometer as f_f and by a rainfall simulator as f_r. One can expect.

(a) $f_f = f_r$ (b) $f_f > f_r$ (c) $f_f < f_r$ (d) no fixed pattern.

3.10 The rainfall on three successive 6-h periods are 1.3, 4.6 and 3.1 cm. If the initial loss is 0.7 cm and the surface runoff resulting from this storm is 3.0 cm, the ϕ index for the storm is

(a) 0.450 cm/h (b) 0.333 cm/h (c) 0.392 cm/h (d) 0.167 cm/h.

3.11 A 6-h storm had 6 cm of rainfall and the resulting runoff was 3 cm. If the ϕ index remains at the same value the runoff due to 12 cm of rainfall in 9 h in the catchment is

(a) 9.0 cm (b) 4.5 cm (c) 6 cm (d) 7.5 cm.

3.12 If for a given basin in a given period, P = precipitation, E = evapotranspiration, R = total runoff and ΔS=increase in the storage of water in the basin, the hydrological water budget equation states

(a) $P = R-E \pm \Delta S$ (b) $R = P+E- \Delta S$
(c) $P = R+E+ \Delta S$ (d) none of these.

4 STREAMFLOW MEASUREMENT

4.1 INTRODUCTION

Streamflow representing the runoff phase of the hydrologic cycle is the most important basic data for hydrologic studies. It was seen in the previous chapters that precipitation, evaporation and evapotranspiration are all difficult to measure exactly and the presently adopted methods have severe limitations. In contrast the measurement of streamflow is amenable to fairly accurate assessment. Interestingly, streamflow is the only part of the hydrologic cycle that can be measured accurately.

A stream can be defined as a flow channel into which the surface runoff from a specified basin drains. Generally, there is considerable exchange of water between a stream and the underground water. Streamflow is measured in units of discharge (m^3/s) occurring at a specified time and constitutes historical data. The measurement of discharge in a stream forms an important branch of *Hydrometry*, the science and practice of water measurement. This chapter deals with only the salient streamflow measurement techniques to provide an appreciation of this important aspect of engineering hydrology. Excellent treatises[1,2,3,4] and a bibliography[5] are available on the theory and practice of streamflow measurement and these are recommended for further details.

Streamflow measurement techniques can be broadly classified into two categories as (*i*) direct determination and (*ii*) indirect determination. Under each category there are a host of methods, the important ones are listed below:

1. Direct determination of stream discharge:
   (a) Area-velocity methods,
   (b) dilution techniqes,
   (c) electromagnetic method, and
   (d) ultrasonic method.

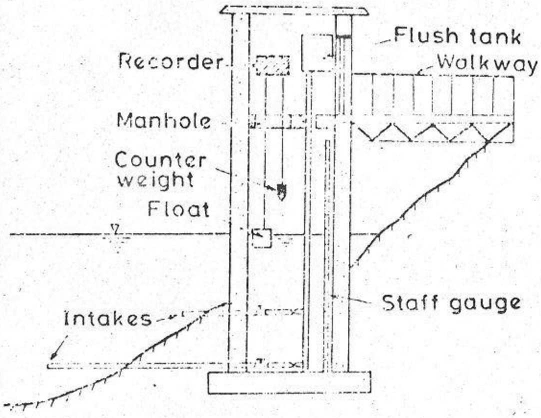

Fig. 4.2 Stilling well installation

during floods. Further, the instrument must be properly housed in a suitable enclosure to protect it from weather elements and vandalism. On account of these, the water-stage-recorder installations prove to be costly in most instances. A water-depth recorder is shown in Fig. 4.3 (Plate I).

Bubble Gauge

In this gauge compressed air or gas is made to bleed out at a very small rate through an outlet placed at the bottom of the river [Figs. 4.4, 4.5 (Plate 1) and 4.6 (Plate 2)]. A pressure gauge measures the gas pressure which in turn is equal to the water column above the outlet. A small change in the water-surface elevation is felt as a change in pressure from the present value at the pressure gauge and this in turn is adjusted by a servo-mechanism to bring the gas to bleed at the original rate under the new head. The pressure gauge reads the new water depth which is transmitted to a recorder.

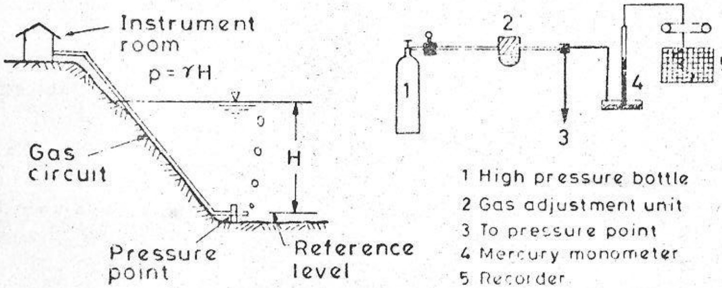

Fig. 4.4 Bubble gauge

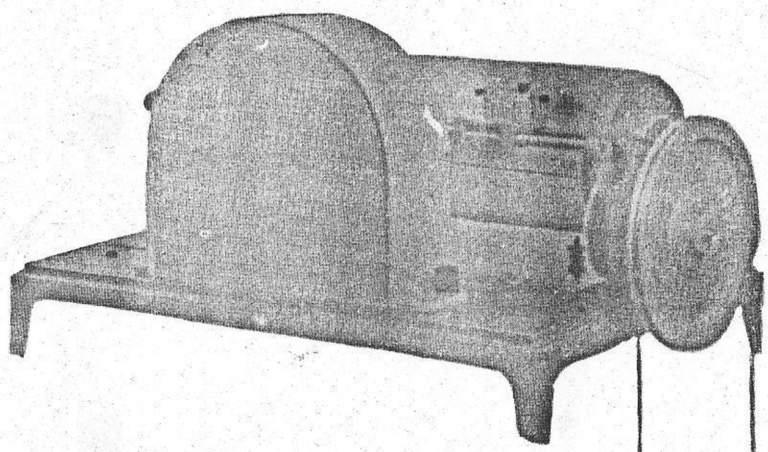

Fig. 4.3 Water-depth recorder – Stevens Type F recorder (*Courtesy*: Leupold and Stevens, Inc. Beaverton, Oregon, USA)

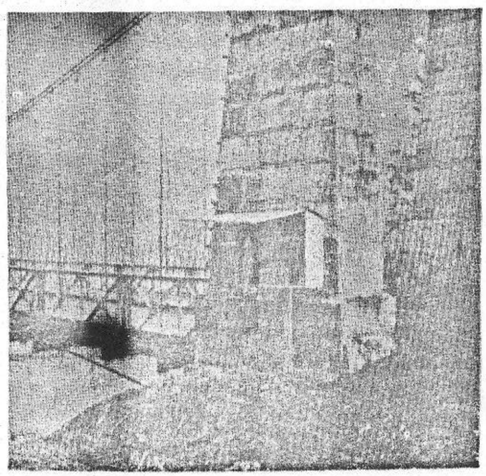

Fig 4.5 Bubble gauge installation—Telemnip (*Courtesy*: Neyrtec, Grenoble, France)

PLATE 2

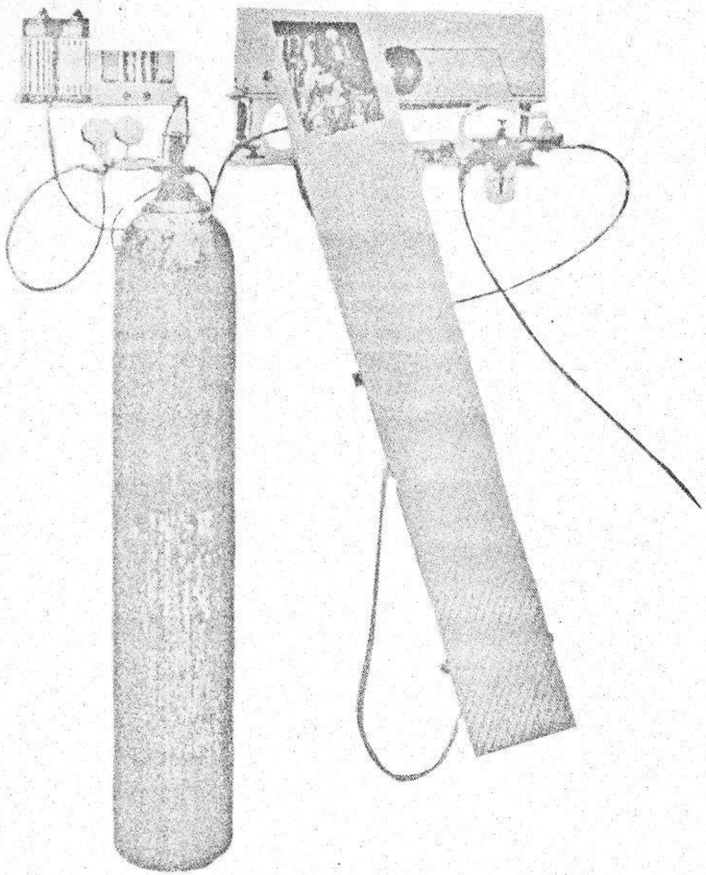

Fig. 4.6 Bubble gauge—Stevens manometer servo
(*Courtesy*: Leupold and Stevens, Inc. Beaverton, Oregon, USA)

The bubble gauge has certain specific advantages over a float operated water stage recorder and these can be listed as under :

1. There is no need for costly stilling wells;
2. a large change in the stage, as much as 30 m, can be measured;
3. the recorder assembly can be quite far away from the sensing point; and
4. due to constant bleeding action there is less likelihood of the inlet getting blocked or choked.

Stage Data

The stage data is often presented in the form of a plot of stage against chronological time (Figure 4.7) known as *stage hydrograph*. In addition to its use in the determination of stream discharge, stage data itself is of importance in flood warning and flood-protection works. Reliable long-term stage data corresponding to peak floods can be analysed statistically to estimate the design peak river stages for use in the design of hydraulic structures, such as bridges, weirs, etc. Historic flood stages are invaluable in the indirect estimation of corresponding flood discharges. In view of these multifarious uses, the river stage forms an important hydrologic parameter chosen for regular observation and recording.

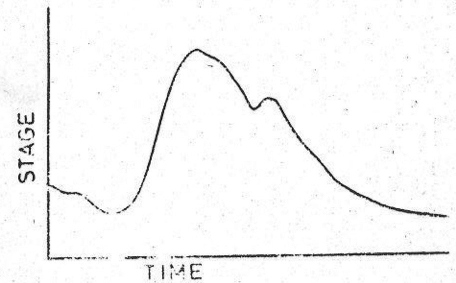

Fig. 4.7 Stage hydrograph

4.3 MEASUREMENT OF VELOCITY

The measurement of velocity is an important aspect of many direct streamflow measurement techniques. A mechanical device, called *current meter*, consisting essentially of a rotating element is probably the most commonly used instrument for accurate determination of the stream-velocity field. Approximate stream velocities can be determined by *floats*.

Current Meters

The most commonly used instrument in hydrometry to measure the velocity at a point in the flow cross-section is the current meter. It consists essentially of a rotating element which rotates due to the reaction of the stream current with an angular velocity proportional to the stream velocity. Historically, Robert Hooke (1663) invented a propeller-type current meter to measure the distance traversed by a ship. The present-day cup-type instrument and the electrical make-and-break mechanism were invented by Henry in 1868. There are two main types of current meters.

1. Vertical-axis meters, and
2. Horizontal-axis meters.

These are discussed below.

Vertical-Axis Meters

These instruments consist of a series of conical cups mounted around a vertical axis [Figs. 4.8 and 4.9 (Plate 3)]. The cups rotate in a horizontal plane and a cam attached to the vertical axial spindle records generated signals proportional to the revolutions of the cup assembly. The Price current meter and Gurley current meter are typical instruments under this category. The normal range of velocities is from 0.15 to 4.0 m/s. The accuracy of these instruments is about 1.50% at the threshold value and improves to about 0.30% at speeds in excess of 1.0 m/s. Vertical-axis

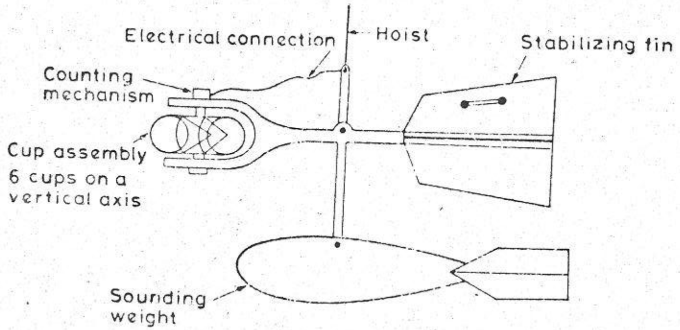

Fig. 4.8 Vertical-axis current meter

instruments have the disadvantage that they cannot be used in situations where there are appreciable vertical components of velocities. For example, the instrument shows a positive velocity when it is lifted vertically in still water.

Horizontal-Axis Meters

These meters consist of a propeller mounted at the end of a horizontal shaft [Fig. 4.10 (Plate 3) and 4.11]. These come in a wide variety of sizes with propeller diameters in the range 6 to 12 cm and can register velocities in the range of 0.15 to 4.0 m/s. Ott-, Neyrtec= [Fig. 4.12 (Plate 4)] and Watt-type meters are typical instruments under this kind. These meters are

PLATE

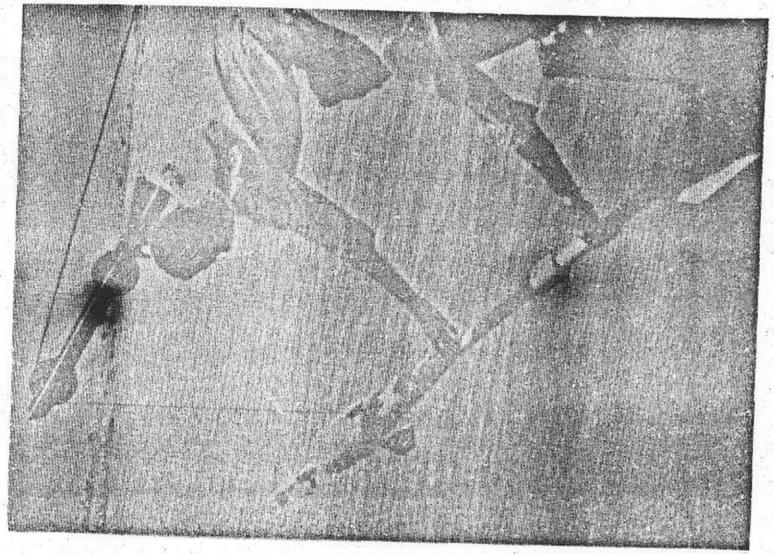

Fig. 4.10 Propeller-type current-meter—Neyrtec type with

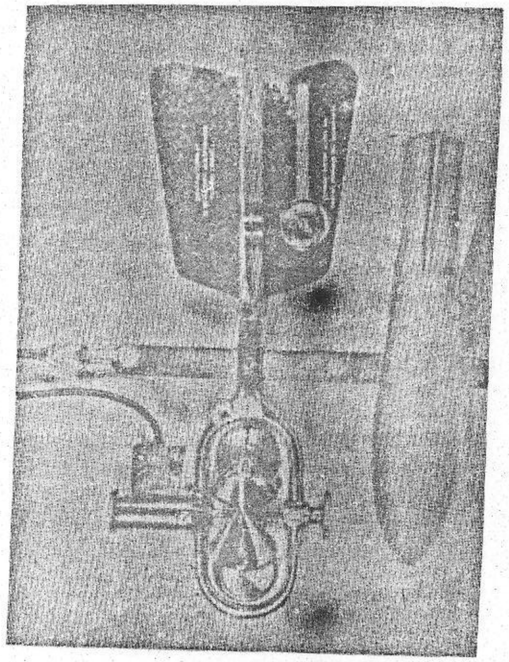

Fig. 4.9 Cup-type current meter with sounding weight—'Lynx' type (*Courtesy:* Lawrence and Mayo (India) New Delhi)

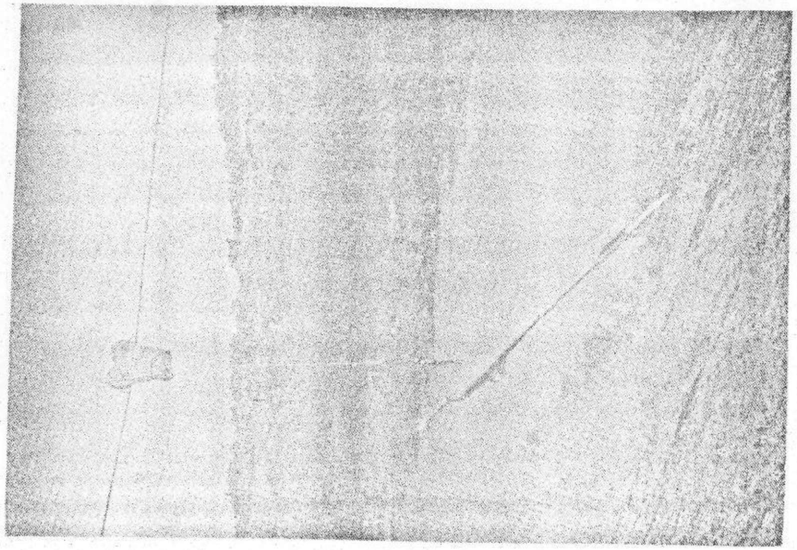

Fig. 4.12(b) Neyrtec type meter in a cableway

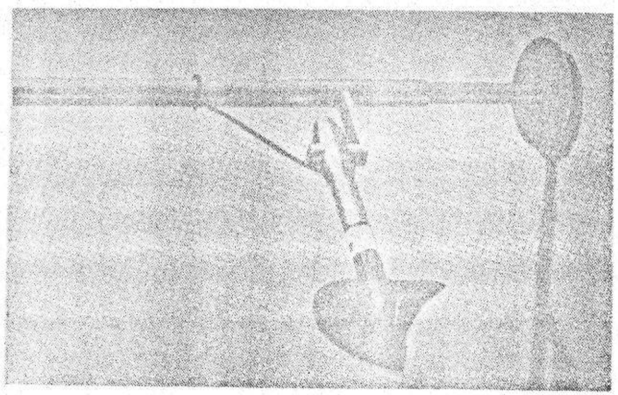

Fig. 4.12(a) Neyrtec type current meter for use in wading (*Courtesy*: Neyrtec, Grenoble, France)

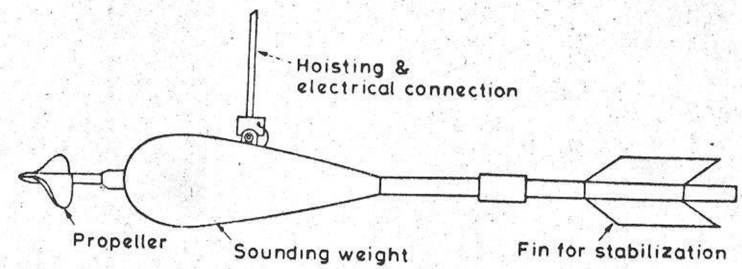

Fig. 4.11 Propeller-type current meter

fairly rugged and are not affected by oblique flows of as much as 15°. The accuracy of the instrument is about 1% at the threshold value and is about 0.25% at a velocity of 0.3 m/s and above.

A current meter is so designed that its rotation speed varies linearly with the stream velocity v at the location of the instrument. A typical relationship is

$$v = a\,N_s + b \tag{4.1}$$

where v = stream velocity at the instrument location in m/s, N_s = revolutions per second of the meter and a, b = constants of the meter. Typical values of a and b for a standard size 12.5 cm dia Price meter (cup-type) is $a = 0.65$ and $b = 0.03$. Smaller meters of 5 cm. diameter cup assembly called *pigmy meters* run faster and are useful in measuring small velocities. The values of the meter constants for them are of the order of $a = 0.30$ and $b = 0.003$. Further, each instrument has a threshold velocity below which Eq. (4.1) is not applicable. The instruments have a provision to count the number of revolutions in a known interval of time. This is usually accomplished by the making and breaking of an electric circuit either mechanically or electro-magnetically at each revolution of the shaft. In older model instruments the breaking of the circuit would be counted through an audible sharp signal ("tick") heard on a headphone. The revolutions per second is calculated by counting the number of such signals in a known interval of time, usually about 100 s. Present-day models employ electro-magnetic counters with digital or analogue displays.

Calibration

The relation between the stream velocity and revolutions per second of the meter as in Eq. (4.1) is called the calibration equation. The calibration equation is unique to each instrument and is determined by towing the instrument in a special tank. A towing tank is a long channel containing still water with arrangements for moving a carriage longitudinally over its surface at constant speed. The instrument to be calibrated is mounted on the carriage with the rotating element immersed to a specified depth in the water body in the tank. The carriage is then towed at a predetermined constant speed (v) and the corresponding average value of revolutions per second (N_s) of the instruments determined. This experiment is repeated over the complete range of velocities and a best-fit linear relation in the form of Eq. (4.1) obtained. The instruments are designed for rugged use and hence the calibration once done lasts for quite some time. However, from the point of view of accuracy it is advisable to check the instrument calibration once in a while and whenever there is a suspicion that the instrument is damaged due to bad handling or accident. In India excellent towing-tank facilities for calibration of current meters exist at the Central Water and Power Research Station, Pune and the Indian Institute of Technology, Madras.

Field Use

The velocity distribution in a stream across a vertical section is logarithmic in nature. In a rough turbulent flow the velocity distribution is given by

$$v = 5.75\, v_* \log_{10}\left(\frac{30y}{k_s}\right) \tag{4.2}$$

where v = velocity at a point y above the bed, v_* = shear velocity and k_s = equivalent sand-grain roughness. To accurately determine the average velocity in a vertical section, one has to measure the velocity at a large number of points on the vertical. As it is time-consuming, certain simplified procedures have been evolved.

1. In shallow streams of depth up to about 3.0 m, the velocity measured at 0.6 times the depth of flow below the water surface is taken as the average velocity in the vertical

$$\bar{v} = v_{0.6} \tag{4.3}$$

This procedure is known as the single-point observation method.

2. In moderately deep streams the velocity is observed at two points: (*i*) at 0.2 times the depth of flow below the free surface ($v_{0.2}$) and (*ii*) at 0.8 times the depth of flow below the free surface ($v_{0.8}$). The average velocity in the vertical $\bar{v}$ is taken as

$$\bar{v} = \frac{v_{0.2} + v_{0.8}}{2} \tag{4.4}$$

3. In rivers having flood flows, only the surface velocity (v_s) is measured within a depth of about 0.5 m below the surface. The average velocity $\bar{v}$ is obtained by using a reduction factor K as

$$\bar{v} = K v_s \tag{4.5}$$

The value of K is obtained from observations at lower stages and lie in the range of 0.85 to 0.95.

In small streams of shallow depth the current meter is held at the requisite depth below the surface in a vertical by an observer who stands in the water. The arrangement, called *wading* is quite fast but is obviously applicable only to small streams.

In rivers flowing in narrow gorges in well-defined channels a cableway is stretched from bank to bank well above the flood level. A carriage moving over the cableway is used as the observation platform.

Bridges, while hydraulically not the best locations, are advantageous from the point of view of accessibility and transportation. Hence, railway and road bridges are frequently employed as gauging stations. The velocity measurement is performed on the downstream portion of the bridge to minimize the instrument damage due to drift and knock against the bridge piers.

For wide rivers, boats are the most satisfactory aids in current meter measurement. A cross-sectional line is marked by distinctive land markings and buoys. The position of the boat is determined by using two theodolites on the bank through an intersection method.

Sounding Weights

Current meters are weighted down by lead weights called *sounding weights* to enable them to be positioned in a stable manner at the required location in flowing water. These weights are of streamlined shape with a fin in the rear (Fig. 4.8) and are connected to the current meter by a hangar bar and pin assembly. Sounding weights come in different sizes and the minimum weight is estimated as

$$W = 50\, \bar{v}\, d \tag{4.6}$$

where W = minimum weight in N, $\bar{v}$ = average stream velocity in the vertical in m/s and d = depth of flow at the vertical in m.

Velocity Measurement by Floats

A floating object on the surface of a stream when timed can yield the surface velocity by the relation

$$v_s = \frac{S}{t} \tag{4.7}$$

where S = distance travelled in time t. This method of measuring velocities while primitive still finds applications in special circumstances, such as : (i) a small stream in flood, (ii) small stream with a rapidly changing water surface and (iii) preliminary or exploratory surveys. While any floating object can be used, normally specially made leakproof and easily identifiable floats are used (Fig. 4.13). A simple float moving on a stream surface is called *surface float*. It is easiest to use and the mean velocity is obtained by multiplying the observed surface velocity by a reduction coefficient as in Eq. (4.5). However, surface floats are affected by surface winds. To get the average velocity in the vertical directly, special floats in which part of the body is under water is used. *Rod float* (Fig. 4.13), in which a cylindrical rod is weighed so that it can float vertically, belongs to this category.

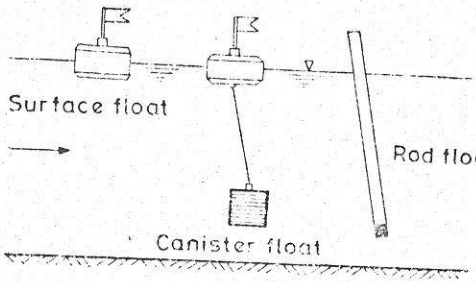

Fig 4.13 Floats

In using floats to observe the stream velocity a large number of easily identifiable floats are released at fairly uniform spacings on the width of the stream at an upstream section. Two sections on a fairly straight reach are selected and the time to cross this reach by each float is noted and the surface velocity calculated.

4.4 AREA-VELOCITY METHOD

This method of discharge measurement consists essentially of measuring the area of cross-section of the river at a selected section called the gauging site and measuring the velocity of flow through the cross-sectional area. The gauging site must be selected with care to assure that the stage-discharge curve is reasonably constant over a long period of about a few years. Towards this the following criteria are adopted:

1. The stream should have a well-defined cross-section which does not change in various seasons.
2. It should be easily accessible all through the year.
3. The site should be in a straight, stable reach.
4. The gauging site should be free from backwater effects in the channel.

At the selected site the section line is marked off by permanent survey markings and the cross-section determined. Towards this the depth at various locations are measured by sounding rods or sounding weights. When the stream depth is large or when quick and accurate depth measurements are needed, an electroacoustic instrument called *echo-depth recorder* is used. In this a high frequency sound wave is sent down by a transducer kept immersed at the water surface and the echo reflected by the bed is also picked up by the same transducer. By comparing the time interval between the transmission of the signal and the receipt of its echo, the distance to the bed is obtained and is indicated or recorded in the

instrument. Echo-depth recorders are particularly advantageous in high-velocity streams, deep streams and in streams with soft or mobile beds.

For purposes of discharge estimation, the cross-section is considered to be divided into a large number of subsections by verticals (Fig. 4.14). The average velocity in these subsections are measured by current meters or floats. It is quite obvious that the accuracy of discharge estimation increases with the number of subsections used. However, the larger the

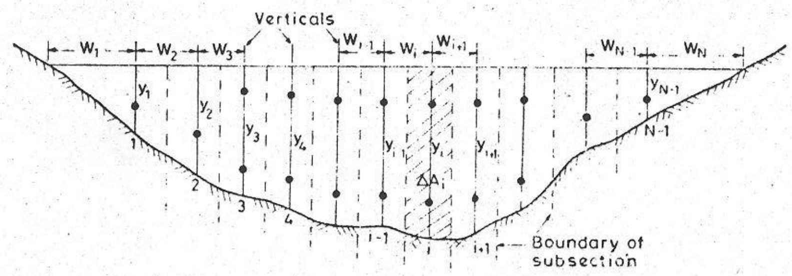

Fig. 4.14 Stream section for area-velocity method

number of segments, the larger is the effort, time and expenditure involved. The following are some of the guidelines to select the number of segments:

1. The segment width should not be greater than 1/15 to 1/20 of the width of the river.
2. The discharge in each segment should be less than 10% of the total discharge.
3. The difference of velocities in adjacent segments should not be more than 20%.

It should be noted that in natural rivers the verticals for velocity measurement are not necessarily equally spaced. The area-velocity method as above using the current meter is often called as the standard current meter method.

Calculation of Discharge

Figure (4.14) shows the cross section of a river in which $N-1$ verticals are drawn. The velocity averaged over the vertical at each section is known. Considering the total area to be divided into $N-1$ semgents, the total discharge is calculated by the *method of mid-sections* as follows:

$$Q = \sum_{i=1}^{N-1} \Delta Q_i \qquad (4.8)$$

where ΔQ_i = discharge in the ith segment

= (depth at the ith segment) × ($\frac{1}{2}$ width to the left + $\frac{1}{2}$ width to right) × (average velocity at the ith vertical)

$$\Delta Q_i = y_i \times \left(\frac{W_i}{2} + \frac{W_{i+1}}{2}\right) \times v_i; \text{ for } i = 2 \text{ to } (N-2) \quad (4.9)$$

For the first and last sections, the segments are taken to have triangular areas and area calculated as

$$\Delta A_1 = \overline{W}_1 \, y_1$$

where $$\overline{W}_1 = \frac{\left(W_1 + \frac{W_2}{2}\right)^2}{2W_1}$$

and $$\Delta A_N = \overline{W}_{N-1} \, y_{N-1}$$

where $$\overline{W}_{N-1} = \frac{\left(W_N + \frac{W_{N-1}}{2}\right)^2}{2\,W_N} \quad (4.10)$$

to get

$$\Delta Q_1 = \bar{v}_1 . \Delta A_1 \quad \text{and} \quad \Delta Q_{N-1} = \bar{v}_N \, \Delta A_{N-1} \quad (4.10)$$

EXAMPLE 4.1 *The data pertaining to a stream-gauging operation at a gauging site are given below.*

Distance from left water edge (m)	0	1.0	3.0	5.0	7.0	9.0	11.0	12.0
Depth (m)	0	1.1	2.0	2.5	2.0	1.7	1.0	0
Revolutions of a current meter kept at 0.6 depth	0	39	58	112	90	45	30	0
Duration of observation (s)	0	100	100	150	150	100	100	0

The rating equation of the current meter is

$$v = 0.51 \, N_s + 0.03 \text{ m/s}$$

Calculate the discharge in the stream.

The calculations are performed in a tabular form.

For the first and last sections,

$$\text{Average width, } \overline{W} = \frac{\left(1 + \frac{2}{2}\right)^2}{2 \times 1} = 2.0 \text{ m}$$

For the rest of segments,

$$\overline{W} = \left(\frac{2}{2} + \frac{2}{2}\right) = 2.0 \text{ m}$$

Since the velocity is measured at 0.6 depth, the measured velocity is the average velocity at that vertical ($\bar{v}$).

The calculation of discharge by the mid-section method is shown in tabular form below:

Distance from left water edge (m)	Average width $\overline{W}$ (m)	Depth y (m)	Velocity $\bar{v}$ (m/s)	Segmental discharge ΔQ_i (m^3/s)
0	0	0	—	—
1	2.00	1.1	0.229	0.504
3	2.00	2.0	0.326	1.304
5	2.00	2.5	0.411	2.055
7	2.00	2.0	0.336	1.344
9	2.00	1.7	0.260	0.844
11	2.00	1.0	0.183	0.363
12	0	0		—

Total discharge $Q = 6.454 \text{ m}^3/\text{s}$

Moving-Boat Method

Discharge measurement of large alluvial rivers, such as the Ganga, by the standard current meter method is very time-consuming even when the flow is low or moderate. When the river is in spate, it is almost impossible to use the standard current meter technique due to the difficulty of keeping the boat stationary on the fast-moving surface of the stream for observation purposes. It is in such circumstance that the newly developed moving-boat techniques prove very helpful.

In this method a special propeller-type current meter which is free to move about a vertical axis is towed in a boat at a velocity v_b at right angles to the stream flow. If the flow velocity is v_f the meter will align itself in the direction of the resultant velocity v_R making an angle θ with the direction of the boat (Fig. 4.15). Further, the meter will register the velocity v_R. If V_b is normal to v_f,

$$v_b = v_R \cos\theta \quad \text{and} \quad v_f = v_R \sin\theta$$

If the time of transit between two verticals is Δt, then the width between the two verticals (Figure 4.8) is

$$W = v_b \, \Delta t$$

The flow in the sub-area between two verticals i and $i+1$ where the depths are y_i and y_{i+1} respectively, by assuming the current meter to measure the average velocity in the vertical, is

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2}\right) W_{i+1} \, v_f$$

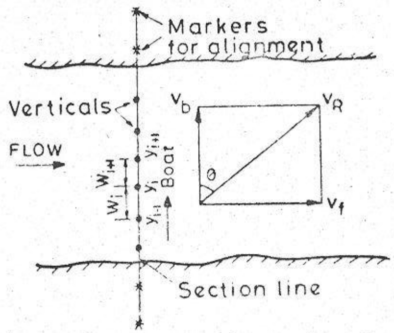

Fig. 4.15 Moving-boat method

i.e.

$$\Delta Q_i = \left(\frac{y_i + y_{i+1}}{2}\right) v_R^2 \sin\theta \cdot \cos\theta \, \Delta t \qquad (4.11)$$

Thus by measuring the depths y_i, velocity v_R and θ in a reach and the time taken to cross the reach Δt, the discharge in the sub-area can be determined. The summation of the partial discharges ΔQ_i over the whole width of the stream gives the stream discharge

$$Q = \Sigma \, \Delta Q_i \qquad (4.12)$$

In field applications a good stretch of the river with no shoals, islands, bars, etc. is selected. The cross-sectional line is defined by permanent land marks so that the boat can be aligned along this line. A motor boat with different sizes of outboard motors for use in different river stages is selected. A special current meter of the propeller-type, in which the velocity and inclination of the meter to the boat director θ in the horizontal plane can be measured, is selected. The current meter is usually immersed at a depth of 0.5 m from the water surface to record surface velocities. To mark the various vertical sections and know the depths at these points, an echo-depth recorder is used.

In a typical run, the boat is started from the water edge and aligned to go across the cross-sectional line. When the boat is in sufficient depth of water, the instruments are lowered. The echo-depth recorder and current meter are commissioned. A button on the signal processor when pressed marks a distinctive mark line on the depth vs time chart of the echo-depth recorder. Further, it gives simultaneously a sharp audio signal to enable the measuring party to take simultaneous readings of the velocity v_R and the inclination θ. A large number of such measurements are taken during the traverse of the boat to the other bank of the river. The operation is repeated in the return journey of the boat. It is important that the boat is kept aligned along the cross-sectional line and this requires considerable skill on the part of the pilot. Typically, a river of about

2 km stretch takes about 15 min for one crossing. A number of crossings are made to get the average value of the discharge.

The surface velocities are converted to average velocities across the vertical by applying a coefficient [Eq. (4.5)]. The depths y_i and time intervals Δt are read from the echo-depth recorder chart. The discharge is calculated by Eqs. (4.11) and (4.12). In practical use additional coefficients may be needed to account for deviations from the ideal case and these depend upon the actual field conditions.

4.5 DILUTION TECHNIQUE OF STREAMFLOW MEASUREMENT

The dilution method of flow measurement, also known as the chemical method depends upon the continuity principle applied to a tracer which is allowed to mix completely with the flow.

Consider a tracer which does not react with the fluid or boundary. Let C_0 be the small initial concentration of the tracer in the streamflow. At section 1 a small quantity (volume V_1) of high concentration C_1 of this tracer is added (Fig. 4.16). Let section 2 be sufficiently far away on the downstream of section 1 so that the tracer mixes throughly with the fluid due to the turbulent-mixing process while passing through the reach. The concentration profile taken at section 2 is schematically shown in Fig. 4.16. The concentration will have a base value of C_0, increases from time t_1 to a peak value and gradually reaches the base value of C_0 at time t_2. The stream-flow is assumed to be steady. By continuity of the tracer material

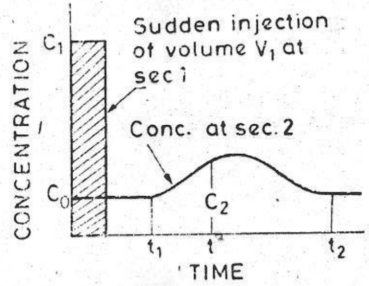

Fig. 4.16 Sudden-injection method

$$M_1 = \text{mass of tracer added at section 1} = V_1 C_1$$

$$= \int_{t_1}^{t_2} Q(C_2 - C_0)\, dt + \frac{V_1}{t_2 - t_1} \int_{t_1}^{t_2} (C_2 - C_0)\, dt$$

Neglecting the second term on the right-hand side as insignificantly small,

$$Q = \frac{V_1 C_1}{\int_{t_1}^{t_2} (C_2 - C_0)\, dt} \qquad (4.13)$$

Thus the discharge Q in the stream can be estimated if for a known M_1 the variation of C_2 with time at section 2 and C_0 are determined. This method is known as *sudden injection* or *gulp* or *integration method.*

Another way of using the dilution principle is to inject the tracer of concentration C_1 at a constant rate Q_t at section 1. At section 2, the concentration gradually rises from the background value of C_0 at time t_1 to a constant value C_2 (Fig. 4.17). At the stready state, the continuity equation for the tracer is

$$Q_t C_1 + Q C_0 = (Q + Q_t) C_2$$

i.e.
$$Q = \frac{Q_t(C_1 - C_2)}{(C_2 - C_0)} \tag{4.14}$$

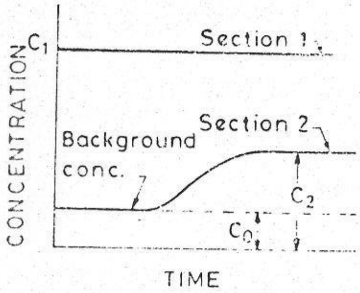

Fig. 4.17 Constant rate injection method

This technique in which Q is estimated by knowing C_1, C_2, C_0 and Q is known as *constant rate injection method* or *plateau gauging*.

It is necessary to emphasise here that the dilution method of gauging is based on the assumption of steady flow. If the flow is unsteady and the flow rate changes appreciably during gauging, there will be a change in the storage volume in the reach and the steady-state continuity equation used to develope Eqs. (4.13) and (4.14) is not valid. Systematic errors can be expected in such cases.

Tracers

The tracer used should have ideally the following properties:

1. It should not be absorbed by the sediment, channel boundary and vegetation. It should not chemically react with any of the above surfaces and also should not be lost by evaporation.
2. It should be non-toxic.
3. It should be capable of being detected in a distinctive manner in small concentrations.
4. It should not be very expensive.

The tracers used are of three main types:

1. Chemicals (common salt and sodium dischromate are typical);
2. fluorescent dyes (Rhodamine—WT and Sulpho-Rhodamine B Extra are typical); and
3. radioactive materials (such as Bromine-82, Sodium-24 and Iodine-132).

Common salt can be detected with an error of $\pm$ 1% up to a concentration of 10 ppm. Sodium dichromate can be detected up to 0.2 ppm concentrations. Fluorescent dyes have the advantage that they can be detected at levels of tens of nanograms per litre ($\sim$ 1 in 10^{11}) and hence require very small amounts of solution for injections. Radioactive tracers are detectable up to accuracies of tens of picocuries per litre ($\sim$ 1 in 10^{14}) and therefore permit large-scale dilutions. However, they involve the use of very sophisticated instruments and handling by trained personnel only. The availability of detection instrumentation, environmental effects of the tracer and overall cost of the operation are chief factors that decide the tracer to be used.

Length of Reach

The length of the reach between the dosing section and sampling section should be adequate to have complete mixing of the tracer with the flow. This length depends upon the geometric dimensions of the channel cross-section, discharge and turbulence levels. An empirical formula suggested by Rimmar (1960) for estimation of mixing length for point injection of a tracer in a straight reach is

$$L = \frac{0.13\, B^2\, C\, (0.7\, C + 2\sqrt{g})}{g\, d} \tag{4.15}$$

where L = mixing length (m), B = average width of the stream (m), d = average depth of the stream (m), C = Chezy coefficient of roughness which varies from 15 to 50 for smooth to rough bed conditions and g = acceleration due to gravity. The value of L varies from about 2 km for a mountain stream carrying a discharge of about 1.0 m³/s to about 100 km for river in a plain with a discharge of about 300 m³/s. The mixing length becomes very large for large rivers and is one of the major constraints of the dilution method. Artificial mixing of the tracer at the dosing station may prove beneficial for small streams in reducing the mixing length of the reach.

Use

The dilution method has the major advantage that the discharge is estimated directly in an absolute way. It is a particularly attractive method for small turbulent streams, such as those in mountainous areas. Where suitable, it can be used as an occasional method for checking the calibration, stage-discharge curves, etc. obtained by other methods.

EXAMPLE 4.2. *A 25 g/l solution of a flourescent tracer was discharged into a stream at a constant rate of 10 cm³/s. The background concentration of the dye in the stream water was found to be zero. At a downstream section sufficiently far away, the dye was found to reach an equilibrium concentration of 5 parts per billion. Estimate the stream discharge.*

By Eq. (4.14) for the constant-rate injection method,

$$Q = \frac{Q_t\,(C_1 - C_2)}{C_2 - C_0}$$

$$Q_t = 10 \text{ cm}^3/\text{s} = 10\times 10^{-6} \text{ m}^3/\text{s}$$

$$C_1 = 0.025,\ C_2 = 5\times 10^{-9},\ C_0 = 0$$

$$Q = \frac{10\times 10^{-6}}{5\times 10^{-9}}(0.025-5\times 10^{-9}) = 50 \text{ m}^3/\text{s}$$

4.6 ELECTROMAGNETIC METHOD

The electromagnetic method is based on the Faraday's principle that an emf is induced in the conductor (water in the present case) when it cuts a normal magnetic field. Large coils buried at the bottom of the channel carry a current I to produce a controlled vertical magnetic field, (Fig. 4.18). Electrodes provided at the sides of the channel section measure the small voltage produced due to flow of water in the channel.

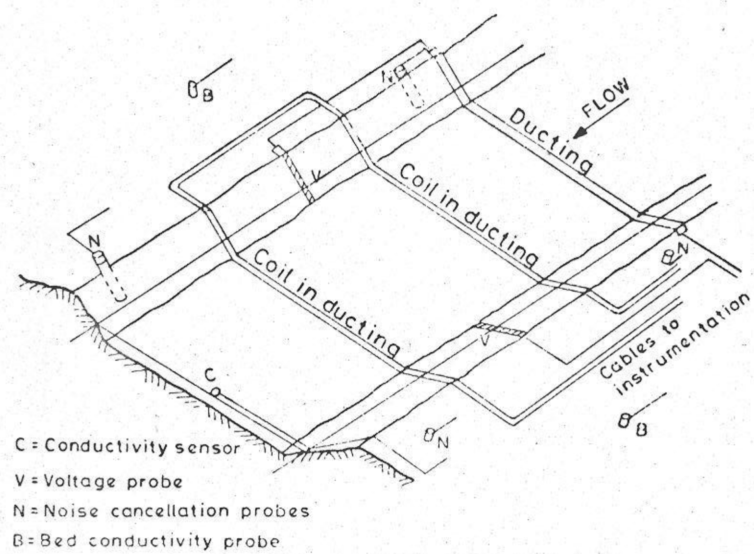

Fig. 4.18 Electromagnetic method

It has been found that the signal output E will be of the order of millivolts and is related to the discharge Q as

$$Q = K_1\left(\frac{E\,d}{I} + K_2\right)^n \tag{4.16}$$

where d = depth of flow, I = current in the coil, and n, K_1 and K_2 are system constants.

The method involves sophisticated and expensive instrumentation and has been successfully tried in a number of installations. The fact that this kind

of set-up gives the total discharge when once it has been calibrated, makes it specially suited for field situations where the cross-sectional properties can change with time due to weed growth, sedimentation, etc. Another specific application is in tidal channels where the flow undergoes rapid changes both in magnitude as well as in direction. Present-day commerically available electromagnetic flowmeters can measure the discharge to an accuracy of $\pm 3\%$, the maximum channel width that can be accommodated being 100 m. The minimum detectable velocity is 0.005 m/s.

4.7 ULTRASONIC METHOD

This is essentially an area-velocity method with the average velocity being measured by using ultrasonic signals. The method was first reported by Swengel (1955); since then it has been perfected and complete systems are available commercially.

Consider a channel carrying a flow with two transducers A and B fixed at the same level h above the bed and on either sides of the channel (Fig. 4.19). These transducers can receive as well as send ultrasonic signals. Let A send an ultrasonic signal to be received at B after an elapse time t_1.

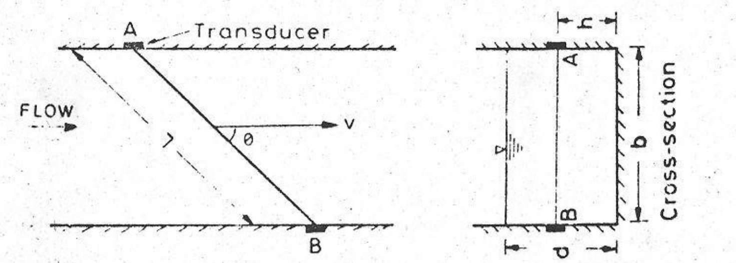

Fig. 4.19 Ultrasonic method

Similarly, let B send a signal to be received at A after an elapse time t_2. If C = velocity of sound in water,

$$t_1 = L/(C+v_P) \tag{4.17}$$

where L = length of path from A to B and v_P = component of the flow velocity in the sound path $= v\cos\theta$. Similarly, from Fig. 4.19 it is easy to see that

$$t_2 = \frac{L}{(C-v_P)} \tag{4.18}$$

Thus
$$\frac{1}{t_1} - \frac{1}{t_2} = \frac{2\,v_P}{L} = \frac{2\,v\cos\theta}{L}$$

or
$$v = \frac{L}{2\cos\theta}\left(\frac{1}{t_1} - \frac{1}{t_2}\right) \tag{4.19}$$

Thus for a given L and θ, by knowing t_1 and t_2, the average velocity along the path AB, i.e. v can be determined. It may be noted that v is the average velocity at a height h above the bed and is not the average velocity V for the whole cross-section. However, for a given channel cross-section v can be related to V and by calibration a relation between v/V and h can be obtained. For a given set-up, as the area of cross-section is fixed, the discharge is obtained as a product of area and mean velocity V. Estimation of discharge by using one signal path as above is called *single-path gauging*. Alternatively, for a given depth of flow, multiple single paths can be used to obtain v for different h values. Mean velocity of flow through the cross-section is obtained by averaging these v values. This technique is known as *multi-path gauging*.

Ultrasonic flowmeters using the above principal have frequencies of the order of 500 kHz. Sophisticated electronics are needed to transmit, detect and evaluate the mean velocity of flow along the path. In a given installation a calibration (usually performed by the current-meter method) is needed to determine the system constants. Currently available commercial systems have been installed successfully at many places and accuracies of about 2% for the single-path method and 1% for the multipath method are reported. The systems are currently available for rivers up to 500 m width.

The specific advantages of the ultrasonic system of river gauging are:

1. It is rapid and gives high accuracy;
2. it is suitable for automatic recording of data;
3. it can handle rapid changes in the magnitude and direction of flow, as in tidal rivers; and
4. the cost of installation is independent of the size of rivers.

The accuracy of this method is limited by the factors that affect the signal velocity and averaging of flow velocity, such as (i) unstable cross-section, (ii) fluctuating weed growth, (iii) high loads of suspended solids, (iv) air entrainment and (v) salinity and temperature changes.

4.8 INDIRECT METHODS

Under this category are included those methods which make use of the relationship between the flow discharge and the depths at specified locations. The field measurement is restricted to the measurement of these depths only. Two broad classifications of these indirect methods are:

1. Flow measuring structures, and
2. slope area methods.

Flow-Measuring Structures

Use of structures like notches, weirs, flumes and sluice gates for flow measurement in hydraulic laboratories is well known. These conventional structures are used in field conditions also but their use is limited by the ranges of head, debris or sediment load of the stream and the back-water effects produced by the installations. To overcome many of these limitations a wide variety of flow measuring structures with specific advantages are in use.

The basic principle governing the use of a weir, flume or similar flow-measuring structure is that these structures produce a unique control section in the flow. At these structures, the discharge Q is a function of the water-surface elevation measured at a specified upstream location,

$$Q = f(H) \tag{4.20}$$

where H = water surface elevation measured from a specified datum. Thus, for example, for weirs, Eq. (4.20) takes the form

$$Q = KH^n \tag{4.21}$$

where H = head over the weir and K, n = system constants. Equation (4.20) is applicable so long as the downstream water level is below a certain limiting water level known as the modular limit. Such flows which are independent of the downstream water level are known as *free flows*. If the tailwater conditions do affect the flow, then the flow is known as *drowned* or *submerged flow*. Discharges under drowned condition are obtained by applying a reduction factor to the free flow discharges. For example, the sumberged flow over a weir (Fig. 4.20) is estimated by the Villemonte formula,

$$Q_s = Q_1 \left[1 - \left(\frac{H_2}{H_1}\right)^n\right]^{0.385} \tag{4.22}$$

where Q_s = submerged discharge, Q_1 = free flow discharge under head H_1, H_1 = upstream water surface elevation measured above the weir crest, H_2 = downstream water surface elevation measured above the weir crest, n = exponent of head in the free flow head discharge relationship [Eq. (4.21)]. For a rectangular weir n = 1.5.

The various flow measuring structures can be broadly considered under three categories:

1. *Thin-plate structures* are usually made from a vertically set metal plate. The V-notch, rectangular full width and contracted notches are typical examples under this category.
2. *Long-base weirs*, also known as *broad-crested weirs* are made of concrete or masonry and are used for large discharge values.
3. *Flumes* are made of concrete, masonry or metal sheets depending on their use and location. They depend primarily on the width constriction to produce a control section.

Details of the discharge characteristics of flow-measuring structures are available in Refs. 1, 2 and 6.

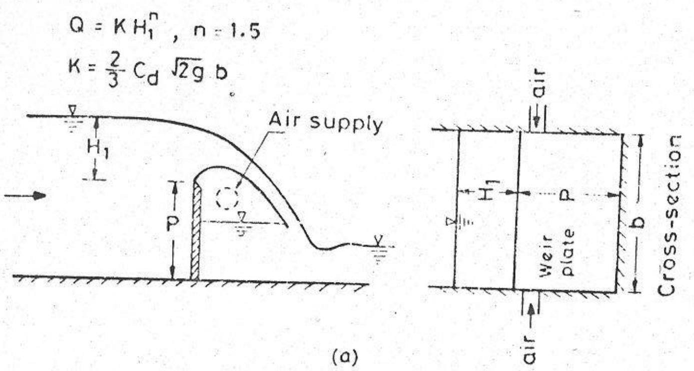

Fig. 4.20 Flow over a weir: (a) Free flow

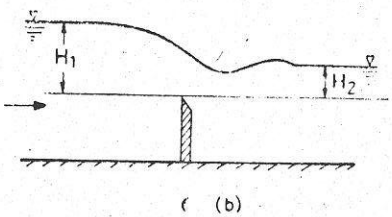

Fig. 4.20 (b) Submerged flow

Slope-Area Method

The resistance equation for uniform flow in an open channel, e.g. Manning's formula can be used to relate the depths at either ends of a reach to the discharge. Figure 4.21 shows the longitudinal section of the flow in a river between two sections, 1 and 2. Knowing the water-surface elevations at

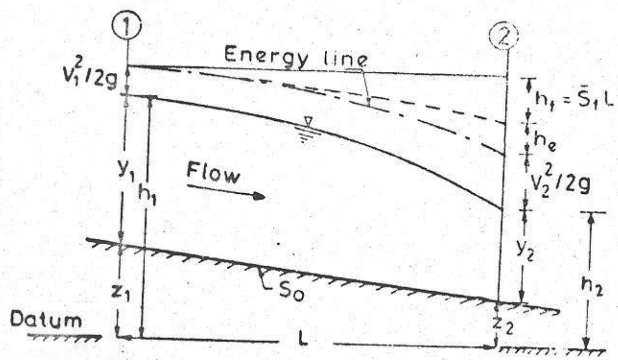

Fig. 4.21 Slope-area method

the two sections, it is required to estimate the discharge. Applying the energy equation to sections 1 and 2,

$$Z_1+y_1+\frac{V_1^2}{2g}=Z_2+y_2+\frac{V_2^2}{2g}+h_L$$

where h_L = head loss in the reach. The head loss h_L can be considered to be made up of two parts (*i*) frictional loss h_f and (*ii*) eddy loss h_e. Denoting $Z+y=h$ = water-surface elevation above the datum,

$$h_1+\frac{V_1^2}{2g}=h_2+\frac{V_2^2}{2g}+h_e+h_f$$

or

$$h_f=(h_1-h_2)+\left(\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right)-h_e \tag{4.23}$$

If L = length of the reach, by Manning's formula for uniform flow,

$$\frac{h_f}{L}=S_f=\text{energy slope}=\frac{Q^2}{K^2}$$

where K = conveyance of the channel $=\frac{1}{n}AR^{2/3}$.

In nonuniform flow, an average conveyance is used to estimate the average energy slope and

$$\frac{h_f}{L}=\bar{S}_f=\frac{Q^2}{K^2} \tag{4.24}$$

where $K=\sqrt{K_1K_2}$; $K_1=\frac{1}{n_1}A_1R_1^{2/3}$ and $K_2=\frac{1}{n_2}A_2R_2^{2/3}$

n = Manning's roughness coefficient

The eddy loss h_e is estimated as

$$h_e=K_e\left|\frac{V_1^2}{2g}-\frac{V_2^2}{2g}\right| \tag{4.25}$$

where K_e = eddy-loss coefficient having values as below.

Cross-section characteristic of the reach	Value of K_e Expansion	Contraction
Uniform	0	0
Gradual transition	0.3	0.1
Abrupt transition	0.8	0.6

Equation (4.23), (4.24) and (4.25) together with the continuity equation $Q = A_1 V_1 = A_2 V_2$ enable the discharge Q to be estimated for known values of h, channel cross-sectional properties and n.

The discharge is calculated by a trial and error procedure using the following sequence of calculations:

1. Assume $V_1 = V_2$. This leads to $V_1^2/2g = V_2^2/2g$ and by Eq. (4.23)'
   $h_f = h_1 - h_2 = F =$ fall in the water surface between sections 1 and 2.
2. Using Eq. (4.24) calculate discharge Q.
3. Compute $V_1 = Q/A_1$ and $V_2 = Q/A_2$. Calculate velocity heads and eddy loss h_e.
4. Now calculate a refined value of h_f by Eq. (4.23) and go to step (2). Repeat the calculations till two successive calculations give values of discharge (or h_f) differing by a negligible margin.

This method of estimating the discharge is known as the slope-area method. It is a very versatile indirect method of discharge estimation and requires (*i*) the selection of a reach in which cross-sectional properties including bed elevations are known at its ends, (*ii*) the value of Manning's n and (*iii*) water-surface elevations at the two end sections.

EXAMPLE 4.3 *During a flood flow the depth of water in a 10 m wide rectangular channel was found to be 3.0 and 2.9 m at two sections 200 m apart. The drop in the water-surface elevation was found to be 0.12 m. Assuming Manning's coefficient to be 0.025, estimate the flood discharge through the channel.*

Using suffixes 1 and 2 to denote the upstream and downstream section respectively, the cross-sectional properties are calculated as follows:

Section 1	Section 2
$y_1 = 3.0$ m	$y_2 = 2.90$ m
$A_1 = 30$ m²	$A_2 = 29$ m²
$P_1 = 16$ m	$P_2 = 15.8$ m
$R_1 = 1.875$ m	$R_2 = 1.835$ m
$K_1 = \frac{1}{0.025} \times 30 \times (1.875)^{2/3} = 1824.7$	$K_2 = \frac{1}{0.025} \times 29 \times (1.835)^{2/3} = 1738.9$

Average K for the reach $= \sqrt{K_1 K_2} = 1781.3$

To start with $h_f =$ fall $= 0.12$ m is assumed.

Eddy loss $h_e = 0$

The calculations are shown in Table 4.1.

$$\bar{S}_f = h_f/L = h_f/200$$

$$Q = K\sqrt{\bar{S}_f} = 1781.3\sqrt{\bar{S}_f}$$

$$\frac{V_1^2}{2g} = \left(\frac{Q}{39}\right)^2 \Big/ 19.62, \quad \frac{V_2^2}{2g} = \left(\frac{Q}{29}\right)^2 \Big/ 19.62$$

$$h_f = (h_1 - h_2) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right)$$

$$h_f = \text{fall} + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) = 0.12 + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \text{(E.1)}$$

TABLE 4.1 CALCULATIONS FOR EXAMPLE 4.3

Trial	h_f (trial)	S_f (units of 10^{-4})	Q (m³/s)	$V_1^2/2g$ (m)	$V_2^2/2g$ (m)	h_f by Eq. (1.e) (m)
1	0.1200	6.000	43.63	0.1078	0.1154	0.1124
2	0.1124	5.615	42.21	0.1009	0.1080	0.1129
3	0.1129	5.645	42.32	0.1014	0.1085	0.1129

(The last column is h_f by Eq. (E.1) and its value is adopted for the next trial.

The discharge in the channel is 42.32 m³/s.

Flood Discharge by Slope-Area Method

The slope-area method is of particular use in estimating the flood discharges in a river by past records of stages at different sections. Floods leave traces of peak elevations called high-water marks in their wake. Floating vegetative matter, such as grass, straw and seeds are left stranded at high water levels when the flood subsides and form excellent marks. Other high-water marks include silt lines on river banks, trace of erosion on the banks called wash lines and silt or stain lines on buildings. In connection with the estimation of very high floods, interviews with senior citizens living in the area, who can recollect from memory certain salient flood marks are valuable. Old records in archives often provide valuable information on flood marks and dates of occurrence of those floods. Various such information relating to a particular flood are cross-checked for consistency and only reliable data are retained. The slope-area method is then used to estimate the magnitude of the flood.

The selection of the reach is probably the most important aspect of the slope–area method. The following criteria can be listed towards this:

1. The quality of high-water marks must be good.
2. The reach should be straight and uniform as far as possible. Gradually contracting sections are preferred to an expanding reach.
3. The recorded fall in the water-surface elevation should be larger than the velocity head. It is preferable if the fall is greater than 0.15 m.
4. The longer the reach, the greater is the accuracy in the estimated discharge. A length greater than 75 times the mean depth provides an estimate of the reach length required

The Manning's roughness coefficient n for use in the computation of discharge is obtained from standard tables.[6] Sometimes a relation between n and the stage is prepared from measured discharges at a neighbouring gauging station and an appropriate value of n selected from it, with extrapolation if necessary.

4.9 STAGE–DISCHARGE RELATIONSHIP

As indicated earlier the measurement of discharge by the direct method involves a two step procedure; the development of the stage-discharge relationship which forms the first step is of utmost importance. Once the stage–discharge (G–Q) relationship is established, the subsequent procedure consists of measuring the stage (G) and reading the discharge (Q) from the (G–Q) relationship. This second part is a routine operation. Thus the aim of all current-meter and other direct-discharge measurements is to prepare a stage–discharge relationship for the given channel gauging section. The stage–discharge relationship is also known as the *rating curve.*

The measured value of discharges when plotted against the corresponding stages give a relationship that represents the integrated effect of a wide range of channel and flow parameters. The combined effect of these parameters is termed *control.* If the (G–Q) relationship for a gauging section is constant and does not change with time, the control is said to be *permanent.* If it changes with time, it is called *shifting control.*

Permanent Control

A majority of streams and rivers, especially nonalluvial rivers exhibit permanent control. For such a case, the relationship between the stage and the discharge is a single-valued relation which is expressed as

$$Q = C_r\,(G-a)^{\beta} \qquad (4.26)$$

in which Q = stream discharge, G = gauge height (stage), a = a constant which represent the gauge reading corresponding to zero discharge, C_r and β are rating curve constants. This relationship can be expressed graphically

by plotting the observed stage against the corresponding discharge values in an arithmetic or logarithmic plot [Fig. 4.22(a) and (b)]. Logarithmic plotting is advantageous as Eq. (4.26) plots as a straight line in logarithmic coordinates. In Fig. 4.22(b) the straight line is drawn to best represent the data plotted as Q vs (G–a). Coefficients C_r and β need not be the same for the full range of stages.

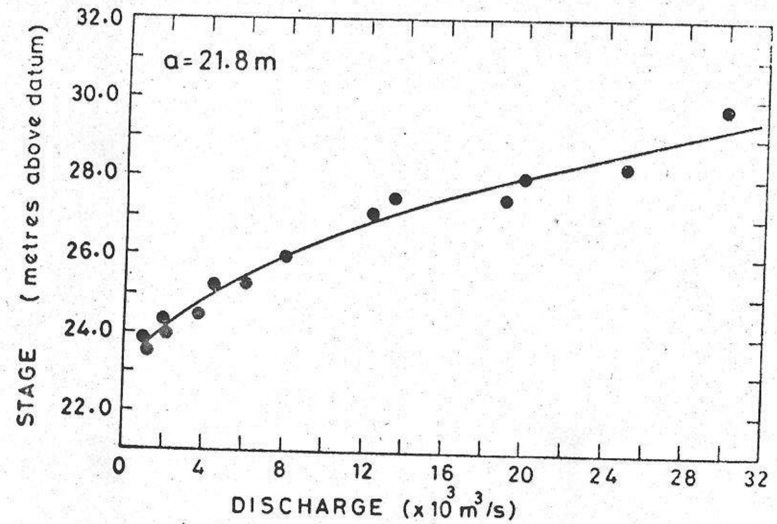

Fig. 4.22 (a) Stage-discharge curve: arithmetic plot

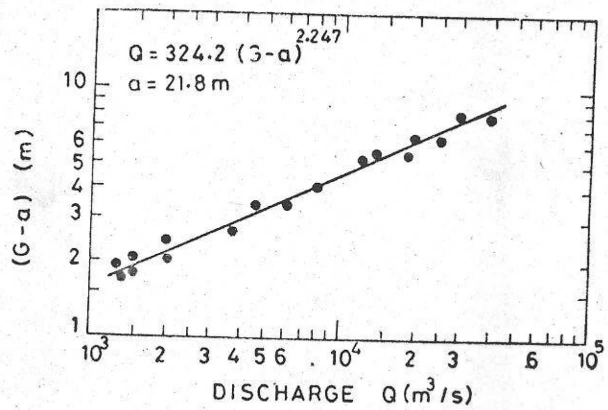

Fig. 4.22 (b) Stage-discharge curve: logarithmic plot

The best values of C_r and β in Eq. (4.26) for a given range of stage are obtained by the least-square-error method. Thus by taking logarithms,

$$\log Q = \beta \log (G-a) + \log C_r \qquad (4.27)$$

$$\text{or } Y = \beta X + b \quad (4.27a)$$

in which $Y = \log Q$, $X = \log (G-a)$ and $b = \log C_r$.

For the best-fit straight line of N observations of X and Y,

$$\beta = \frac{N(\Sigma XY) - (\Sigma X)(\Sigma Y)}{N(\Sigma X^2) - (\Sigma X)^2} \quad (4.28)$$

$$\text{and } b = \frac{\Sigma Y - \beta(\Sigma X)}{N}$$

In the above it should be noted that a is an unknown and its determination poses some difficulties. The following alternative methods are available for its determination:

1. Plot Q vs G on an arithmetic graph paper and draw a best-fit curve. By extrapolating the curve by eye judgement find a as the value of G corresponding to $Q = 0$. Using this value of a, plot log Q vs log $(G-a)$ and verify whether the data plots as a straight line. If not, select another value in the neighbourhood of previously assumed value and by trial and error find an acceptable value of a which gives a straight line plot of log Q vs log $(G-a)$.

2. A graphical method due to Running[7] is as follows: The Q vs G data are plotted to an arithmetic scale and a smooth curve through the plotted points are drawn. Three points A, B and C on the curve are selected such that their discharges are in geometric progression, (Fig. 4.23) i.e.

$$\frac{Q_A}{Q_B} = \frac{Q_B}{Q_C}$$

At A and B vertical lines are drawn and then horizontal lines are drawn at B and C to get D and E as intersection points with the verticals. Two straight lines ED and BA are drawn to intersect at F. The

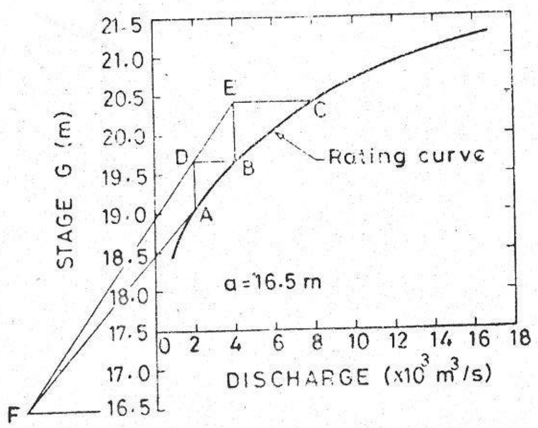

Fig. 4.23 Running's method for estimation of the constant

ordinate at F is the required value of a, the gauge height corresponding to zero discharge. This method assumes the lower part of the stage-discharge curve to be a parabola.

3. Plot Q vs G to an arithmetic scale and draw a smooth good-fitting curve by eye-judgement. Select three discharges Q_1, Q_2 and Q_3 such that $Q_1/Q_2 = Q_2/Q_3$ and note from the curve the corresponding values of gauge readings G_1, G_2 and G_3. From Eq. (4.27)

$$(G_1 - a)/(G_2 - a) = (G_2 - a)/(G_3 - a)$$

$$\text{i.e.} \quad a = \frac{G_1 G_3 - G_2^2}{(G_1 + G_3) - 2G_2} \quad (4.29)$$

4. A number of optimization procedures that are based on the use of computers are available to estimate the best value of a. A trial-and-error search for a which gives the best value of the correlation coefficient is one of them.

Shifting Control

The control that exists at a gauging section giving rise to a unique stage-discharge relationship can change due to: (*i*) changing characteristics caused by weed growth, dredging or channel encroachment, (*ii*) aggradation or degradation phenomenon in an alluvial channel, (*iii*) variable backwater effects affecting the gauging section and (*iv*) unsteady flow effects of a rapidly changing stage. There are no permanent corrective measures to tackle the shifting controls due to causes (*i*) and (*ii*) listed above. The only recourse in such cases is to have frequent current-meter gaugings and to update the rating curves. Shifting controls due to causes (*iii*) and (*iv*) are described below.

Backwater Effect

If the shifting control is due to variable backwater curves, the same stage will indicate different discharges depending upon the backwater effect. To remedy this situation another gauge, called the secondary gauge or *auxiliary gauge* is installed some distance downstream of the gauging section and readings of both gauges are taken. The difference between the main gauge and the secondary gauge gives the *fall* (F) of the water surface in the reach. Now, for a given main-stage reading, the discharge under variable backwater condition is a function of the fall F, i.e.

$$Q = f(G, F) \quad (4.30)$$

Schematically, this functional relationship is shown in Fig. 4.24. Instead of having a three-parameter plot, the observed data is normalized with respect to a constant fall value. Let F_0 be a normalizing value of the fall

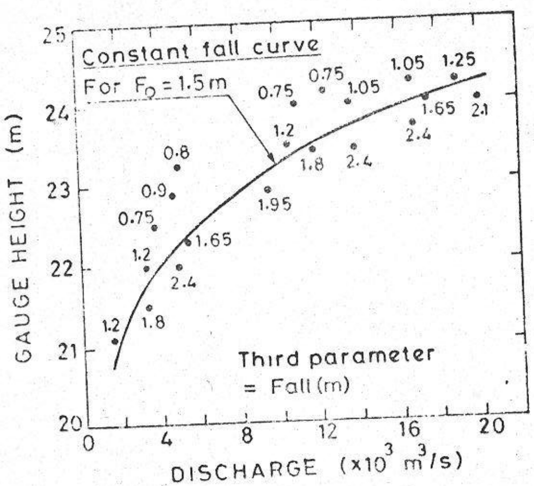

Fig. 4.24 Backwater effect on a rating curve—normalized curve

taken to be constant at all stages, F the actual fall at a given stage when the actual discharge is Q. These two fall values are related as

$$\frac{Q}{Q_0} = \left(\frac{F}{F_0}\right)^m \tag{4.31}$$

in which Q_0 = normalized discharge at the given stage when the fall is equal to F_0 and m = an exponent with a value close to 0.5. From the observed data, a convenient value of F_0 is selected. An approximate Q_0 vs G curve for a constant F_0 called *constant fall curve* is drawn. For each observed data, Q/Q_0 and F/F_0 values are calculated and plotted as Q/Q_0 vs F/F_0 (Fig. 4.25). This is called the *adjustment curve*. Both the constant-fall curve and the adjustment curve are refined, by trial and error to get

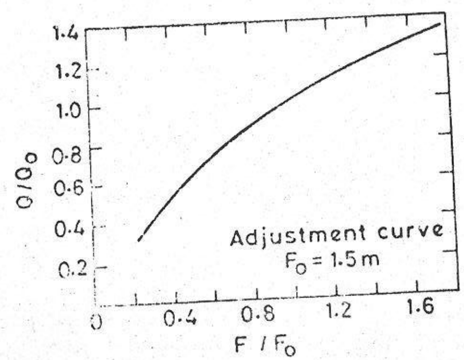

Fig. 4.25 Backwater effect on a rating curve—adjustment curve

the best-fit curves. When finalized, these two curves provide the stage-discharge information for gauging purposes. For example, if the observed stage is G_1 and fall F_1, first by using the adjustment curve the value of Q_1/Q_0 is read for a known value of F_1/F_0. Using the constant fall-rating curve, Q_0 is read for the given stage G_1 and the actual discharge calculated as $(Q_1/Q_0) \times Q_0$.

Unsteady-Flow Effect

When a flood wave passes a gauging station in the advancing portion of the wave the approach velocities are larger than in the steady flow at corresponding stages. Thus for the same stage, more discharge than in a steady uniform flow occurs. In the retreating phase of the flood wave the converse situation occurs with reduced approach velocities giving lower discharges than in an equivalent steady flow case. Thus the stage–discharge relationship for an unsteady flow will not be a single-valued relationship as in steady flow but it will be a looped curve as in Fig. 4.26. It may be noted that at the same stage, more discharge passes through the river during rising stages than in falling ones. Since the conditions for each flood may be different, different floods may give different loops.

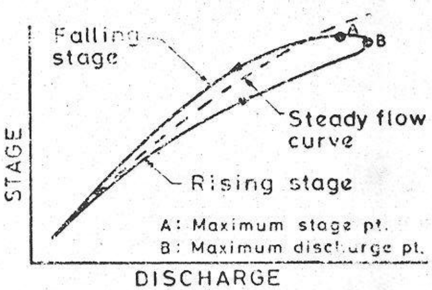

Fig. 4.26 Loop rating curve

If Q_n is the normal discharge at a given stage under steady uniform flow and Q_M is the measured (actual) unsteady flow the two are related as[6]

$$\frac{Q_M}{Q_n} = \sqrt{1 + \frac{1}{V_w S_0}\frac{dh}{dt}} \tag{4.32}$$

where S_0 = channel slope = water surface slope at uniform flow, dh/dt = rate of change of stage and V_w = velocity of the flood wave. For natural channels, V_w is usually assumed equal to 1.4 V, where V = average velocity for a given stage estimated by applying Manning's formula and the energy slope S_f. Also, the energy slope is used in place of S_0 in the denominator of Eq. (4.32). If enough field data about the flood magnitude and dh/dt are available, the term $(1/V_w S_0)$ can be calculated and plotted against the stage for use in Eq. (4.32). For estimating the actual discharge at an observed stage, Q_M/Q_n is calculated by using the observed data of dh/dt. Here Q_n is the discharge corresponding to the observed stage relationship for steady flow in the channel reach.

4.10 EXTRAPOLATION OF RATING CURVE

Most hydrological designs consider extreme flood flows. As an example, in the design of hydraulic structures, such as barrages, dams and bridges one needs maximum flood discharges as well as maximum flood levels. While the design flood discharge magnitude can be estimated from other considerations, the stage-discharge relationship at the project site will have to be used to predict the stage corresponding to design-flood discharges. Rarely will the available stage–discharge data include the design-flood range and hence the need for extrapolation of the rating curve.

Before attempting extrapolation, it is necessary to examine the site and collect relevant data on changes in the river cross-section due to flood plains, roughness and backwater effects. The reliability of the extrapolated value depends on the stability of the gauging section control. A stable control at all stages leads to reliable results. Extrapolation of the rating curve in an alluvial river subjected to aggradation and degradation is unreliable and the results should always be confirmed by alternate methods. There are many techniques of extending the rating curve and two well-known methods are described here.

Conveyance Method

The conveyance of a channel in nonuniform flow is defined by the relation

$$Q = K\sqrt{S_f} \tag{4.33}$$

where Q = discharge in the channel, S_f = slope of the energy line and K = conveyance. If Manning's formula is used,

$$K = \frac{1}{n} A R^{2/3} \tag{4.34}$$

where n = Manning's roughness, A = area of cross-section and, R = hydraulic radius. Since A and R are functions of the stage, the values of K for various values of stage are calculated by using Eq. (4.34) and plotted against the stage. The range of the stage should include values beyond the level up to which extrapolation is desired. Then a smooth curve is fitted to the plotted points [Fig. 4.27(a)]. Using the available discharge and stage data, values of S_f are calculated by using Eq. (4.33) as $S_f = Q^2/K^2$ and are plotted against the stage. A smooth curve is fitted through the plotted points [Fig. 4.27(b)]. This curve is then extrapolated keeping in mind that S_f approaches a constant value at high stages.

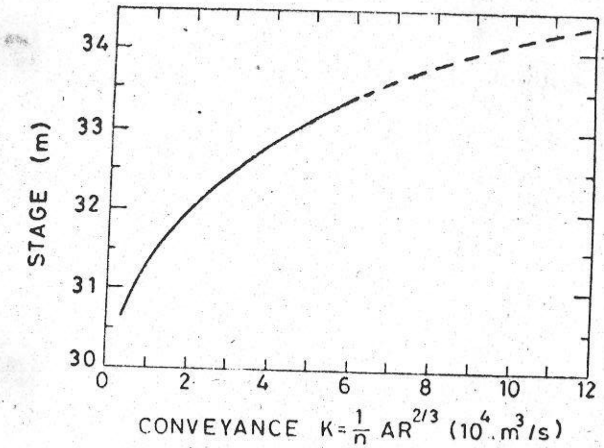

Fig. 4.27 (a) Conveyance method of rating curve extension: K vs stage

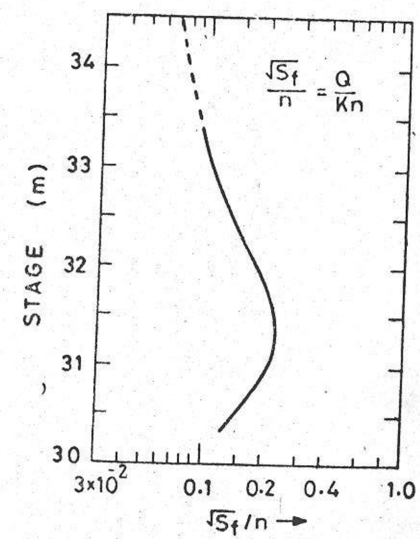

Fig. 4.27 (b) Conveyance method of rating curve extension: S_f vs stage

Using the conveyance and slope curves, the discharge at any stage is calculated as $Q = K\sqrt{S_f}$ and a stage–discharge curve covering the desired range of extrapolation is constructed. With this extrapolated-rating curve, the stage corresponding to a design-flood discharge can be obtained.

Logarithmic-Plot Method

In this technique the stage–discharge relationship given by Eq. (4.26) is made use of. The stage is plotted against the discharge on a log–log paper. A best-fit linear relationship is obtained for data points lying in the high-stage range and the line is extended to cover the range of extrapolation. Alternatively, coefficients of Eq. (4.26) are obtained by the least-square-error method by regressing X on Y in Eq. (4.27a). For this Eq. (4.27a) is written as

$$X = \alpha Y + C \tag{4.35}$$

where $X = \log (G\text{-}a)$ and $Y = \log Q$. The coefficients α and C are obtained as,

$$\alpha = \frac{N(\Sigma XY) - (\Sigma Y)(\Sigma X)}{N(\Sigma Y^2) - (\Sigma Y)^2} \tag{4.35a}$$

and

$$C = \frac{(\Sigma X) - \alpha (\Sigma Y)}{N} \tag{4.35b}$$

The relationship governing the stage and discharge is now

$$(G - a) = C_1 Q^{\alpha} \tag{4.36}$$

where $C_1 = \text{antilog } C$.

By the use of Eq. (4.36) the value of the stage corresponding to a design flood discharge is estimated.

4.11 HYDROMETRY STATIONS

As the measurement of discharge is of paramount importance in applied hydrologic studies, considerable expenditure and effort are being expended in every country to collect and store this valuable historic data. The WMO recommendations for the minimum number of hydrometry stations in various geographical regions are given in Table 4.2.

TABLE 4.2 WMO CRITERIA FOR HYDROMETRY STATION DENSITY

S. No.	Region	Minimum density (km²/station)	Tolerable density under difficult conditions (km²/station)
1.	Flat region of temperate, mediterranean and tropical zones	1,000-2,500	3,000–10,000
2.	Mountainous regions of temperate, medeterranean and tropical zones	300-1,000	1,000– 5,000
3.	Arid and polar zones	5,000-20,000	

Hydrometry stations must be sited in adequate numbers in the catchment area of all major streams so that the water potential of an area can be assessed as accurately as possible.

As per WMO norms, India needs 1700 hydrometry stations. Currently there are about 410 key-gauging stations maintained by the Central Water Commission in a few of which the modern moving-boat method facilities exist. In addition to the above, the state governments maintain nearly 800 discharge observation stations. There are also a large number of stations in the country where only gauge readings are taken.

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PROBLEMS

4.1 The following data were collected during a stream-gauging operation in a river. Compute the discharge.

Distance from left water edge (m)	Depth (m)	Velocity (m/s) at 0.2 *d*	Velocity (m/s) at 0.8 *d*
0.0	0.0	0.0	0.0
1.5	1.3	0.6	0.4
3.0	2.5	0.9	0.6
4.5	1.7	0.7	0.5
6.0	1.0	0.6	0.4
7.5	0.4	0.4	0.3
9.0	0.0	0.0	0.0

4.2 The velocity distribution in a stream is usually approximated as $v/v_a = (y/a)^m$, where v and v_a are velocities at heights y and a above the bed respectively and m is a coefficient with a value between 1/5 to 1/8. (i) Obtain an expression for $v/\bar{v}$, where $\bar{v}$ is the mean velocity in terms of the depth of flow. (ii) If $m = 1/6$ show that (a) the measured velocity is equal to the mean velocity if the velocity is measured at 0.6 depth from the water surface and (b) $\bar{v} = \frac{1}{2}(v_{0.2} + v_{0.82})$ where $v_{0.2}$ and $v_{0.82}$ are the velocities measured at 0.2 and 0.82 depths below the water surface respectively.

4.3 The following are the data obtained in a stream-gauging operation. A current meter with a calibration equation $V = (0.32\,N + 0.032)$ m/s where N = revolutions per second was used to measure the velocity at 0.6 depth. Using the mid-section method, calculate the discharge in the stream.

Distance from right bank (m)	0	2	4	6	9	12	15	18	20	22	23	24
Depth (m)	0	0.50	1.10	1.95	2.25	1.85	1.75	1.65	1.50	1.25	0.75	0
Number of revolutions	0	80	83	131	139	121	114	109	92	85	70	0
Time (s)	0	180	120	120	120	120	120	120	120	120	150	0

4.4 In the moving-boat method of discharge measurement the magnitude (V_R) and direction (θ) of the velocity of the stream relative to the moving boat are measured. The depth of the stream is also simultaneously recorded. Estimate the discharge in a river that gave the following moving-boat data. Assume the mean velocity in a vertical to be 0.95 times the surface velocity measured by the instrument.

Section	V_R (m/s)	θ (degrees)	Depth (m)	Remark
0	—	—	—	Right bank
1	1.75	55	1.8	θ is the angle made by
2	1.84	57	2.5	V_R with the boat
3	2.00	60	3.5	direction
4	2.28	64	3.8	
5	2.30	65	4.0	The various sections
6	2.20	63	3.8	are spaced at a constant
7	2.00	60	3.0	distance of 75 m apart
8	1.84	57	2.5	
9	1.70	54	2.0	Left Bank
10	—	—	—	

4.5 The dilution method with the sudden-injection procedure was used to measure the discharge of a stream. The data of concentration measurements are given below.

A fluorescent dye weighing 300 N used as a tracer was suddenly injected at station A at 07 h.

Time (h)	07	08	09	10	11	12	13	14	15	16	17	18
Concentration at station B in parts per 10^9 by weight	0	0	3.0	10.5	18.0	18.0	12.0	9.0	6.0	4.5	1.5	0

Estimate the stream discharge.

4.6 A 500 g/l solution of sodium dichromate was used as chemical tracer. It was dosed at a constant rate of 4 l/s and at a downstream section the equilibrium concentration was measured as 4 parts per million (ppm). Estimate the discharge in the stream.

4.7 A 200 g/l solution of common salt was discharged into a stream at a constant rate of 25 l/s. The background concentration of the salt in the stream water was found to be 10 ppm. At a downstream section where the solution was believed to have been completely mixed, the salt concentration was found to reach an equilibrium value of 45 ppm. Estimate the discharge in the stream.

4.8 It is proposed to adopt the dilution method of stream gauging for a river whose hydraulic properties at average flow are as follows: Width = 45 m, depth = 2.0 m, discharge = 85 m³/s, Chezy coefficient = 20 to 30. Determine the safe mixing length that has to be adopted for this stream.

4.9 During a high flow water-surface elevations of a small stream were noted at two sections A and B, 10 km apart. These elevations and other salient hydraulic properties are given below.

Section	Water-surface elevation (m)	Area of cross-section (m²)	Hydraulic radius (m)	Remarks
A	104.771	73.293	2.733	A is upstream of B
B	104.500	93.375	3.089	n = 0.020

The eddy loss coefficients of 0.3 for gradual expansion and 0.1 for gradual contraction are appropriate. Estimate the discharge in the stream.

4.10 A small stream has a trapezoidal cross section with base width of 12 m and side slope 2 horizontal: 1 vertical in a reach of 8 km. During a flood the high water levels recorded at either ends of the reach are as below.

Section	Elevation of bed	Water surface elevation	Remarks
Upstream	100.20	102.70	Manning's n = 0.030
Downstream	98.60	101.30	

Estimate the discharge in the stream.

4.11 Downstream of a main gauging station, an auxiliary gauge was installed and the following readings were obtained.

Main gauge	Auxiliary gauge	Discharge (m^3/s)
121.00	120.50	300
121.00	119.50	580

What discharge is indicated when the main gauge reading is 121.00 and the auxiliary gauge reads 120.10. [Hint: Use Eq. (4.31).]

4.12 The following are the coordinates of a smooth curve drawn to best represent the stage–discharge data of a river.

Stage (m)	20.80	21.42	21.95	22.37	23.00	23.52	23.90
Discharge (m^3/s)	100	200	300	400	600	800	1000

Determine the stage corresponding to zero discharge.

4.13 The stage–discharge data of a river are given below. Establish the rating curve by assuming the stage value for zero discharge as 20.50. Determine the stage of the river corresponding to a discharge of 2600 m^3/s.

Stage (m)	Discharge (m^3/s)	Stage (m)	Discharge (m^3/s)
21.95	100	24.05	780
22.45	220	24.55	1010
22.80	295	24.85	1220
23.00	400	25.40	1300
23.40	490	25.15	1420
23.75	500	25.55	1550
23.65	640	25.90	1760

4.14 During a flood the water surface at a section in a river was found to increase at a rate of 11.2 cm/h. The slope of the river is 1/3600 and the normal discharge for the river stage read from a steady-flow rating curve is 160 m^3/s. If the velocity of the flood wave can be assumed as 2.0 m/s, determine the actual discharge.

QUESTIONS

4.1 In a river carrying a discharge of 142 m^3/s, the stage at a station A was 3.6 m and the water surface slope was 1 in 6000. If during a flood the stage at A was 3.6 m and the water surface slope was 1/3000 the flood discharge (in m^3/s) was approximately.

(a) 100 (b) 284 (c) 71 (d) 200.

4.2 In a triangular channel the top width and depth of flow were 2.0 and 0.9 m respectively. Velocity measurements on the centreline at 18 and 72 cm below water surface indicated velocities of 0.60 m/s and 0.40 m/s respectively. The discharge in the channel (in m^3/s) is

(a) 0.90 (b) 1.80 (c) 0.45 (d) none of these.

4.3 In the moving-boat method of stream-flow measurement, the essential measurements are :

(a) the velocity recorded by the current meter, the depths and the speed of the boat
(b) the velocity and direction of the current meter, the depths and the time interval between depth readings
(c) the depth, time interval between readings, speed of boat and velocity of the stream
(d) the velocity and direction of the current meter and the speed of the boat.

4.4 If a gauging section is having shifting control due to backwater effects, then

(a) a loop rating curve results
(b) the section is useless for stream-gauging purposes
(c) the discharge is determined by area–velocity methods
(d) a secondary gauge downstream of the section is needed.

4.5 The stage discharge relation in a river during the passage of a flood wave is measured. If Q_R = discharge at a stage when the water surface was rising and Q_F = discharge at the same stage when the water surface was falling, then

(a) $Q_F = Q_R$ (b) $Q_R > Q_F$ (c) $Q_R < Q_F$ (d) Q_R/Q_F = constant at all stages.

4.6 The relation between the stage G and discharge Q in a river is usually expressed as

(a) $Q = C_r G^{\beta}$ (b) $Q = C_r(G-a)$ (c) $Q = C_r(G-a)^{\beta}$ (d) $G = C_r(Q-a)^{\beta}$.

4.7 The dilution method of stream gauging is ideally suited for measuring discharges in

(a) a large alluvial river
(b) flood flow in a mountain stream
(c) steady flow in a small turbulent steam
(d) a stretch of a river having heavy industrial pollution loads.

4.8 The slope–area method is extensively used in

(a) development of rating curve
(b) estimation of flood discharge based on high-water marks
(c) cases where shifting control exist
(d) cases where backwater effect is present.

4.9 For a given stream the rating curve applicable to a section is available. To determine the discharge in this stream, the following data are needed

(a) current meter readings at various verticals at the section
(b) slope of the water surface at the section

(c) stage at the section
(d) surface velocity at various sections.

4.10 During a flood in a wide, rectangular channel it is found that at a section the depth of flow increases by 50% and at this depth the water-surface slope is half its original value in a given interval of time. This marks an approximate change in the discharge of

(a) −33% (b) + 39% (c) + 20% (d) no change.

5

RUNOFF

5.1 INTRODUCTION

Runoff means the draining or flowing off of precipitation from a catchment area through a surface channel. It thus represents the output from the catchment in a given unit of time.

Consider a catchment area receiving precipitation. For a given precipitation, the evapotranspiration, initial loss, infiltration and detention-storage requirements will have to be first satisfied before the commencement of runoff. When these are satisfied, the excess precipitation moves over the land surfaces to reach smaller channels. This portion of the runoff is called *overland flow* and involves building up of a storage over the surface and draining off of the same. Usually the lengths and depths of overland flow are small and the flow is in the laminar regime. Flows from several small channels join bigger channels and flows from these in turn combine to form a larger stream, and so on, till the flow reaches the catchment outlet. The flow in this mode where it travels all the time over the surface as overland flow and through the channels as open-channel flow and reaches the catchment outlet is called *surface runoff*.

A part of the precipitation that infilters moves laterally through upper crusts of the soil and returns to the surface at some location away from the point of entry into the soil. This component of runoff is known variously as *interflow*, *through flow*, *storm seepage*, *subsurface*, *storm flow* or *quick return flow* (Fig. 5.1). The amount of interflow depends on the geological conditions of the catchment. A fairly pervious soil overlying a hard impermeable surface is conducive to large interflows. Depending upon the time delay between the infiltration and the outflow, the interflow is sometimes classified into *prompt interflow*, i.e. the interflow with the least time lag and *delayed interflow*.

Another route for the infiltered water is to undergo deep percolation and

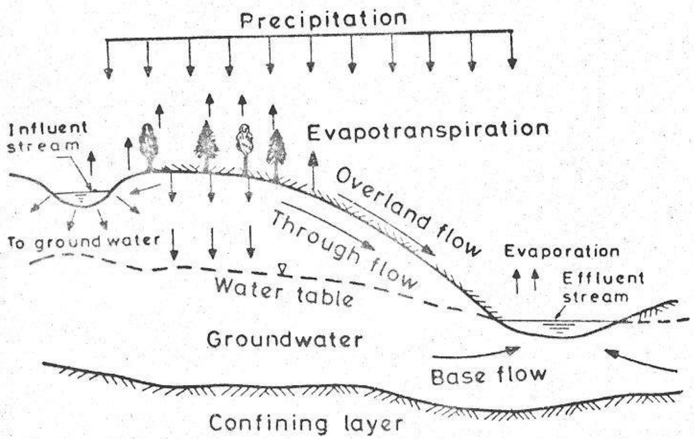

Fig. 5.1 Different routes of runoff

reach the groundwater storage in the soil. The groundwater follows a complicated and long path of travel and ultimately reaches the surface. The time lag, i.e. the difference in time between the entry into the soil and outflows from it is very large, being of the order of months and years. This part of runoff is called *groundwater runoff* or *groundwater flow*. Groundwater flow provides the dry-weather flow in perennial streams.

Based on the time delay between the precipitation and the runoff, the runoff is classified into two categories; as

1. Direct runoff, and
2. base flow.

These are discussed below.

Direct Runoff

It is that part of runoff which enters the stream immediately after the precipitation. It includes surface runoff, prompt interflow and precipitation on the channel surface. In the case of snow-melt, the resulting flow entering the stream is also a direct runoff. Sometimes terms such as *direct storm runoff* and *storm runoff* are used to designate direct runoff.

Base Flow

The delayed flow that reaches a stream essentially as groundwater flow is called *base flow*. Many times delayed interflow is also included under this category.

Runoff, representing the response of a catchment to precipitation reflects the integrated effects of a wide range of catchment, climate and precipitation characteristics. True runoff is therefore stream flow in the natural

condition, i.e. without human intervention. Such a stream flow unaffected by works of man, such as structures for storage and diversion on a stream is called *virgin flow*. When there exist storage or diversion works on a stream, the flow in the downstream channel is affected by structures and hence does not represent the true runoff unless corrected for storage effects and the diversion of flow and return flow.

EXAMPLE 5.1 *The following table gives values of measured discharges at a stream-gauging site in a year. Upstream of the gauging site a weir built across the stream diverts 3.0 and 0.50 Mm³ (million m³) of water per month for irrigation and for use in an industry respectively. The return flows from irrigation is estimated at 0.8 Mm³ and from the industry at 0.30 Mm³ reaching the stream upstream of the gauging site. Estimate the virgin flow. If the catchment area is 120 km² and the average annual rainfall is 185 cm, determine the runoff-rainfall ratio.*

Month	1	2	3	4	5	6	7	8	9	10	11	12
Gauged flow (Mm^3)	2.0	1.5	0.8	0.6	2.1	8.0	18.0	22.0	14.0	9.0	7.0	3.0

In a month the virgin flow volume R_v is obtained as

$$R_v = V_s + V_d - V_r$$

where V_s = volume of flow measured

V_d = volume diverted out of the stream

V_r = volume returned to the stream

The calculations are performed in the following tabular form :

$V_r = 0.80 + 0.30 = 1.10\ Mm^3$, $V_d = 3.0 + 0.5 = 3.5\ Mm^3$.

Volume (in Mm^3)

Month	1	2	3	4	5	6	7	8	9	10	11	12
V_s	2.0	1.5	0.8	0.6	2.1	8.0	18.0	22.0	14.0	9.0	7.0	3.0
V_d	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5	3.5
V_r	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1	1.1
R_v	4.4	3.9	3.2	3.0	4.5	10.4	20.4	24.4	16.4	11.4	9.4	5.4

Total R_v = 116.8 Mm^3

∴ Annual virgin flow = annual runoff = 116.8 Mm^3

Area of the catchment 120 km^2 = $1.2 \times 10^8\ m^2$

$$\text{Annual runoff} = \frac{1.168 \times 10^8 \text{ m}^3}{1.2 \times 10^8 \text{ m}^2} = 0.973 \text{ m}$$

$$= 97.3 \text{ cm}$$

$$\text{Rainfall} \quad = 185 \text{ cm}$$

$$\therefore \text{ Runoff/rainfall} = \frac{97.3}{185} = 0.526$$

5.2 HYDROGRAPH

A plot of the discharge in a stream plotted against time chronologically is called a hydrograph. Depending upon the unit of time involved, we have:

1. Annual hydrographs showing the variation of daily or weekly or 10 daily mean flows over a year;
2. monthly hydrographs showing the variation of daily mean flows over a month;
3. seasonal hydrographs depicting the variation of the discharge in a particular season such, as the monsoon season or dry season; and
4. flood hydrographs or hydrographs due to a storm representing stream flow due to a storm over a catchment.

Each of these types have particular applications. Annual and seasonal hydrographs are of use in (i) calculating the surface water potential of stream, (ii) reservoir studies and (iii) drought studies. Flood hydrographs are essential in analysing stream characteristics associated with floods. This chapter is concerned with the estimation and use of long-term runoffs. The study of storm hydrograph forms the subject matter of the next chapter.

Water Year

In annual runoff studies it is advantageous to consider a water year beginning from the time when the precipitation exceeds the average evapotranspiration losses. In India, June 1st is the beginning of a water year which ends on May 31st of the following calender year. In a water year a complete cycle of climatic changes is expected and hence the water budget will have the least amount of carry over.

5.3 RUNOFF CHARACTERISTICS OF STREAMS

A study of the annual hydrographs of streams enables one to classify streams into three classes as (i) perennial, (ii) intermittent and (iii) ephemeral.

A perennial stream is one which always carries some flow (Fig. 5.2), There is considerable amount of groundwater flow throughout the year.

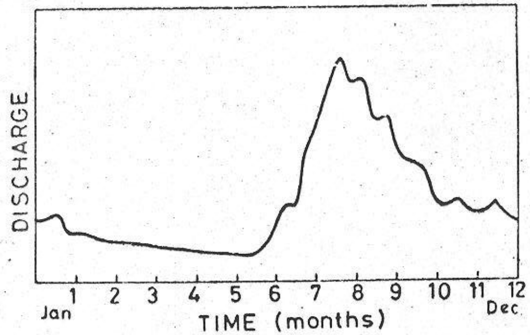

Fig. 5.2 Perennial stream

Even during dry seasons the water table will be above the bed of the stream.

An intermittent stream has limited contribution from the groundwater. During the wet season the water table is above the stream bed and there is a contribution of the base flow to the stream flow. However, during dry seasons the water table drops to a level lower than that of the stream bed and the stream dries up. Excepting for an occasional storm which can produce a short-duration flow, the stream remains dry for the most part of the dry months (Fig. 5.3).

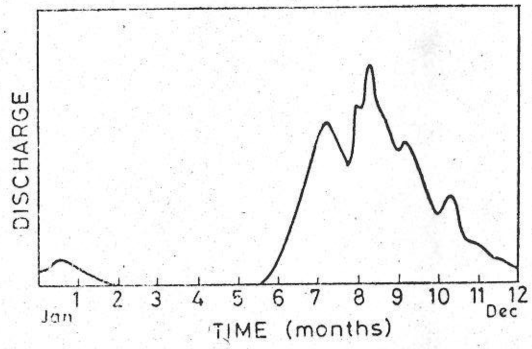

Fig. 5.3 Intermittent stream

An ephemeral stream is one which does not have any base-flow contribution. The annual hydrograph of such a river show series of short-duration spikes marking flash flows in response to storms (Fig. 5.4). The stream becomes dry soon after the end of the storm flow. Typically an ephemeral stream does not have any well-defined channel. Most rivers in arid zones are of the ephemeral kind.

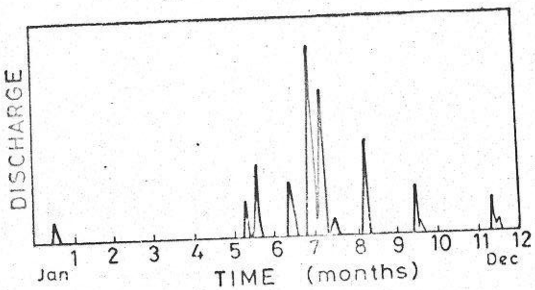

Fig. 5 4 Ephemeral stream

The flow characteristics of a stream depend upon:

1. The ranifall characteristics, such as magnitude intensity, distribution in time and space and its variability;
2. catchment characteristics such as soil, vegetation, slope, geology, shape and drainage density; and
3. climatic factors which influence evapotranspiration.

The interrelationship of these factors is extremely complex. However, at the risk of oversimplification, the following salient points can be noted:

1. The seasonal variation of rainfall is clearly reflected in the runoff. High stream discharges occur during monsoon months and low flow which is essentially due to base flow is maintained during the rest of the year.
2. The shape of the storm hydrograph and hence the peak flow is essentially controlled by the storm and physical characteristics of the catchment. Evapotranspiration plays a minor role in this.
3. The annual runoff volume (yield) of a stream is mainly controlled by the amount of rainfall and evapotranspiration. The geology of the catchment is significant to the extent of deep percolation losses.

5.4 YIELD (ANNUAL RUNOFF VOLUME)

The total quantity of water that can be expected from a stream in a given period such as a year is called the *yield* of the river. It is usual for the yield to be referred to the period of a year and then it represents the annual runnoff volume. In this book the term yield is used to mean the annual runoff volume unless otherwise specified. The calculation of yield is of fundamental importance in all water-resources development studies. The various methods used for the estimation of yield can be listed as below:

1. Correlation of stream flow and rainfall,

2. empirical equations, and
3. watershed simulations.

A brief review of these methods follow.

Rainfall–Runoff Correlation

The relationship between rainfall and the resulting runoff is quite complex and is influenced by a host of factors relating the catchment and climate. Further, there is the problem of paucity of data which forces one to adopt simple correlations for the adequate estimation of runoff. One of the most common methods is to correlate runoff, R with rainfall, P values. Plotting of R values against P and drawing a best-fit line can be adopted for very rough estimates. A better method is to fit a linear regression line between R and P and to accept the result if the correlation coefficient is nearer unity. The equation for straight-line regression between runoff R and rainfall P is

$$R = a\,P + b \qquad (5.1)$$

and the values of the coefficients a and b are given by

$$a = \frac{N(\Sigma\, P\,R) - (\Sigma\, P)\,(\Sigma\, R)}{N(\Sigma\, P^2) - (\Sigma\, P)^2} \qquad (5.2)$$

and

$$b = \frac{\Sigma\, R - a\,\Sigma\, P}{N} \qquad (5.3)$$

in which N = number of observation sets R and P. The coefficient of correlation r can be calculated as

$$r = \frac{N(\Sigma\, P\,R) - (\Sigma\, P)\,(\Sigma\, R)}{\sqrt{[N(\Sigma\, P^2) - (\Sigma\, P)^2] \times [N(\Sigma\, R^2) - (\Sigma\, R)^2]}} \qquad (5.4)$$

The value of r lies between 0 to + 1 as R can have only positive correlation with P. A value of $0.6 < r < 1.0$ indicates good correlation. Further it should be noted that $R \geqslant 0$.

For large catchments, it is found advantageous to have an exponential relationship as

$$R = \beta\, P^m \qquad (5.5)$$

where β and m are constants, instead of the linear relationship given by Eq. (5.1). In that case Eq. (5.5) is reduced to a linear form by logarithmic transformation as

$$\ln R = m\ \ln p + \ln \beta \qquad (5.6)$$

and the coefficients m and ln β determined by using the method indicated earlier.

Since rainfall records of longer duration than the runoff data are normally available for a catchment, the regression equation [(Eq. (5.1) or (5.5)] can be used to generate synthetic runoff data by using rainfall data. While this may be adequate for preliminary studies, for accurate results sophisticated methods are adopted for synthetic generation of runoff data. Many improvements of the above basic rainfall-runoff correlation by considering additional parameters such as soil moisture or antecedent rainfall have been attempted. Antecedent rainfall influences the initial soil moisture and hence the infiltration rate at the start of a storm. For calculation of the annual runoff from the annual rainfall a commonly used antecedent precipitation index, P_a is given by

$$P_a = a\,P_i + b\,P_{i-1} + c\,P_{i-2} \tag{5.7}$$

where P_i, P_{i-1} and P_{i-2} are the annual precipitation in the ith, $(i-1)$th and $(i-2)$th year, i = current year, a, b and c are coefficients with their sum equal to unity. The coefficients are found by trial and error to produce best results.

There are many other types of antecedent precipitation indices in use to achieve good correlations of rainfall and runoff. The use of coaxial chart with a defined antecedent precipatation index is given by Linsley et al.[1]

EXAMPLE 5.2 *Given below are the monthly rainfall P and the corresponding runoff R values covering a period of 18 months for a catchment. Develop a correlation equation between R and P.*

Month	P (cm)	R (cm)	Month	P (cm)	R (cm)
1	5	0.5	10	30	8.0
2	35	10.0	11	10	2.3
3	40	13.8	12	8	1.6
4	30	8.2	13	2	0.0
5	15	3.1	14	22	6.5
6	10	3.2	15	30	9.4
7	5	0.1	16	25	7.6
8	31	12.0	17	8	1.5
9	36	16.0	18	6	0.5

For the given data,

$\Sigma P = 348$ $\qquad \Sigma R = 104.3$ $\qquad N = 18$

$\Sigma P^2 = 9534$ $\qquad \Sigma R^2 = 1040.51$ $\qquad \Sigma PR = 3083.3$

$(\Sigma P)^2 = 121104$ $\qquad (\Sigma R)^2 = 10878.49$

For the correlation equation $R = aP + b$, by Eq. (5.2)

$$a = \frac{N(\Sigma PR) - (\Sigma P)(\Sigma R)}{N(\Sigma P^2) - (\Sigma P)^2}$$

$$= \frac{(18 \times 3083.3) - (348 \times 104.3)}{(18 \times 9534) - 121104} = 0.380$$

By Eq. (5.3),

$$b = \frac{\Sigma R - a\,\Sigma P}{N}$$

$$= \frac{104.3 - (0.380 \times 348)}{18} = -1.55$$

Hence $R = 0.38\,P - 1.55$.

In this equation both R and P are in cm and $R \geq 0$. Correlation coefficient r by Eq. (5.4) is

$$r = \frac{N(\Sigma PR) - (\Sigma P)(\Sigma R)}{\sqrt{[N(\Sigma P^2) - (\Sigma P)^2 \times [N(\Sigma R^2) - (\Sigma R^2)]}}$$

$$= \frac{(18 \times 3083.3) - (348 \times 104.3)}{\sqrt{[(18 \times 9534) - 121104] \times [(18 \times 1040.51) - 10878.49]}}$$

$= 0.964$

Empirical Equations

The importance of estimating the water availability from the available hydrologic data for purposes of planning water-resource projects was recognised by engineers even in the last century. With a keen sense of observation in their region of their activity many engineers of the past have developed empirical runoff estimation formulae. However, these formulae are applicable only to the region in which they were derived. These formulae are essentially rainfall-runoff relations with additional third or fourth parameters to account for climatic or catchement characteristics. Some of the important formulae used in various parts of India are given below.

Binnie's Percentages

Sir Alexander Binnie measured the runoff from a small catchment near Nagpur (Area of 16 km²) during 1869 and 1872 and developed curves of cumulative runoff against cumulative rainfall. The two curves were found to be similar. From these he established percentages of runoff from rainfall. These percentages have been used in Madhya Pradesh and Vidarbha region of Maharashtra for the estimation of yield.

Barlow's Tables

Barlow, the first Chief Engineer of the Hydro-Electric Survey of India (1915) on the basis of his study in small catchments (area~130 km²) in Uttar Pradesh expressed runoff R as

$$R = K_b\, P \tag{5.8}$$

where K_b = runoff coefficient which depends upon the type of catchment and nature of monsoon rainfall. Values of K_b are given in Table 5.1.

TABLE 5.1 BARLOW'S RUNOFF COEFFICIENT K_b IN PERCENTAGE
(Developed for use in UP)

Class	Description of catchment	Values of K_b (percentage)		
		Season 1	Season 2	Season 3
A	Flat, cultivated aud absorbent soils	7	10	15
B	Flat, partly cultivated, stiff soils	12	15	18
C	Average catchment	16	20	32
D	Hills and plains with little cultivation	28	35	60
E	Very hilly, steep and hardly any cultivation	36	45	81

83

Season 1: Light rain, no heavy downpour
Season 2: Average or varying rainfall, no continuous downpour
Season 3: Continuous downpour

Strange's Tables

Strange (1928) studied the then available data on rainfall and runoff in the border areas of present-day Maharashtra and Karnataka and obtained the values of the runoff coefficient

$$K_s = R/P \tag{5.9}$$

as a function of the catchment character. For purposes of calculating the yield from the total monsoon rainfall, the catchments were characterized as "good", "average" and "bad". Values of K_s for these catchments are shown in Table 5.2. Strange also gave a table for calculating the daily runoff from daily rainfall. In this the runoff coefficient depends not only on the amount of rainfall but also on the state of the ground. Three categories of the original ground state as 'dry', 'damp' and 'wet' are used by him. 123

Inglis and DeSouza Formula

As a result of careful stream gauging in 53 sites in Western India, Inglis and DeSouza (1929) evolved two regional formulae between annual runoff 154 R in cm and annual rainfall P in cm as follows:

TABLE 5.2 EXTRACT OF STRANGE'S TABLES OF RUNOFF COEFFICIENT K_s IN PER CENT
(For use in border areas of Maharashtra and Karnataka)

Total monsoon rainfall (cm)	Runoff coefficient K_s per cent		
	Good catchment	Average catchment	Bad catchment
25	4.3	3.2	2.1
50	15.0	11.3	7.5
75	26.3	19.7	13.1
100	37.5	28.0	18.7
125	47.6	35.7	23.8
150	58.9	44.1	29.4

1. **For Ghat regions of western India**

$$R = 0.85\, P - 30.5 \tag{5.10}$$

2. **For Deccan plateau**

$$R = \frac{1}{254}\, P\,(P - 17.8) \tag{5.11}$$

Khosla's Formula

Khosla (1960) analysed the rainfall, runoff and temperature data for various catchments in India and USA to arrive at an empirical relationship between runoff and rainfall. The time period is taken as a month. His relationship for monthly runoff is

$$R_m = P_m - L_m \tag{5.12}$$

and $L_m = 0.48\, T_m$ for $T_m > 4.5°C$

where R_m = monthly runoff in cm and $R_m \geqslant 0$
P_m = monthly rainfall in cm
L_m = monthly losses in cm
T_m = mean monthly temperature of the catchment in °C

For $T_m \leqslant 4.5°C$, the loss L_m may provisionally be assumed as

T°C	4.5	−1	−6.5
L_m (cm)	2.17	1.78	1.52

Annual runoff = ΣR_m

Khosla's formula is indirectly based on the water-balance concept and th mean monthly catchment temperature is used to reflect the losses due to evapotranspiration. The formula has been tested on a number of catchments in India and is found to give fairly good results for the annual yield for use in preliminary studies. The formula can also be used to generate synthetic runoff data from historical rainfall and temperature data.

EXAMPLE 5.3 *For a catchment in UP, India, the mean monthly rainfall and temperatures are given. Calculate the annual runoff and annual runoff coefficient by Khosla's formula.*

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Temp. (°C)	12	16	21	27	31	34	31	29	28	29	19	14
Rainfall (cm)	4	4	2	0	2	12	32	29	16	2	1	2

In using Khosla's formula (Eq. 5.12),

$$R_m = P_m - L_m$$

if the loss L_m is higher than P_m then R_m is taken to be zero. The values of R_m calculated by Eq. (5.12) are

Month	Jan	Feb	Mar	Apr	May	June	July	Aug	Sep	Oct	Nov	Dec
Runoff (cm)	0	0	0	0	0	0	17.1	15.1	2.6	0	0	0

Annual runoff = Total = 34.8 cm

$$\text{Annual runoff coefficient} = \frac{\text{Annual runoff}}{\text{Annual rainfall}} = \frac{34.8}{116.0} = 0.30$$

Watershed Simulation

The hydrologic water-budget equation for the determination of runoff for a given period is written as

$$R = R_s + G_o = P - E_{et} - \Delta S \tag{5.13}$$

in which R_s = surface runoff, P = precipitation, E_{et} = actual evapotranspiration, G_o = net groundwater outflow and ΔS = change in the soil-moisture storage. The sum of R_s and G_o is considered to be given by the total runoff R, i.e. streamflow.

Starting from an initial set of values, one can use Eq. (5.13) to calculate R by knowing values of P and functional dependence of E_{et}, ΔS and infiltration rates with catchment and climatic conditions. For accurate results, the functional dependence of various parameters governing the runoff in the catchment and values of P at short time intervals are needed. Calculations can then be done sequentially to obtain the runoff at any time. However, the calculation effort involved is enormous if attempted manually. With the availability of digital computers the use of water budgeting as above to determine the runoff has become feasible. This technique of predicting the runoff, which is the catchment response to a given rainfall input is called *deterministic watershed simulation.* In this the mathematical relationships describing the interdependence of various parameters in the system are first prepared and this is called the ***model.*** The model is then calibrated i.e. the numerical values of various coefficients determined, by simulating the known rainfall-runoff records. The accuracy of the model is further checked by reproducing the results of another string of rainfall data for which runoff values are known. This phase is known as *validation* or *verification* of the model. After this, the model is ready for use.

Crawford and Linsley (1959) pioneered this technique by proposing a watershed simulation model known as the Stanford Watershed Model (SWM). This underwent successive refinements and the 'Stanford Watershed Model-IV (SWM-IV) suitable for use on a wide variety of conditions was proposed in 1966. The flow chart of SWM-IV is shown in Figure 5.5. The main inputs are hourly precipitation and daily evapotranspiration in addition to physical description of the catchment. The model considers the soil in three zones with distinct properties to simulate evapotranspiration, infiltration, overland flow, channel flow, interflow and baseflow phases of the runoff phenomenon. For calibration about 5 years of data are needed. In the calibration phase, the initial guess value of parameters are adjusted on a trial-and-error basis until the simulated response matches the recorded values. Using an additional length of rainfall-runoff of about 5 years duration, the model is verified for its ability to give proper response. A detailed description of the application of SWM to an Indian catchment is given in Ref. 2.

SWM-IV has been tested in a number of applications and it has been found to give satisfactory results for the yield and not so satisfactory results in predicting peak values. However, it requires considerable familiarity with the model to arrive at optimal values in the calibrating stage. An improved version called Hydrocomp Simulation Program (HSP) (1966) gives a package of three simulation modules to solve a variety of water-shed-simulation problems. Another model called the SSARR model (Streamflow Synthesis and Reservoir Regulation Model) developed by Rockwood (1968) for the Columbia river basin, USA has been successfully tested on large watersheds. The Kentucky Watershed model (KWM) (1970) is a revised and updated version of SWM-IV. KWM is used with an optimization programme called OPSET which generates best-fit parameter estimates. The successful use of KWM to catchments up to 1200 km² size have been reported.

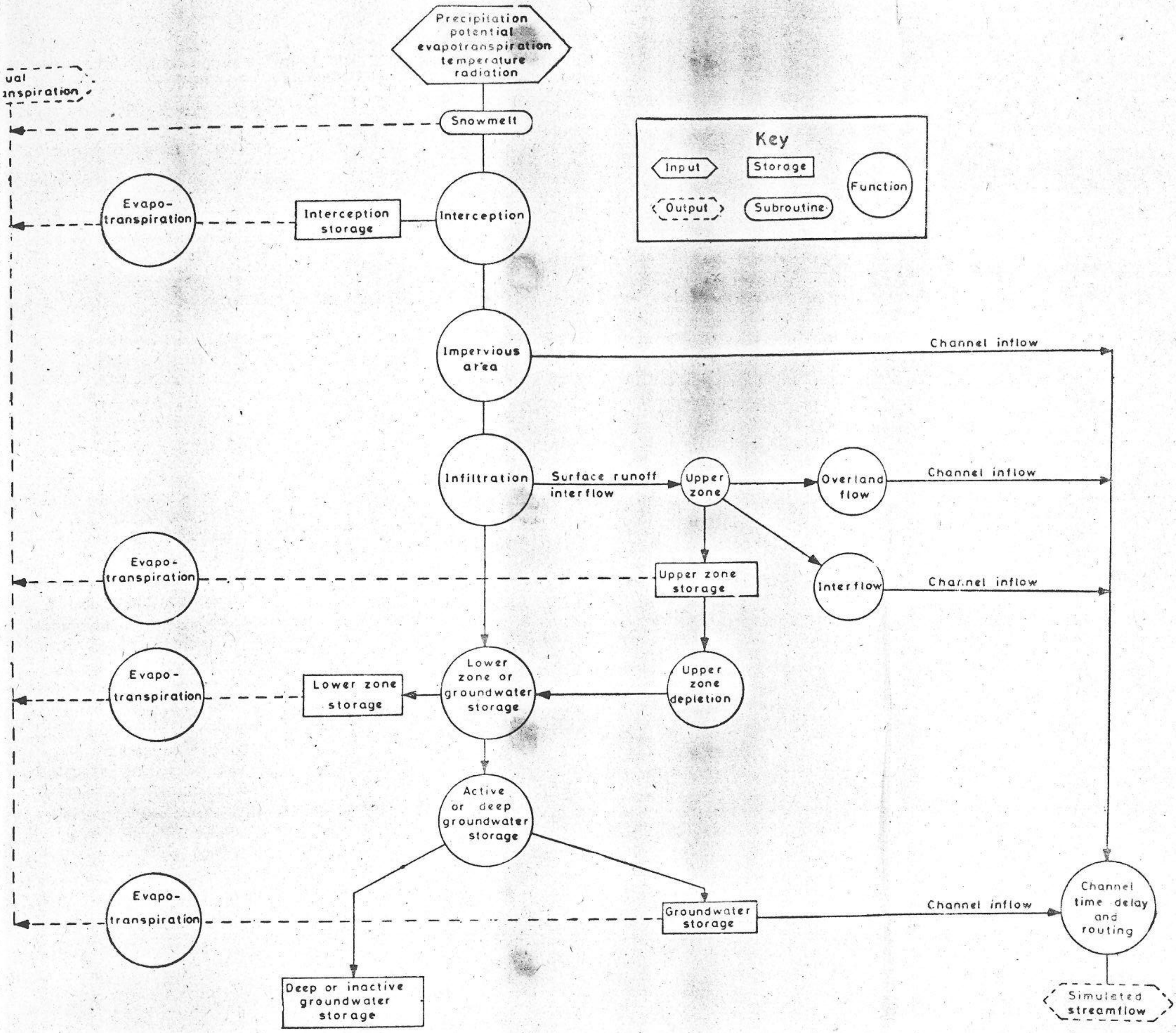

Fig. 5.5 Flow chart of SWM-IV

5.5 FLOW-DURATION CURVE

It is well-known that the streamflow varies over a water year. One of the popular methods of studying this streamflow variability is through flow-duration curves. A flow-duration curve of a stream is a plot of discharge against the per cent of time the flow was equalled or exceeded. This curve is also known as *discharge-frequency curve.*

The streamflow data is arranged in a descending order of discharges, using class intervals if the number of individual values is very large. The data used can be daily, weekly, ten daily or monthly values. If N number of data points are used in this listing, the plotting position of any discharge (or class value) Q is

$$P_p = \frac{m}{N+1} \times 100\% \qquad (5.14)$$

where m is the order number of the discharge (or class value), P_p = percentage probability of the flow magnitude being equalled or exceeded. The plot of the discharge Q against P_p is the flow duration curve (Fig. 5.6). Arithmetic scale paper, or semi-log or log-log paper is used depending upon the range of data and use of the plot. The flow duration curve represents the cumulative frequency distribution and can be considered to represent the streamflow variation of an average year. The ordinate Q_p at any percentage probability P_p represents the flow magnitude in an average year that can be expected to be equalled or exceeded P_p per cent of time and is termed as P_p% *dependable flow.* In a perennial river Q_{100} = 100% dependable flow is a finite value. On the other hand in an intermittent or ephemeral river the streamflow is zero for a finite part of an year and as such Q_{100} is equal to zero.

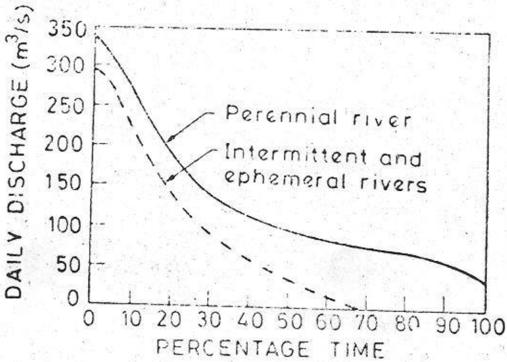

Fig. 5.6 Flow duration curve

The following characteristics of the flow duration curve are of interest.

1. The slope of a flow duration curve depends upon the interval of data selected. For example, a daily stream flow data gives a steeper curve

than a curve based on monthly data for the same stream. This is due to the smoothening of small peaks in monthly data.

2. The presence of a reservoir in a stream considerably modifies the virgin-flow duration curve depending on the nature of flow regulation Figure 5.7 shows the typical reservoir regulation effect.

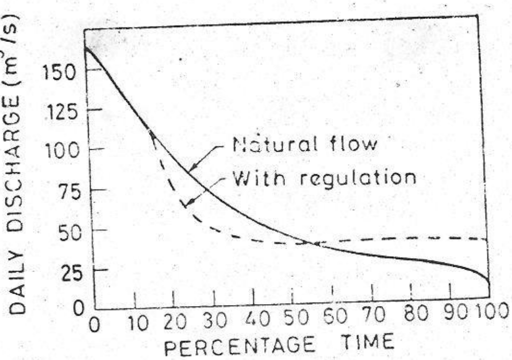

Fig. 5.7 **Reservoir regulation effect**

3. The virgin-flow duration curve when plotted on a log probability paper plots as a straight line at least over the central region. From this property, various coefficients expressing the variability of the flow in a stream can be developed for the description and comparison of different streams.
4. The chronological sequence of occurrence of the flow is masked in the flow-duration curve. A discharge of say 1000 m^3/s in a stream will have the same percentage P_p whether it occurred in January or June. This aspect, a serious handicap, must be kept in mind while interpreting a flow-duration curve.
5. The flow-duration curve plotted on a log-log paper (Fig. 5.8) is useful in comparing the flow characteristics of different streams. A steep slope of the curve indicates a stream with a highly variable discharge. On the other hand, a flat slope indicates a slow response of the catchment to the rainfall and also indicates small variability. At the lower end of the curve, a flat portion indicates considerable base flow. A flat curve on the upper portion is typical of river basins having large flood plains and also of rivers having large snowfall during a wet season.

Flow-duration curves find considerable use in water-resources planning and development activities. Some of the important uses are :

1. In evaluating various dependable flows in the planning or water-resources engineering projects;
2. in evaluating the characteristics of the hydropower potential of a river;

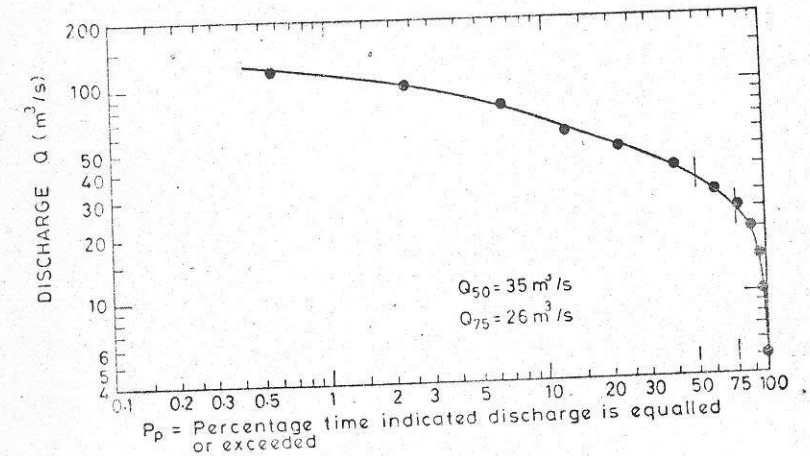

Fig. 5.8 Flow duration curve—Example 5.4

3. in the design of drainage systems;
4. in flood-control studies;
5. in computing the sediment load and dissolved solids load of a stream; and
6. in comparing the adjacent catchments with a view to extend the streamflow data.

EXAMPLE 5.4 *The daily flows of a river for three consecutive years are shown in Table 5.3. For convenience the discharges are shown in class intervals and the number of days the flow belonged to the class is shown. Calculate the 50 and 75% dependable flows for the river.*

The data are arranged in descending order of class value. In Table 5.3, column 5 shows the total number of days in each class. Column 6 shows the cumulative total of column 5, i.e. the number of days the flow is equal to or greater than the class interval. This gives the value of m. The percentage probability P_p, the probability of flow in the class interval being equalled or exceeded is given by Eq. (5.14),

$$P_p = \frac{m}{(N+1)} \times 100\%.$$

In the present case $N = 1096$. The smallest value of the discharge in each class interval is plotted against P_p on a log-log paper (Fig. 5.8). From this figure Q_{50} = 50% dependable flow = 35 m³/s and Q_{75} = 75% dependable flow = 26 m³/s.

5.6 FLOW-MASS CURVE

The flow-mass curve is a plot of the cumulative discharge volume against

TABLE 5.3 CALCULATION OF FLOW DURATION CURVE FROM DAILY FLOW DATA—EXAMPLE 5.4

Daily mean discharge (m³/s)	No. of days flow in each class interval 1961-62	1962-63	1963-64	Total of columns 2, 3, 4 1961-64	Cumulative Total m	$P_p = \left(\frac{m}{N+1}\right) \times 100\%$
1	2	3	4	5	6	7
140–120.1	0	1	5	6	6	0.55
120–100.1	2	7	10	19	25	2.28
100– 80.1	12	18	15	45	70	6.38
80– 60.1	15	32	15	62	132	12.03
60– 50.1	30	29	45	104	236	21.51
50– 40.1	70	60	64	194	430	39.19
40– 30.1	84	75	76	235	665	60.62
30– 25.1	61	50	61	172	837	76.30
25– 20.1	43	45	38	126	963	87.78
20– 15.1	28	30	25	83	1046	95.35
15– 10.1	15	18	12	45	1091	99.45
10– 5.1	5	—	—	5	1096	99.91
Total	365	365	366	N=1096		

time plotted in chronological order. The ordinate of the mass curve, V at any time t is thus

$$V = \int_{t_0}^{t} Q\, dt \tag{5.15}$$

where t_0 is the time at the beginning of the curve and Q is the discharge rate. Since the hydrograph is a plot of Q vs t, it is easy to see that the flow-mass curve is an integral curve (summation curve) of the hydrograph. The flow-mass curve is also known as Rippl's mass curve after Rippl (1882) who suggested its use first. Figure 5.9 shows a typical flow-mass curve. Note that the abscissa is chronological time in months in this figure. It can also be in days, weeks or months depending on the data being analysed. The ordinate is in units of volume in million m³. Other units employed for ordinate include m³/s. day (cumec day), ha.m and cm over a catchment area.

The slope of the mass curve at any point represents $\frac{dV}{dt} = Q$ = rate of flow at that instant. If two points M and N are connected by a straight

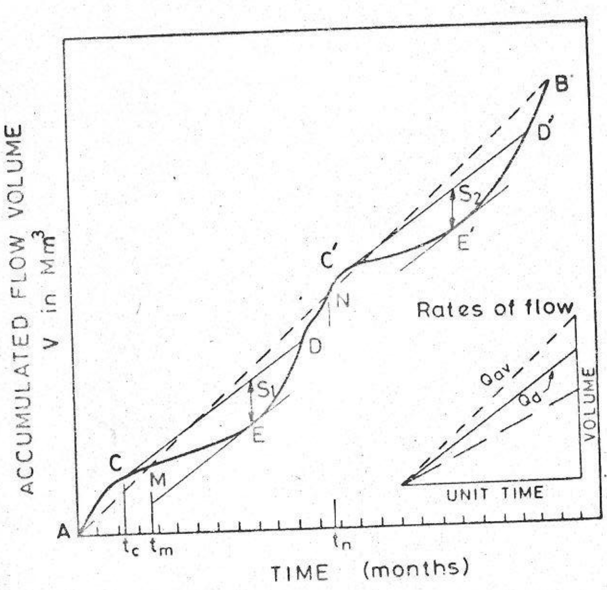

Fig. 5.9 Flow mass curve

line, the slope of the line represents the average rate of flow that can be maintained between the times t_m and t_n if a reservoir of adequate storage is available. Thus the slope of the line AB joining the first and the last points of a mass curve represents the average discharge over the whole period of plotted record.

Calculation of Storage Volume

Consider a reservoir on the stream whose mass curve is plotted in Fig. 5.9. If it is assumed that the reservoir is full at the beginning of a dry period, i.e. when the inflow rate is less than the withdrawal (demand) rate, the maximum amount of water drawn from the storage is the cumulative difference between supply and demand volumes from the beginning of the dry season. Thus the storage required S is

$$S = \text{maximum of } (\Sigma V_D - \Sigma V_S)$$

where V_D = demand volume, V_S = supply volume. The storage, S which is the maximum cumulative deficiency in any dry season is obtained as the maximum difference in the ordinate between mass curves of supply and demand. The minimum storage volume required by a reservoir is the largest of such S values over different dry periods.

Consider the line CD of slope Q_d drawn trangential to the mass curve at a high point on a ridge. This represents a constant rate of withdrawal Q_d

from a reservoir and is called *demand line*. If the reservoir is full at C (at time t_c), then from point C to E the demand is larger than the supply rate as the slope of the flow-mass curve is smaller than the demand line CD. Thus the reservoir will be depleting and the lowest capacity is reached at E. The difference in the ordinates between the demand line CD and a line EF drawn parallel to it and tangential to the mass curve at E (S_1 in Fig 5.9) represents the volume of water needed as storage to meet the demand from the time the reservoir was full. If the flow data for a large time period is available, the demand lines are drawn tangentially at various other ridges (e.g. $C'D'$ in Fig. 5.9) and the largest of the storages obtained is selected as the minimum storage required by a reservoir. Example 5.5 explains this use of the mass curve. Example 5.6 indicates storage calculations by arithmetic calculations by adopting the mass-curve principle.

EXAMPLE 5.5 *The following table gives the mean monthly flows in a river during 1981. Calculate the minimum storage required to maintain a demand rate of 40 m³/s.*

Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
Mean Flow (m³/s)	60	45	35	25	15	22	50	80	105	90	80	70

From the given data the monthly flow volume and accumulated volumes are calculated as in table 5.4. The actual number of days in the month are

TABLE 5.4 CALCULATION OF MASS CURVE–EXAMPLE 5.5

Month	Mean flow (m³/s)	Monthly flow volume (cumec · day)	Accumulated volume (cumec · day)
Jan	60	1860	1860
Feb	45	1260	3120
Mar	35	1085	4205
April	25	750	4955
May	15	465	5420
June	22	660	6080
July	50	1550	7630
Aug	80	2480	10,110
Sep	105	3150	13,260
Oct	90	2790	16,050
Nov	80	2400	18,450
Dec	70	2170	20,620

used in calculating the monthly flow volume. Volumes are calculated in units of cumec. day (= 8.64×10^4 m³).

A mass curve of accumulated flow volume against time is plotted (Fig. 5.10). In this figure all the months are assumed to be of average duration of 30.4 days. A demand line with slope of 40 m³/s is drawn tangential to the "hump" at beginning of the curve; line AB in Fig. 5.10. A line parallel to this line is drawn tangential to the mass curve at the "valley" portion; line $A'B'$. The vertical distance S_1 between these two parallel lines is the storage required to maintain the demand. The value of S_1 is found to be 2100 m³/s. days = 181.4 million m³.

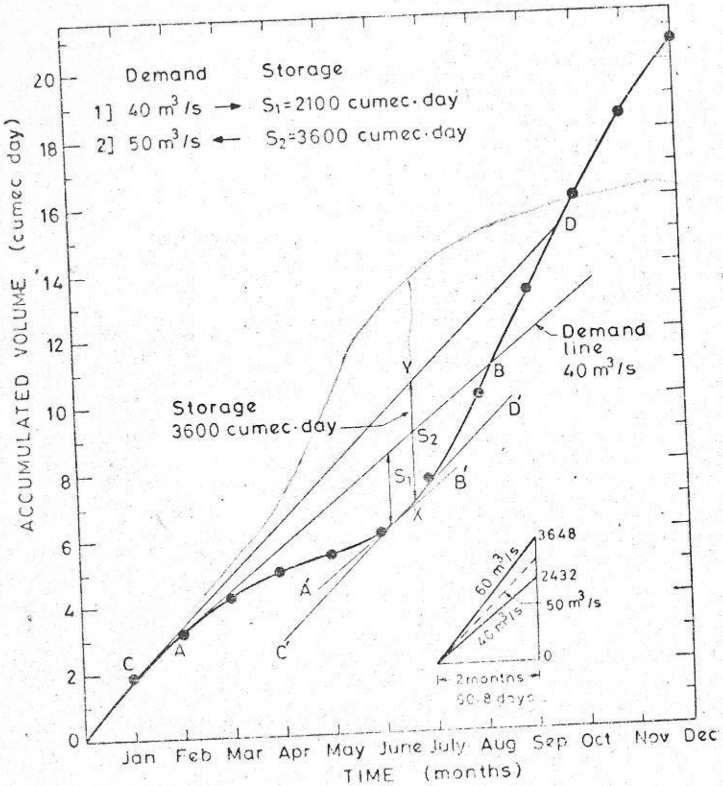

Fig. 5.10 Flow mass curve—Example 5.5

EXAMPLE 5.6 *Work out the Example 5.5 through arithmetic calculation without the use of the mass curve.*

The inflow and demand volumes in each month are calculated in Table 5.5. Column 6 indicates the departure of the inflow volume from the demand. The negative values indicates the excess of demand over the inflow and thus these have to be met by storage. Column 7 indicates the

cumulative excess demand (i.e. the cumulative excess negative departure). The maximum value in this column represents the minimum storage necessary to meet the demand requirements. The storage requirement in the present case is 1920 m³/s. day. Note that the difference between this and the value obtained by using the mass curve is due to the curvilinear variation of inflow volumes obtained by drawing a smooth mass curve. The arithmetic calculation assumes linear variations of the mass-curve ordinate between two adjacent time units.

TABLE 5.5 CALCULATION OF STORAGE—EXAMPLE 5.6

Month	Mean flow rate (m³/s)	Volume flow (cumec. day)	Demand rate (m³/s)	Demand volume (cumec. day)	Departure col. 3–5 (m³/s day)	Cumulative excess demand volume (cumec. day)	Cumulative excess flow volume (m³/s)
1	2	3	4	5	6	7	8
Jan	60	1860	40	1240	+ 620		620
Feb	45	1260	40	1120	+ 140		760
March	35	1085	40	1240	− 155	−155	—
April	25	750	40	1200	− 450	−605	
May	15	465	40	1240	− 775	−1380	—
June	22	660	40	1200	− 540	−1920	—
July	50	1550	40	1240	+ 310	—	310
Aug	80	2480	40	1240	+ 1240	—	1550
Sept	105	3150	40	1200	+ 1950	—	3500
Oct	90	2790	40	1240	+ 1550	—	6050
Nov	80	2400	40	1200	+ 1200	—	7250
Dec	70	2170	[illegible]	1240	+ 930	—	8180

Column 8 indicates the cumulative excess inflow volume starting from each demand withdrawal from storage. This indicates the filling up of reservoir and the volumes in excess of storage (in this case 1920 m³/s. day) represent spillover. The calculation of this column is necessary to know whether the reservoir fills up after a demand, and if so, when. In this example, the cumulative excess inflow volume will reach +1920 m³/s in the beginning of September. The reservoir will be full again and will start spilling after that time.

Calculation of Maintainable Demand

The converse problem of determining the maximum demand rate that can be maintained by a given storage volume can also be solved by using a mass curve. In this case tangents are drawn from the "ridges" of the mass curves across the next "valley" at various slopes. The demand line that requires just the given storage ($u_1 v_1$ in Fig. 5.11) is the proper demand that can be sustained by the reservoir in that dry period. Similar demand lines are drawn at other "valleys" in the mass curve (e.g. $u_2 v_2$ in Fig. 5.11) and the demand rates determined. The smallest of the various demand rates thus found denotes the maximum firm demand that can be sustained by the given storage. It may be noted that this problem involves a trial-and-error procedure for its solution. Example 5.7 indicates this use of the mass curve.

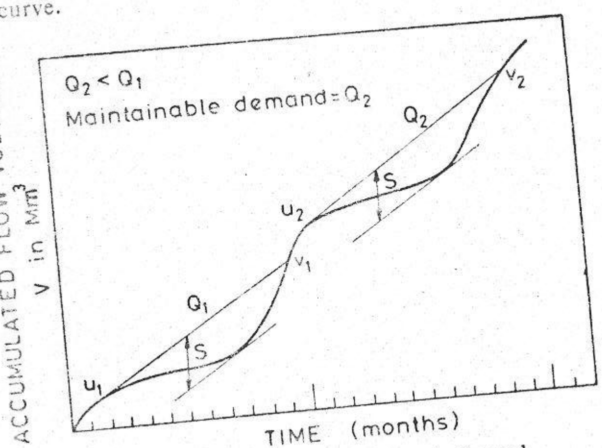

Fig. 5.11 Determination of maintainable demand

The following salient points in the use of the mass curve are worth noting:

1. The vartical distance between two successive tangents to a mass curve at the ridges (points v_1 and u_2 in Fig. 5.11) represent the water "wasted" over the spillway.
2. A demand line must intersect the mass curve if the reservoir is to refill. Nonintersection of the demand line and mass curve indicates insufficient inflow.

EXAMPLE 5.7 *Using the mass curve of Example 5.5 obtain that maximum uniform rate that can be maintained by a storage of 3600 m³/s. days.*

An ordinate *XY* of magnitude 3600 m³/s is drawn in Fig. 5.10 at an approximate lowest position in the dip of the mass curve and a line passing through *Y* and tangential to the "hump" of the mass curve at *C* is drawn (line *CYD* in Fig. 5.10). A line parallel to *CD* at the lowest position of the mass curve is now drawn and the vertical interval between the two measured. If the original guess location of *Y* is correct, this vertical distance will be

3600 m³/s. day. If not, a new location for *Y* will have to be chosen and the above procedure repeated.

The slope of the line *CD* corresponding to the final location of *XY* is the required demand rate. In this example this rate is found to be 50 m³/s.

Variable Demand

In the examples given above a constant demand rate was assumed to simplify the problem. In practice, it is more likely that the demand rate varies with time to meet various end uses, such as irrigation, power and water-supply needs. In such cases a mass curve of demand, also known as *variable use line* is prepared and superposed on the flow-mass curve with proper matching of time. For example, the demand for the month of February must be against the inflow for the same month. If the reservoir is full at first point of intersection of the two curves, the maximum intercept between the two curves represents the needed storage of the reservoir (Fig. 5.12). Such a plot is sometimes known as *regulation diagram* of a reservior.

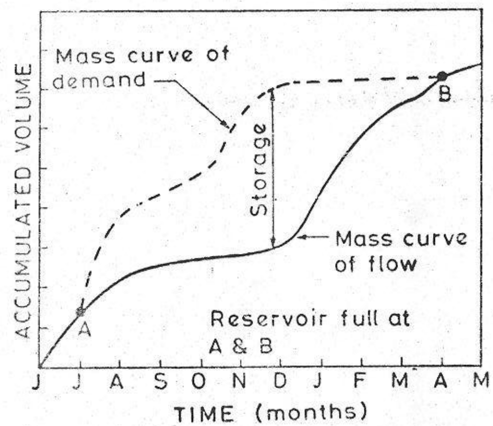

Fig. 5.12 Variable use line

In the analysis of problems related to reservoirs it is necessary to account for evaporation, leakage and other losses from the revervoir. These losses in the volume of water in a known interval of time can either be included in demand rates or deducted from inflow rates. In the latter method, which is generally preferred, the mass curve may have negative slopes at some points. Example 5.8 gives an arithmetic calculation procedure for calculating storage under variable demand. Further details about the mass curve method and a useful technique, called the *sequent peak Algorithm* for the reservoir storage estimation are described in Appendix—B.

EXAMPLE 5.8 *For a proposed reservoir the following data were collected. The prior water rights required the release of natural flow or 5 m³/s, whichever is less. Assuming an average reservoir area of 20 km², estimate the* *storage required to meet these demands. (Assume the runoff coefficient of the area submerged by the reservoir = 0.5.)*

Month	Mean flow (m^3/s)	Demand (million m^3)	Monthly evaporation (cm)	Monthly rainfall (cm)
Jan	25	22.0	12	2
Feb	20	23.0	13	2
March	15	24.0	17	1
April	10	26.0	18	1
May	4	26.0	20	1
June	9	26.0	16	13
July	100	16.0	12	24
Aug	108	16.0	12	19
Sept	80	16.0	12	19
Oct	40	16.0	12	1
Nov	30	18.0	11	6
Dec	30	22.0	17	2

Use actual number of days in a month for calculating the monthly flow and an average month of 30.4 days for prior right release.

Prior right release = $5 \times 30.4 \times 8.64 \times 10^4 = 13.1\ Mm^3$

when $Q > 5.0\ m^3/s$.

$$\text{Evaporation volume} = \frac{E}{100} \times 20 \times 10^6 = 0.2\ E\ Mm^3$$

$$\text{Rainfall volume} = \frac{P}{100} \times (1-0.5) \times 20 = 0.1\ P\ Mm^3$$

Inflow volume : $I \times$ (No. of days in the month) $\times 8.64 \times 10^4\ m^3$

The calculations are shown in Table 5.6 and the required storage capacity is $64.5\ Mm^3$.

The mass-curve method assumes a definite sequence of events and this is its major drawback. In practice, the runoff is subject to considerable time variations and definite sequential occurrences represent only an idealized situation. The mass-curve analysis is thus adequate for small projects or preliminary studies of large storage projects. The latter ones require sophisticated methods such as *time-series analysis* of data for the final design.

5.7 SURFACE WATER RESOURCES OF INDIA

TABLE 5.6 CALCULATION OF RESERVOIR STORAGE—EXAMPLE 5.8

Month	Inflow volume (Mm^3)	Withdrawal: Demand (Mm^3)	Withdrawal: Prior rights (Mm^3)	Withdrawal: Evaporation (Mm^3)	Withdrawal: Rainfall (Mm^3)	Total withdrawal (3+4+5+6) (Mm^3)	Departure (Mm^3)	Cum. excess demand (Mm^3)	Cum. excess flow volume (Mm^3)
1	2	3	4	5	6	7	8	9	10
Jan	67.0	22.0	13.1	2.4	−0.2	37.3	+29.7	—	29.7
Feb	48.4	23.0	13.1	2.6	−0.2	38.5	+ 9.9	—	39.6
Mar	40.2	24.0	13.1	3.4	−0.1	40.4	− 0.2	− 0.2	—
Apr	25.9	26.0	13.1	3.6	−0.1	42.6	−16.7	−16.9	—
May	10.7	26.0	10.7	4.0	−0.1	40.6	−29.9	−46.8	
June	23.3	26.0	13.1	3.2	−1.3	41.0	−17.7	−64.5	
July	267.8	16.0	13.1	2.4	−2.4	29.1	+238.7	—	238.7
Aug	289.3	16.0	13.1	2.4	−1.9	29.6	259.7		498.4
Sept	207.4	16.0	13.1	2.4	−1.9	29.6	177.8		676.2
Oct	107.1	16.0	13.1	2.4	−0.1	31.4	75.7		751.9
Nov	77.8	16.0	13.1	2.2	−0.6	30.7	47.1		799.0
Dec	80.4	22.0	13.1	3.4	−0.2	38.3	42.1		841.1

One of the foremost requirements for the efficient planning of the water resources of a country is the availability of requisite data pertaining to streamflow. Unfortunately, reliable discharge data are hard to obtain, especially in developing countries. For India, reliable streamflow covering the entire river system of the country are not available. Based on Khosla's formula (Eq. 5.12), the total annual flow in all the rivers of India has been estimated as 1672.5 billion m^3. Rao[3] (1975) has made an estimate based on a comprehensive analysis of available data. According to his estimate, the total annual runoff from the river system of India is 1645 billion m^3, which is remarkably close to the earlier estimate.

The total catchment area of all the rivers in India is approximately 3.05 million km^2. This can be considered to be made up of three classes of catchments:

1. Large catchments with basin area larger than 20000 km^2;
2. medium catchments with area between 20000 to 2000 km^2; and
3. minor catchments with area less than 2000 km^2.

Rao's assessment[3] indicates that large catchments occupy nearly 85% of the country's total drainage area and produce nearly 85% of the runoff. The medium and minor catchments account for 7% and 8% of annual runoff respectively. In the major river basins of the country two mighty rivers the Brahmaputra and Ganga together constitute 71.5% of the total runoff in their class and contribute 61% of the country's river flow. Further, these two rivers rank eighth and tenth respectively in the list of the world's 10 largest rivers (Table 5.7). It is interesting to see from Table 5.7 that these 10 rivers account for nearly 50% of the world's annual runoff.

TABLE 5.7 WORLD'S TEN LARGEST RIVERS

S. No.	Name	Annual runoff (billion m³)
1	Amazon	6307
2	Platt	1358
3	Congo	1245
4	Orinoco	1000
5	Yangtze	927
6	Mississippi	593
7	Yenisei	550
8	Brahmaputra	510
9	Mekong	500
10	Ganga	493

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2. Wagner, T.P. and R.K. Linsley, "Application of Stanford Watershed Model to an Indian catchment", Irrigation and Power, *J. of C.B.I.P., India*, Vol. 32, No. 4, Oct. 1975, pp. 465-475.
3. Rao, K.L., *India's Water Wealth*, Orient Longman, New Delhi, 1975.

PROBLEMS

5.1 Long-term observations at a streamflow-measuring station at the outlet of a catchment in a mountainous area gives a mean annual discharge of 65 m³/s. An isohyetal map for the annual rainfall over the catchment gives the following areas

closed by isohyets and the divide of the catchment:

Isohyet (cm)	Area (km²)	Isohyet (cm)	Area (km²)
140-135	50	120-115	600
135-130	300	115-110	400
130-125	450	110-105	200
125-120	700		

Calculate (a) the mean annual depth of rainfall over the catchment, (b) the mean annual runoff and (c) the runoff coefficient.

5.2 A small stream with a catchment area of 70 km² was gauged at a location some distance downstream of a reservoir. The data of the mean monthly gauged flow, rainfall and upstream diversion are given. The regenerated flow reaching the stream upstream of the gauging station can be assumed to be constant at a value of 0.20 Mm³/month. Obtain the rainfall-runoff relation for this stream. What virgin flow can be expected for a monthly rainfall value of 15.5 cm ?

Month	Monthly rainfall (cm)	Gauged monthly yield (Mm³)	Upstream Utilization (Mm³)
1	5.2	1.09	0.60
2	8.6	2.27	0.70
3	7.1	1.95	0.70
4	9.2	2.80	0.70
5	11.0	3.25	0.70
6	1.2	0.28	0.30
7	10.5	2.90	0.70
8	11.5	2.98	0.70
9	14.0	3.80	0.70
10	3.7	0.84	0.30
11	1.6	0.28	0.30
12	3.0	0.40	0.30

5.3 The following table shows the observed annual rainfall and the corresponding annual yield for a small catchment. Develop the rainfall-runoff correlation equation for this catchment and find the correlation coefficient. What yield can be expected from this catchment for an annual rainfall of 100 cm ?

Year	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975
Annual rainfall (cm)	90.5	111.0	38.7	129.5	145.5	99.8	147.6	50.9	120.2	90.3	65.2	75.9
Yield (cm)	30.1	50.2	5.3	61.5	74.8	39.9	64.7	6.5	46.1	36.2	24.6	20.0

5.4 The rainfall-runoff relation for a 650 sq. km drainage basin under dry soil conditions is given below.

Rainfall (cm)	1.5	5.0	7.5	10.0	12.5
Runoff	0.0	2.0	4.0	6.5	8.8

A storm of 2 hr effective duration occurred over this basin at a time when the soil was dry. The resulting hydrograph was as follows:

Time (h)	0	3	6	9	12	15	18
Flow (m^3/s)	0	300	500	400	250	75	0

Determine the runoff from this basin due to the storm. Assume zero base flow. What was the average coefficient of runoff during this storm ?

5.5 The mean monthly rainfall and temperature of a catchment near Bangalore are given below. Estimate the annual runoff and the annual runoff coefficient by using Khosla's runoff formula.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp (°C)	24	27	32	33	31	26	24	24	23	21	20	21
Rainfall (cm)	0.7	0.9	1.1	4.5	10.7	7.1	11.1	13.7	16.4	15.3	6.1	1.3

5.6 Discharges in a river are considered in 10 class intervals. Three consecutive years of data of the discharge in the river are given below. Draw the flow-duration curve for the river and determine the 75% dependable flow.

Discharge range (m^3/s)	<6	6.0-9.9	10-14.9	15-24.9	25-39	40-99	100-149	150-249	250-349	>350
No. of occurrences	20	137	183	232	169	137	121	60	30	6

5.7 The average monthly inflow into a reservoir in a dry year is given below:

Month	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Jan	Feb	Mar	Apr	May
Mean Monthly flow (m^3/s)	20	60	200	300	200	150	100	80	60	40	30	25

If a uniform discharge at 90 m^3/s is desired from this reservoir what minimum storage capacity is required ? [Hint : Assume the next year to have similar flows as the present year.]

5.8 For the data given in Prob. 5.7, plot the mass curve and find:

(a) The minimum storage required to sustain a uniform demand of 70 m^3/s;

(b) if the reservoir capacity is 7500 cumec. day, estimate the maximum uniform rate of withdrawal possible from this reservoir.

5.9 The following table gives the monthly inflow and contemplated demand from a proposed reservoir. Estimate the minimum storage that is necessary to meet the demand.

Month	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sept	Oct	Nov	Dec
Monthly inflow (Mm^3)	50	40	30	25	20	30	200	225	150	90	70	60
Monthly demand (Mm^3)	70	75	80	85	130	120	25	25	40	45	50	60

5.10 For the reservoir in Prob. 5.9, the mean monthly evaporation and rainfall are given below.

Month	Jan	Feb	Mar	Apr	May	Jun	July	Aug	Sep	Oct	Nov	Dec
Evaporation (cm)	6	8	13	17	22	22	14	11	13	12	7	5
Rainfall (cm)	1	0	0	0	0	19	43	39	22	6	2	1

If the average reservoir area can be assumed to be 30 km^2, estimate the change in the storage requirement necessitated by this additional data. Assume the runoff coefficient of the area flooded by the reservoir as equal to 0.4.

QUESTIONS

5.1 A mean annual runoff of 1 m³/s from a catchment of area 31.54 km² represents an effective rainfall of
(a) 100 cm (b) 1 cm (c) 100 mm (d) 3.17 cm.

5.2 Direct runoff is made up of
(a) surface runoff, prompt interflow and channel precipitation
(b) surface runoff, infiltration and evapotranspiration
(c) overland flow only
(d) rainfall and evaporation.

5.3 The term *base flow* denotes
(a) delayed groundwater flow reaching a stream
(b) delayed groundwater and snowmelt reaching a stream
(c) delayed groundwater and interflow
(d) the annual minimum flow in a stream.

5.4 *Virgin flow* is
(a) the flow in the river downstream of a gauging station
(b) the flow in the river upstream of a gauging station
(c) the flow unaffected by works of man
(d) the flow that would exist in the stream if there were no abstractions to the precipitation.

5.5 The water year in India starts from the first day of
(a) January (b) April (c) June (d) September.

5.6 An ephemeral stream
(a) is one which always carries some flow
(b) does not have any base flow contribution
(c) is one which has limited contribution of groundwater in wet season
(d) is one which carries only snow-melt water.

5.7 Khosla's formula for monthly runoff R_m due to a monthly rainfall P_m is $R_m = P_m - L_m$, where L_m is
(a) a constant
(b) monthly loss and depends on the mean monthly catchment temperature
(c) a monthly loss coefficient depending on the antecedent precipitation index
(d) a monthly loss depending on the infiltration characteristics of the catchment.

5.8 The flow-duration curve is a plot of
(a) accumulated flow against time
(b) discharge against time in chronological order
(c) the base flow against the percentage of times the flow is exceeded
(d) the stream discharge against the percentage of times the flow is equalled or exceeded.

5.9 In a flow-mass curve study the demand line drawn from a ridge in the curve did not intersect the mass curve again. This represents that
(a) the reservoir was not full at the beginning
(b) the storage was not adequate
(c) the demand cannot be met by the inflow as the reservoir will not refill
(d) the reservoir is wasting water by spill.

5.10 If in a flow-mass curve, a demand line drawn tangent to the lowest point in a valley of the curve does not intersect the mass curve at an earlier time period, it represents that
(a) the storage is inadequate

(b) the reservoir will not be full at the start of the dry period
(c) the reservoir is full at the beginning of the dry period
(d) the reservoir is wasting water by spill.

5.11. The flow-mass curve is an integral curve of
(a) the hydrograph (b) the hyetograph
(c) the flow duration curve (d) the S-curve.

5.12 The total rainfall in a catchment of area 1200 km² during a 6-h storm is 16 cm while the surface runoff due to the storm is 1.2×10^8 m³. The ϕ index is
(a) 0.1 cm/h (b) 1.0 cm/h (c) 0.2 cm/h
(d) cannot be estimated with the given data.

6

HYDROGRAPHS

6.1 INTRODUCTION

While long-term runoff concerned with the estimation of yield was discussed in the previous chapter, the present chapter examines in detail the short-term runoff phenomenon. The storm hydrograph is the focal point of the present chapter.

Consider a concentrated storm producing a fairly uniform rainfall of duration, T_r over a catchment. After the initial losses and infiltration losses are met, the rainfall excess reaches the stream through overland and channel flows. In the process of translation a certain amount of storage is built up in the overland and channel-flow phases. This storage gradually depletes after the cessation of the rainfall. Thus there is a time lag between the occurrence of rainfall in the basin and the time when that water passes the gauging station at the basin outlet. The runoff measured at the stream-gauging station will give a typical hydrograph as shown in Fig. 6.1. The duration of the rainfall is also marked in this figure to indicate the time lag in the rainfall and runoff. The hydrograph of this kind which results due to an isolated storm is typically single-peaked skew distribution of discharge and is known variously as *storm hydrograph*, *flood hydrograph* or simply *hydrograph*. It has three characteristic regions : (i) the rising limb AB, joining point A, the starting point of the rising curve and point B, the point of inflection, (ii) the crest segment BC between the two points of inflection with a peak P in between, (iii) the falling limb or *depletion curve* CD starting from the second point of inflection C. Other points of interest are t_{pk}, the time to peak from the starting point A, the time interval from the centre of mass of rainfall to the centre of mass of hydrograph called *lag time* T_L, the peak discharge Q_p and the time base of the hydrograph T_B.

The hydrograph is the response of a given catchment to a rainfall input.

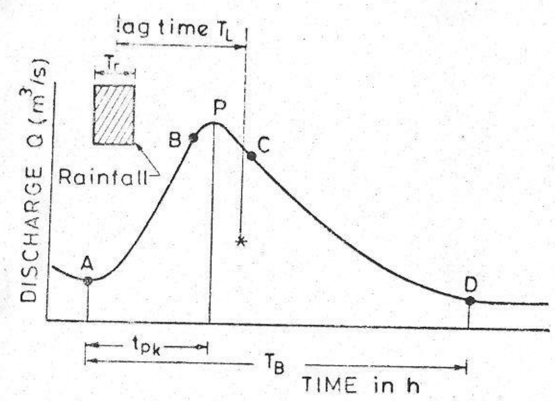

Fig. 6.1 Elements of a flow hydrograph

It consists of flow in all the three phases of runoff, viz. surface runoff, interflow and base flow, and embodies in itself the integrated effects of a wide variety of catchment and rainfall parameters having complex interactions. Thus two different storms in a given catchment produce hydrographs differing from each other. Similarly, identical storms in two catchments produce hydrographs that are different. The interactions of various storms and catchments are in general extremely complex. If one examines the record of a large number of flood hydrographs of a stream, it will be found that many of them will have kinks, multiple peaks, etc. resulting in shapes much different from the simple single-peaked hydrograph of Fig. 6.1. These complex hydrographs are the result of storm and catchment peculiarities and their complex interactions. While it is theoretically possible to resolve a complex hydrograph into a set of simple hydrographs for purposes of hydrograph analysis, the requisite data of acceptable quality are seldom available. Hence, simple hydrographs resulting from isolated storms are preferred for hydrograph studies.

6.2 FACTORS AFFECTING FLOOD HYDROGRAPH

The factors that affect the shape of the hydrograph can be broadly grouped into climatic factors and physiographic factors. Each of these two groups contains a host of factors and the important ones are listed in Table 6.1. Generally, the climatic factors control the rising limb and the recession limb is independent of storm characteristics, being determined by catchment characteristics only. Many of the factors listed in Table 6.1 are interdependent. Further, their effects are very varied and complicated. As such only important effects are listed below in qualitative terms only.

TABLE 6.1 FACTORS AFFECTING FLOOD HYDROGRAPH

Physiographic factors	Climatic factors
1. Basin characteristics : (a) shape (b) size (c) slope (d) nature of the valley (e) elevation (f) drainage density 2. Infiltration characteristics : (a) landuse and cover (b) soil type and geological conditions (c) lakes, swamps and other storage 3. Channel characteristics : cross-section, roughness and storage capacity.	1. Storm characteristics : precipitation, intensity, duration, magnitude and movement of storm. 2. Initial loss 3. Evapotranspiration

Shape of the Basin

The shape of the basin influences the time taken for water from the remote parts of the catchment to arrive at the outlet. Thus the occurrence of the peak and hence the shape of the hydrograph are affected by the basin shape. Fan-shaped, i.e. nearly semi-circular shaped catchments give high peak and narrow hydrographs while elongated catchments give broad- and low-peaked hydrographs. Figure 6.2 shows schematically the hydrographs from

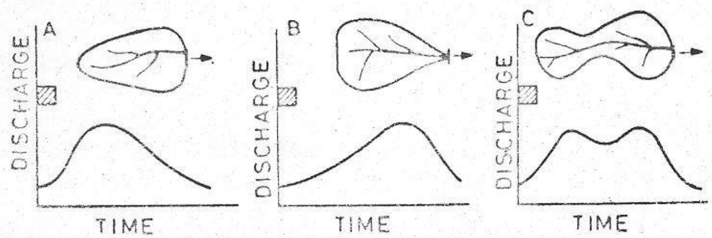

Fig. 6.2 Effect of catchment shape on the hydrograph

three catchments having identical infiltration characteristics due to identical rainfall over the catchment. In catchment A the hydrograph is skewed to the left, i.e. the peak occurs relatively quickly. In catchment B, the hydrograph is skewed to the right, the peak occurring with a relatively longer lag. Catchment C indicates the complex hydrography produced by a composite shape.

Size

Small basins behave different from the large ones in terms of the relative importance of various phases of the runoff phenomenon. In small catchments the overland flow phase is predominant over the channel flow. Hence the land use and intensity of rainfall have important role on the peak flood. On large basins these effects are suppressed as the channel flow phase is more predominant. The peak discharge is found to vary as A^n where A is the catchment area and n is an exponent whose value is less than unity, being about 0.5. The time base of the hydrographs from larger basins will be larger than those of corresponding hydrographs from smaller basins. The duration of the surface runoff from the time of occurrence of the peak is proportional to A^m, where m is an exponent less than unity and is of the order of magnitude of 0.2.

Slope

The slope of the main stream controls the velocity of flow in the channel. As the recession limb of the hydrograph represents the depletion of storage, the stream channel slope will have a pronounced effect on this part of the hydrograph. Large stream slopes give rise to quicker depletion of storage and hence result in steeper recession limbs of hydrographs. This would obviously result in a smaller time base.

The basin slope is important in small catchments where the overland flow is relatively more important. In such cases the steeper slope of the catchment results in larger peak discharges.

Drainage Density

The drainage density is defined as the ratio of the total channel length to the total drainage area. A large drainage density creates situation conducive for quick disposal of runoff down the channels. This fast response is reflected in a pronounced peaked discharge. In basins with smaller drainage densities, the overland flow is predominant and the resulting hydrograph is squat with a slowly rising limb (Fig. 6.3).

Land Use

Vegetation and forests increase the infiltration and storage capacities of the soils. Further, they cause considerable retardance to the overland flow. Thus the vegetal cover reduces the peak flow. This effect is usually very pronounced in small catchments of area less than 150 km². Further, the effect of the vegetal cover is prominent in small storms. In general, for two catchments of equal area, other factors being identical, the peak discharge is higher for a catchment that has a lower density of forest cover. Of the various factors that control the peak discharge, probably the only factor that can be manipulated is land use and thus it represents the only practical means of exercising long-term natural control over the flood hydrograph of a catchment.

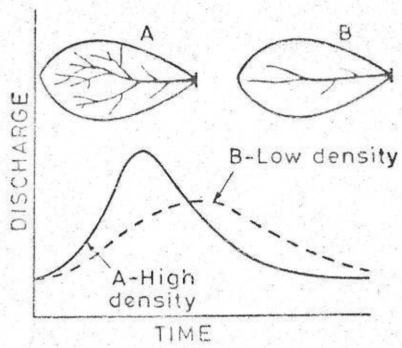

Fig. 6.3 Role of drainage density on the hydrograph

Climatic Factors

Among climatic factors the intensity, duration and direction of storm movement are the three important ones affecting the shape of a flood hydrograph. For a given duration, the peak and volume of the surface runoff are essentially proportional to the intensity of rainfall. This aspect is made use of in the unit hydrograph theory of estimating peak-flow hydrographs, as discussed in subsequent sections of this chapter. In very small catchments, the shape of the hydrograph can also be affected by the intensity.

The duration of a storm of given intensity also has a direct proportional effect on the volume of runoff. The effect of duration is reflected in the rising limb and peak flow. Ideally, if a rainfall of given intensity i lasts sufficiently long enough, a state of equilibrium discharge proportional to iA is reached.

If the storm moves from upstream of the catchment to the downstream end, there will be a quicker concentration of flow at the basin outlet. This results in a peaked hydrograph. Conversely, if the storm movement is up the catchment, the resulting hydrograph will have a lower peak and longer time base. This effect is further accentuated by the shape of the catchment, with long and narrow catchments having hydrographs most sensitive to the storm-movement direction.

6.3 COMPONENTS OF A HYDROGRAPH

As indicated earlier, the essential components of a hydrograph are: (i) the rising limb, (ii) the crest segment, and (iii) the recession limb. A few salient features of these components are described below.

Rising Limb

The rising limb of a hydrograph, also known as *concentration curve* represents the increase in discharge due to the gradual building up of storage in channels and over the catchment surface. The initial losses and high infiltration losses during the early period of a storm cause the discharge to rise rather slowly in the initial periods. As the storm continues, more and more flow from distant parts reach the basin outlet. Simultaneously the infiltration losses also decrease with time. Thus under a uniform storm over the catchment, the runoff increases rapidly with time. As indicated earlier, the basin and storm characteristics control the shape of the rising limb of a hydrograph.

Crest Segment

The crest segment is one of the most important parts of a hydrograph as it contains the peak flow. The peak flow occurs when the runoff from various parts of the catchment simultaneously contribute the maximum amount of flow at the basin outlet. Generally for large catchments, the peak flow occurs after the cessation of rainfall, the time interval from the centre of mass of rainfall to the peak being essentially controlled by basin and storm characteristics. Multiple-peaked complex hydrographs in a basin can occur when two or more storms occur in close succession. Estimation of the peak flow and its occurrence, being very important in flood-flow studies are dealt in detail elsewhere in this book.

Recession Limb

The recession limb which extends from the point of inflection at the end of the crest segment to the commencement of the natural groundwater flow represents the withdrawal of water from the storage built up in the basin during the earlier phases of the hydrograph. The starting point of the recession limb, i.e. the point of inflection represents the condition of maximum storage. Since the depletion of storage takes place after the cessation of rainfall, the shape of this part of the hydrograph is independent of storm characteristics and depends entirely on the basin characteristics:

The storage of water in the basin exists as (i) surface storage, which includes both surface detention and channel storage, (ii) interflow storage, and (iii) groundwater storage, i.e. base-flow storage. Barnes (1940) showed that the recession of a storage can be expressed as

$$Q_t = Q_0 K_r^t \tag{6.1}$$

in which Q_0 and Q_t are discharges at a time interval of t days with Q_0 being the initial discharge; K_r is a recession constant of value less than unity. Equation (6.1) can also be expressed in an alternative form of the exponential decay as

$$Q_t = Q_0 e^{-at} \tag{6.2}$$

where $a = -\ln K_r$

The recession constant K_r can be considered to be made up of three components to take care of the three types of storages as

$$K_r = K_{rs} \cdot K_{ri} \cdot K_{rb} \qquad (6.3)$$

where K_{rs} = recession constant for surface storage, K_{ri} = recession constant for interflow and K_{rb} = recession constant for base flow. Typically the values of these recession constants, when t is in days, are

K_{rs} = 0.05 to 0.20

K_{ri} = 0.50 to 0.85

K_{rb} = 0.85 to 0.99

If the interflow is not significant K_{ri} can be assumed to be unity. When Eq. (6.1) or (6.2) is plotted on a semilog paper with the discharge on the log-scale, it plots as a straight line and from this the value of K_r can be found.

EXAMPLE 6.1 *The recession portion of a flood hydrograph is given below. The time is indicated from the arrival of peak. Assuming the interflow component to be negligible, calculate the baseflow and surface flow recession coefficients.*

Time from peak (days)	Discharge (m³/s)	Time from peak (days)	Discharge (m³/s)
0.0	90	3.5	5.0
0.5	66	4.0	3.8
1.0	34	4.5	3.0
1.5	20	5.0	2.6
2.0	13	5.5	2.2
2.5	9	6.0	1.8
3.0	6.7	6.5	1.6
		7.0	1.5

The data are plotted on a semilog paper with discharge on the log-scale (Fig. 6.4). The part of the curve AB that plots as a straight line indicates the base flow. The surface flow terminates at point B, 5 days after the peak. From Eq. (6.1),

$$Q_t/Q_0 = K_{rb}^t$$

$$\text{Log } K_{rb} = \frac{1}{t} \log (Q_t/Q_0)$$

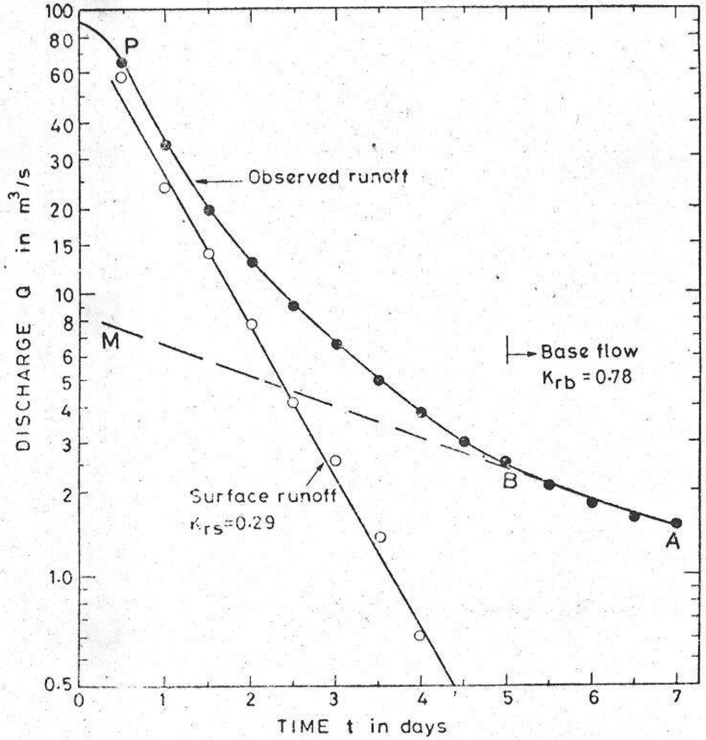

Fig. 6.4 Storage recession curve—Example 6.1

From Fig. 6.4, taking

$$Q_0 = 6.6 \text{ m}^3/\text{s}, \ t = 2 \text{ days}, \ Q_t = 4.0 \text{ m}^3/\text{s}$$

leading to

$$\log K_{rb} = \frac{1}{2} \log \left(\frac{4.0}{6.6}\right)$$

$$K_{rb} = 0.778, \text{ says } 0.78$$

From the curve PA, the base flow recession ABM is subtracted to get the surface runoff. Figure 6.1 shows the surface runoff depletion plots as a straight line. Taking, Q_0 = 26 m³/s, t = 2 days, Q_t = 2.25 m³/s leads to K_{rs} = 0.29.

6.4 BASE-FLOW SEPARATION

In many hydrograph analyses a relationship between the surface-flow hydrograph and the effective rainfall (i.e. rainfall minus losses) is sought to be established. The surface-flow hydrograph is obtained from the total storm hydrograph by separating the quick-response flow from the slow-response runoff. It is usual to consider the interflow as a part of the surface flow in view of its quick response. Thus only the base flow is to be deducted from the total storm hydrograph to obtain the surface flow hydrograph. There are three methods of base-flow separation that are in common use.

Method I—Straight-Line Method

In this method the separation of the base flow is achieved by joining with a straight line the beginning of the surface runoff to a point on the recession limb representing the end of the direct runoff. In Fig. 6.5 point *A* represents the beginning of the direct runoff and it is usually easy to identify in view of the sharp change in the runoff rate at that point.

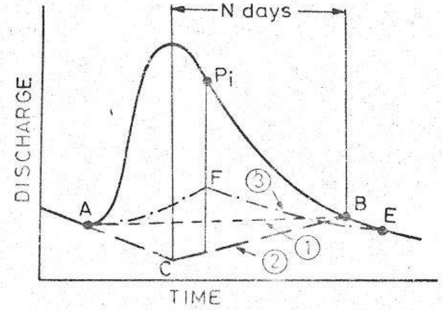

Fig. 6.5 Base-flow separation methods

Point *B*, marking the end of the direct runoff is rather difficult to locate exactly. An empirical equation for the time interval *N* (days) from the peak to the point *B* is

$$N = 0.83\ A^{0.2} \tag{6.4}$$

where A = drainage area in km² and N is in days. Points *A* and *B* are joined by a straight line to demarcate to the base flow and surface runoff. It should be realised that the value of *N* obtained as above is only approximate and the position of *B* should be decided by considering a number of hydrographs for the catchment. This method of base-flow separation is the simplest of all the three methods.

Method II

In this method the base-flow curve existing prior to the commencement of the surface runoff is extended till it intersects the ordinate drawn at the peak (point *C* in Fig. 6.5). This point is joined to point *B* by a straight line. Segments *AC* and *CB* demarcate the base flow and surface runoff. This is probably the most widely used base-flow separation procedure.

Method III

In this method the base-flow recession curve after the depletion of the flood water is extended backwards till it intersects the ordinate at the point of inflection (line *EF* in Fig. 6.5). Points *A* and *F* are joined by an arbitrary smooth curve. This method of base-flow separation is realistic in situations where the groundwater contributions are significant and reach the stream quickly.

It is seen that all the three methods of base-flow separation are rather arbitrary. The selection of any one of them depends upon the local practice and successful predictions achieved in the past. The surface-runoff hydrograph obtained after the base-flow separation is also known as *direct runoff hydrograph* (DRH).

6.5 EFFECTIVE RAINFALL

For purposes of correlating DRH with the rainfall which produced the flow, the hydrograph of the rainfall is also pruned by deducting the losses. Figure 6.6 shows the hyetograph of a storm. The initial loss and infiltration losses are subtracted from it. The resulting hyetograph is known as *effective rainfall hyetograph* (ERH). It is also known as *hyetograph of rainfall excess* or *supra rainfall*.

Both DRH and ERH represent the same total quantity but in different units. Since ERH is usually in cm/h plotted against time, the area of ERH multiplied by the catchment area gives the total volume of direct runoff which is the same as the area of DRH. The initial loss and infiltration losses are estimated based on the available data of the catchment.

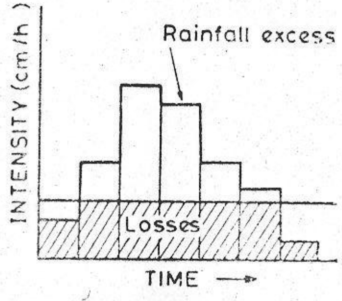

Fig. 6.6 Effective rainfall hyetograph

Example 6.2 *Rainfall of magnitude 3.8 cm and 2.8 cm occurring on two consecutive 4-h durations on a catchment of area 27 km² produced the following hydrograph of flow at the outlet of the catchment. Estimate the rainfall excess and φ index.*

Time from start of rainfall (h)	−6	0	6	12	18	24	30	36	42	48	54	60	66
Observed flow (m³/s)	6	5	13	26	21	16	12	9	7	5	5	4.5	4.5

The hydrograph is plotted to scale (Fig. 6.7). It is seen that the storm hydrograph has a base-flow component. For using the simple straight-line method of base-flow separation, by Eq. (6.4)

$$N = 0.83 \times (27)^{0.2} = 1.6 \text{ days} = 38.5 \text{ h}$$

However, by inspection, DRH starts at $t = 0$, has the peak at $t = 12$ h and ends at $t = 48$ h (which gives a value of $N = 48 - 12 = 36$ h). As $N = 36$ h appears to be more satisftory than $N = 38.5$ h, in the present case DRH is

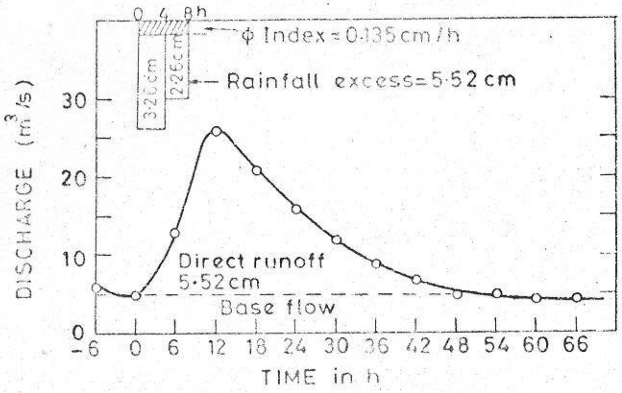

Fig. 6.7 Base flow separation—Example 6.2

assumed to exist from $t = 0$ to 48 h. A straight line base flow separation gives a constant value of 5 m³/s for the base flow.

$$\text{Area of DRH} = (6\times60\times60)\left[\frac{1}{2}(8) + \frac{1}{2}(8+21) + \frac{1}{2}(21+16) + \frac{1}{2}(16+11) + \frac{1}{2}(11+7) + \frac{1}{2}(7+4) + \frac{1}{2}(4+2) + \frac{1}{2}(2)\right]$$

$$= 3600 \times 6 \times (8+21+16+11+7+4+2)$$

$$= 1.4904 \times 10^6 \text{ m}^3$$

= total direct runoff due to storm

$$\text{Runoff depth} = \frac{\text{runoff volume}}{\text{catchment area}} = \frac{1.4904\times10^6}{27\times10^6} = 0.0552 \text{ m}$$

$$= 5.52 \text{ cm} = \text{rainfall excess}$$

Total rainfall = 3.8 + 2.8 = 6.6 cm

Duration = 8 h

$$\phi\text{ index} = \frac{6.6-5.52}{8} = 0.135 \text{ cm/h}$$

6.6 UNIT HYDROGRAPH

The problem of predicting the flood hydrograph resulting from a known storm in a catchment has received considerable attention. A large number of methods are proposed to solve this problem and of them probably the most popular and widely used method is the unit-hydrograph method. This method was first suggested by Sherman in 1932 and has undergone many refinements since then.

A unit hydrograph is defined as the hydrograph of direct runoff resulting from one unit depth (1 cm) of rainfall excess occurring uniformly over the basin and at a uniform rate for a specified duration (*D* h). The term unit here refers to a unit depth of rainfall excess which is usually taken as 1 cm. The duration, being a very important characteristic, is used as a prefix to a specific unit hydrograph. Thus one has a 6-h unit hydrograph, 12-h unit hydrograph, etc. and in general a *D*-h unit-hydrograph applicable to a given catchment. The definition of a unit hydrograph implies the following :

1. The unit hydrograph represents the lumped response of the catchment to a unit rainfall excess of *D*-h duration to produce a direct-runoff hydrograph. It relates only the direct runoff to the rainfall excess. Hence the volume of water contained in the unit hydrograph must be equal to the rainfall excess. As 1 cm depth of rainfall excess is considered the area of the unit hydrograph is equal to a volume given by 1 cm over the catchment.
2. The rainfall is considered to have an average intensity of *excess rainfall* (ER) of $1/D$ cm/h for the duration of the storm.
3. The distribution of the storm is considered to be uniform all over the catchment.

Figure 6.8 shows a typical 6-h unit hydrograph. Here the duration of the rainfall excess is 6 h.

Area under the unit hydrograph = 12.92×10^6 m³

Hence

Catchment area of the basin = 1292 km²

Two basic assumptions constitute the foundations for the unit-hydrograph theory. These are: (i) the time invariance and (ii) the linear response.

Time Invariance

This first basic assumption is that the direct-runoff response to a given effective rainfall in a catchment is time-invariant. This implies that the DRH for a given ER in a catchment is always the same irrespective of when it occurs.

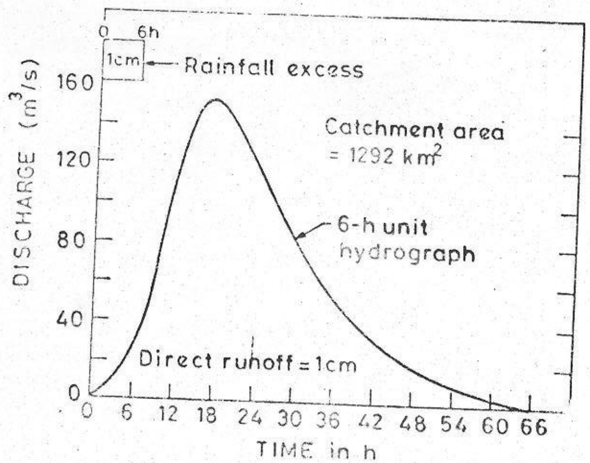

Fig. 6.8 Typical 6-h unit hydrograph

Linear Response

The direct-runoff response to the rainfall excess is assumed to be linear. This is the most important assumption of the unit-hydrograph theory. Linear response means that if an input $x_1(t)$ causes an output $y_1(t)$ and an input $x_2(t)$ causes an output $y_2(t)$, then an input $x_1(t)+x_2(t)$ gives an output $y_1(t)+y_2(t)$. Consequently, if $x_2(t) = rx_1(t)$, then $y_2(t) = ry_1(t)$. Thus if the rainfall excess in a duration D is r times the unit depth, the resulting DRH will have ordinates bearing ratio r to those of the corresponding D-h unit hydrograph. Since the area of the resulting DRH should increase by the ratio r, the base of the DRH will be the same as that of the unit hydrograph.

The assumption of linear response in a unit hydrograph enables the method of superposition to be used to derive DRHs. Accordingly, if two rainfall excesses of D-h duration each occur consecutively, their combined effect is obtained by superposing the respective DRHs with due care being taken to account for the proper sequence of events. These aspects resulting from the assumption of linear response are made clearer in the following two illustrative examples.

EXAMPLE 6.3 *Given below are the ordinates of a 6-h unit hydrograph for a catchment. Calculate the ordinates of the DRH due to a rainfall excess of 3.5 cm occurring in 6 hr.*

Time (h)	0	3	6	9	12	15	18	24	30	36	42	48	54	60	69
UH ordinate (m³/s)	0	25	50	85	125	160	185	160	110	60	36	25	16	8	0

The desired ordinates of the DRH are obtained by multiplying the ordinates of the unit hydrograph by a factor of 3.5 as in Table 6.2. The resulting DRH as also the unit hydrograph are shown in Fig. 6.9(a). Note that the time base of DRH is not changed and remains the same as that of the unit hydrograph. The intervals of coordinates of the unit hydrograph (shown in column 1) are not in any way related to the duration of the rainfall excess and can be any convenient value.

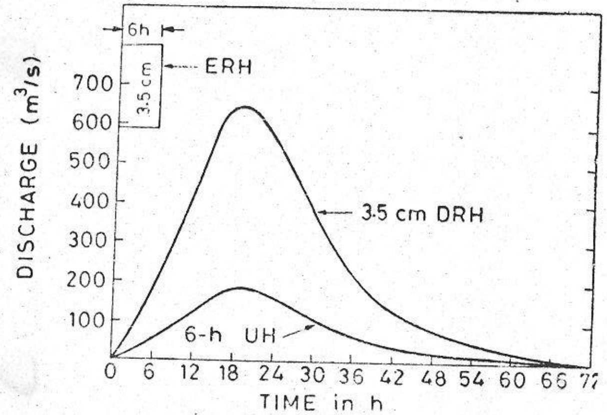

Fig. 6.9(a) 3.5 cm DRH derived from 6-h unit hydrograph—Example 6.3

TABLE 6.2 CALCULATION OF DRH DUE TO 3.5 cm ER—*Example* 6.3

Time (h)	Ordinate of 6-h unit hydrograph (m³/s)	Ordinate of 3.5 cm DRA (m³/s)
1	2	3
0	0	0
3	25	87.5
6	50	175.0
9	85	297.5
12	125	437.5
15	160	560.0
18	185	647.5
24	160	560.0
30	110	385.0
36	60	210.0
42	36	126.0
48	25	87.5
54	16	56.0
60	8	28.0
69	0	0

EXAMPLE 6.4 *Two storms each of 6-h duration and having rainfall excess values of 3.0 and 2.0 cm respectively occur successively. The 2-cm ER rain follows the 3-cm rain. The 6-h unit hydrograph for the catchment is the same as given in Example 6.3. Calculate the resulting DRH.*

First, the DRHs due to 3.0- and 2.0-cm ER are calculated, as in Example 6.3 by multiplying the ordinates of the unit hydrograph by 3 and 2 respectively. Noting that the 2-cm DRH occurs after the 3-cm DRH, the ordinates of the 2-cm DRH are lagged by 6 hrs as shown in column 4 of Table 6.3. Columns 3 and 4 give the proper sequence of the two DRHs. Using the method of superposition, the ordinates of the resulting DRH are obtained by combining the ordinates of the 3- and 2-cm DRHs at any instant. By this process the ordinates of the 5 cm DRH are obtained in column 5. Figure 6.9(b) shows the component 3- and 2-cm DRHs as well as the composite 5-cm DRH obtained by the method of superposition.

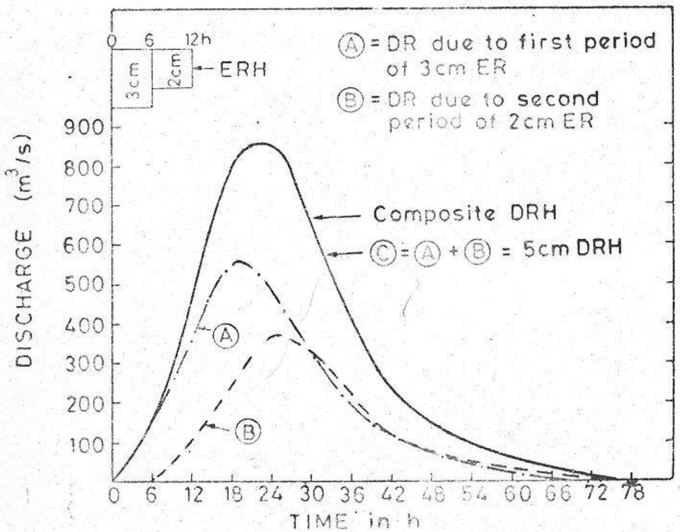

Fig. 6.9(b) Principle of superposition—Example 6.4

Application of Unit Hydrograph

Using the basic principles of the unit hydrograph, one can easily calculate the DRH in a catchment due to a given storm if an appropriate unit hydrograph was available. Let it be assumed that a *D*-h unit-hydrograph and the storm hyetograph are available. The initial losses and infiltration losses are estimated and deducted from the storm hyetograph to obtain the ERH (Sec. 6.5). The ERH is then divided into *M* blocks of *D*-h duration each. The rainfall excess in each *D*-h duration is then operated upon the unit hydrograph successively to get the various DRH curves. The ordinates of these DRHs are suitably lagged to obtain the proper time sequence and are then collected and added at each time element to obtain the required nett DRH due to the storm.

TABLE 6.3 CALCULATION OF DRH BY METHOD OF SUPERPOSITION—*Example 6.4*

Time (h)	Ordinate of 6-h UH (m³/s)	Ordinate of 3-cm DRH (col. 2×3)	Ordinate of 2-cm DRH (col. 2 lagged by 6 h×2)	Ordinate of 5-cm DRH (col. 3+col. 4) (m³/s)	Remarks
1	2	3	4	5	6
0	0	0	0	0	
3	25	75	0	75	
6	50	150	0	150	
9	85	255	50	305	
12	125	375	100	475	
15	160	480	170	650	
18	185	555	250	805	
(21)	(172.5)	(517.5)	(320)	(837.5)	Interpolated value
24	160	480	370	850	
30	110	330	320	650	
36	60	180	220	400	
42	36	108	120	228	
48	25	75	72	147	
54	16	48	50	98	
60	8	24	32	56	
(66)	(2.7)	(8.1)	(16)	(24.1)	Interpolated value
69	0	0	(10.6)	(10.6)	Interpolated value
75	0	0	0	0	

Note: 1. The entries in col. 4 are shifted by 6 h in time.
2. Due to unequal time interval of ordinates a few entries have to be interpolated to complete the table. These interpolated values are shown in parentheses.

Consider Fig. 6.10 in which a sequence of M rainfall excess values $R_1, R_2, \ldots, R_i, \ldots R_m$ each of duration D h is shown. The line $u[t]$ is the ordinate of a D-h unit hydrograph at t h from the beginning.

The direct runoff due to R_1 at time t is

$$Q_1 = R_1 \cdot u[t]$$

The direct runoff due to R_2 at time $(t-D)$ h is

$$Q_2 = R_2 \cdot u[t-D]$$

Similarly, $Q_i = R_i \cdot u[t-(i-1)D]$

and $\quad Q_m = R_m \cdot u[t-(M-1)D]$

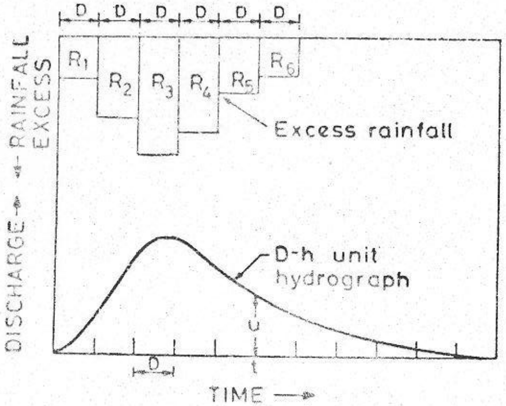

Fig. 6.10 DRH due to an ERH

Thus at any time t, the total direct runoff is

$$Q_t = \sum_{i=1}^{M} Q_i = \sum_{i=1}^{M} R_i \cdot u[t-(i-1)\,D] \qquad (6.5)$$

The arithmetic calculations of Eq. (6.5) are best performed in a tabular manner as indicated in Examples 6.4 and 6.5. After deriving the nett DRH, the estimated base flow is then added to obtain the total flood hydrograph.

EXAMPLE 6.5 *The average storm rainfall values over a catchment in three successive 6-h intervals are known to be 3.5, 7.5 and 5.5 cm. The storm loss rate (ϕ index) for the catchment is estimated at 0.25 cm/h. Using the 6-h unit hydrograph ordinates of Example 6.4, estimate the direct-runoff hydrograph. If the base flow can be assumed to be 15 m³/s at the beginning and increasing by 2.0 m³/s every 12 h, till the end of the direct-runoff hydrograph, estimate the resulting flood hydrograph.*

The effective rainfall hyetograph is calculated as in the following table.

Interval	1st 6 hours	2nd 6 hours	3rd 6 hours
Rainfall depth (cm)	3.5	7.5	5.5
Loss @ 0.25 cm/h for 6 h	1.5	1.5	1.5
Effective rainfall (cm)	2.0	6.0	4.0

The direct runoff hydrograph is next calculated by the method of superposition as indicated in Table 6.4. The ordinates of the unit hydrograph are multiplied by the ER values successively. The second and third set of ordinates are advanced by 6 and 12 h respectively and the ordinates at a

TABLE 6.4 CALCULATION OF FLOOD HYDROGRAPH DUE TO A KNOWN ERH—EXAMPLE 6.5

Time	Ordinates of U.H	Col. 2 ×2.0	Col. 2 ×6.0 (Advanced by 6 h)	Col. 2 ×4.0 (Advanced by 12 h)	Ordinates of final DRH (Col. 3+4+5)	Base flow (m³/s)	Ordinates of flood hydrograph (m³/s) (Col. 6+7)
1	2	3	4	5	6	7	8
0	0	0	0	0	0	15	15
3	25	50	0	0	50	15	65
6	50	100	0	0	100	15	115
9	85	170	150	0	320	15	335
12	125	250	300	0	550	17	567
15	160	320	510	100	930	17	947
18	185	370	750	200	1320	17	1337
(21)	(172.5)	(345)	(960)	(340)	(1645)	(17)	(1662)
24	160	320	1110	500	1930	19	1949
(27)	(135)	(270)	(1035)	(640)	(1945)	19	1964
30	110	220	960	740	1920	19	1939
36	60	120	660	640	1420	21	1441
42	36	72	360	440	872	21	893
48	25	50	216	240	506	23	529
54	16	32	150	144	326	23	349
60	8	16	96	100	212	25	237
66	(2.7)	(5.4)	(48)	(64)	(117)	25	142
69	0	0	—	—	—	—	—
72		0	16	32	48	27	75
75		0	0	—	—	—	—
78		0	0	(10.8)	(11)	27	49
81				0	0	27	27
84						27	27

Note : Due to the unequal time intervals of unit hydrograph ordinates, a few entries, indicated in parentheses have to be interpolated to complete the table.

given time interval added. The base flow is then added to obtain the flood hydrograph Col. 8, Table 6.4.

6.7 DERIVATION OF UNIT HYDROGRAPHS

A number of isolated storm hydrographs caused by short spells of rainfall excess, each of approximately same duration [0.90 to 1.1 D h] are selected from a study of the continuously gauged runoff of the stream. For each of these surface hydrographs, the base flow is separated by adopting one of the methods indicated in Sec. 6.4.

The area under each DRH is evaluated and the volume of the direct runoff obtained is divided by the catchment area to obtain the depth of ER. The ordinates of the various DRHs are divided by the respective ER values to obtain the ordinates of the unit hydrograph.

Flood hydrographs used in the analysis should be selected to meet the following desirable features with respect to the storms responsible for them:

1. The storms should be isolated storms occurring individually.
2. The rainfall should be fairly uniform during the duration and should cover the entire catchment area.
3. The duration of the rainfall should be 1/5 to 1/3 of the basin lag.
4. The rainfall excess of the selected storm should be high. A range of ER values of 1.0 to 4.0 cm is sometimes preferred.

A number of unit hydrographs of a given duration are derived by the above method and then plotted on a common pair of axes as shown in Fig. 6.11. Because of rainfall variations both in space and time and due to some departures from the assumptions of the unit-hydrograph theory, the various unit hydrographs thus developed will not be identical. It is common practice to adopt a mean of such curves as the unit hydrograph of a given duration for the catchment. While deriving the mean curve, the average of peak flows and time to peaks are first calculated. Then a mean curve of best fit, judged by eye, is drawn through the averaged peak to close on an averaged base length. The volume of DRH is calculated and any departure from unity is corrected by adjusting the value of the peak. The averaged ERH of unit depth is customarily drawn in the plot of the unit hydrograph to indicate the type and duration of rainfall causing the unit hydrograph.

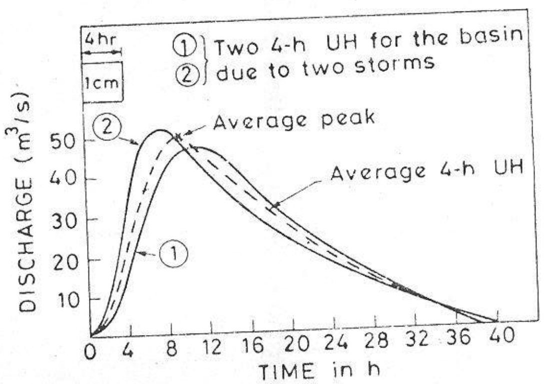

Fig. 6.11 Derivation of an average unit hydrograph

By definition the rainfall excess is assumed to occur uniformly over the catchment during duration D of a unit hydrograph. An ideal duration for a unit hydrograph is one wherein small fluctuations in the intensity of rainfall within this duration do not have any significant effects on the runoff. The catchment has a damping effect on the fluctuations of the rainfall intensity in the runoff-producing process and this damping is a function of the catchment area. This indicates that larger durations are admissible for larger catchments. By experience it is found that the duration of the unit hydrograph should not exceed 1/5 to 1/3 basin lag. For catchments of sizes larger than 250 km² the duration of 6 h is generally satisfactory.

EXAMPLE 6.6 *Following are the ordinates of a storm hydrograph of a river draining a catchment area of 423 km² due to a 6-h isolated storm. Derive the ordinates of a 6-h unit hydrograph for the catchment.*

Time from start of storm (h)	−6	0	6	12	18	24	30	36	42	48
Discharge (m³/s)	10	10	30	87.5	115.5	102.5	85.0	71.0	59.0	47.5

Time from start of storm (h)	54	60	66	72	78	84	90	96	102
Discharge (m³/s)	39.0	31.5	26.0	21.5	17.5	15.0	12.5	12.0	12.0

The storm hydrograph is plotted to scale (Fig. 6.12). Denoting the time from beginning of storm as t, by inspection of Fig. 6.12,

A = beginning of DRH $t = 0$

B = end of DRH $t = 90$ h

P_m = peak $t = 20$ h

Hence

$$N = (90-20) = 70 \text{ h} = 2.91 \text{ days}$$

By Eq. (6.4),

$$N = 0.83\,(423)^{0.2} = 2.78 \text{ days},$$

However, $N = 2.91$ days is adopted for convenience. A straight line joining A and B is taken as the divide line for base-flow separation. The ordinates of DRH are obtained by subtracting the base flow from the ordinates of the storm hydrograph. The calculations are shown in Table 6.5.

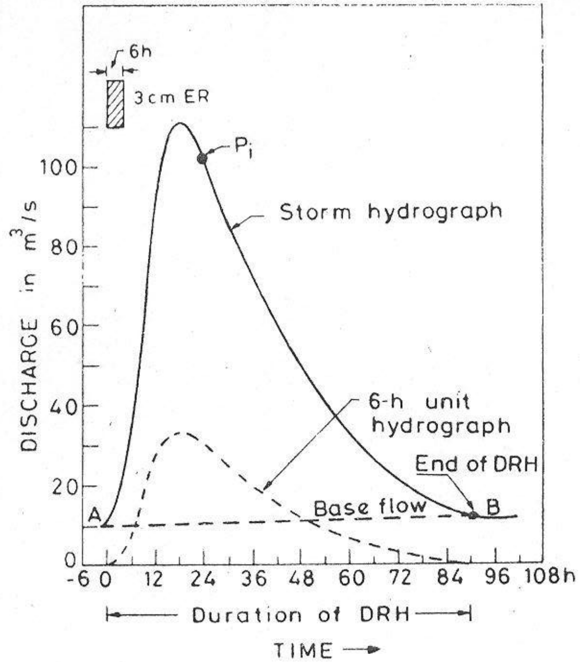

Fig. 6.12 Derivation of unit hydrograph from a storm hydrograph

Volume of DRH $= 60\times60\times6\times$(sum of DRH ordinates)
$= 60\times60\times6\times587 = 12.68\ \text{Mm}^3$

Drainage area $= 423\ \text{km}^2 = 423\ \text{Mm}^2$

Runoff depth = ER depth $= \dfrac{12.68}{423} = 0.03\ \text{m} = 3\ \text{cm}.$

The ordinates of DRH (col. 4) are divided by 3 to obtain the ordinates of the 6-h unit hydrograph, see Table 6.5.

EXAMPLE 6.7 *The peak of a flood hydrograph due to a 3-h effective storm is 270 m³/s. The mean depth of rainfall is 5.9 cm. Assuming an average infiltration loss of 0.3 cm/h and a constant base flow of 20 m³/s, estimate the peak of the 3-h unit hydrograph.*

Duration of rainfall excess = 3 h

Total depth of rainfall = 5.9 cm

Loss @ 0.3 cm/h for 3 h = 0.9 cm

Rainfall excess = 5.9-0.9 = 5.0 cm

TABLE 6.5 CALCULATION OF THE ORDINATES OF A 6-h UNIT HYDROGRAPH—EXAMPLE 6.6

Time from beginning of storm (h)	Ordinate of storm hydro-graph (m³/s)	Base flow (m³/s)	Ordinate of DRH (m³/s)	Ordinate of 6-h unit hydrograph (Col. 4 ÷ 3)
1	2	3	4	5
—6	10.0	10.0	0	0
0	10.0	10.0	0	0
6	30.0	10.0	20.0	6.7
12	87.5	10.5	77.0	25.7
18	111.5	10.5	101.0	33.7
24	102.5	10.5	92.0	30.7
30	85.0	11.0	74.0	24.7
36	71.0	11.0	60.0	20.0
42	59.0	11.0	48.0	16.0
48	47.5	11.5	36.0	12.0
54	39.0	11.5	27.5	9.2
60	31.5	11.5	20.0	6.6
66	26.0	12.0	14.0	4.6
72	21.5	12.0	9.5	3.2
78	17.5	12.0	5.5	1.8
84	15.0	12.5	2.5	0.8
90	12.5	12.5	0	0
96	12.0	12.0	0	0
102	12.0	12.0	0	0
			Sum = 587.0	195.7

Example 6.7 (*Contd.*)

Peak flow:

Peak of flood hydrograph = 270 m³/s

Base flow = 20~~0~~ m³/s

Peak of DRH = 250 m³/s

Peak of 3-h unit hydrograph $= \dfrac{\text{peak of DRH}}{\text{rainfall excess}}$

$= \dfrac{250}{5.0} = 50\ \text{m}^3/\text{s}$

Unit Hydrograph from a Complex Storm

When suitable simple isolated storms are not available, data from complex storms of long duration will have to be used in unit-hydrograph derivation. The problem is to decompose a measured composite flood hydrograph into its component DRHs and base flow. A common unit hydrograph of appropriate duration is assumed to exist. This problem is thus the inverse of the derivation of flood hydrograph through use of Eq. (6.5). Consider a rainfall excess made up of three consecutive durations of *D*-h and ER values of R_1, R_2 and R_3. Figure 6.13 shows the ERH. By base flow separation of the resulting composite flood hydrograph a composite DRH is obtained (Fig. 6.13). Let the ordinates of the composite DRH be

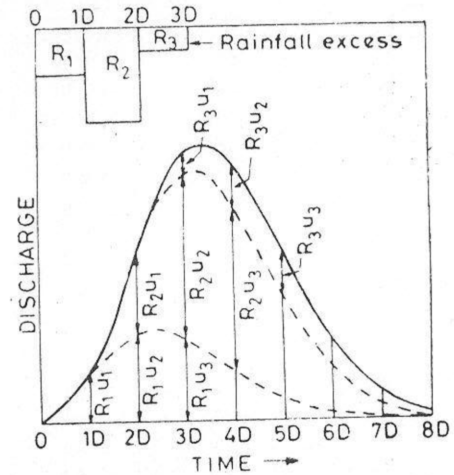

Fig. 6.13 Unit hydrograph from a complex storm

drawn at a time interval of *D* h. At various time intervals 1*D*, 2*D*, 3*D*, ... from the start of the ERH, let the ordinates of the unit hydrograph be u_1, u_2, u_3, ... and the ordinates of the composite DRH be Q_1, Q_2, Q_3, ... , Then

$$
\begin{aligned}
Q_1 &= R_1 u_1 \\
Q_2 &= R_1 u_2 + R_2 u_1 \\
Q_3 &= R_1 u_3 + R_2 u_2 + R_3 u_1 \\
Q_4 &= R_1 u_4 + R_2 u_3 + R_3 u_2 \\
Q_5 &= R_1 u_5 + R_2 u_4 + R_3 u_3 \\
&\cdots\cdots\cdots\cdots
\end{aligned}
\qquad (6.6)
$$

so on.

From the Eq. (6.6) the values of u_1, u_2 u_3, ... can be determined. However, this method suffers from the disadvantage that the errors propogate and increase as the calculations proceed. In the presence of errors the recession limb of the derived *D*-h unit hydrograph can contain oscillations and even negative values. Matrix methods with optimisation schemes are available for solving Eq. (6.6) in a digital computer.

6.8 UNIT HYDROGRAPHS OF DIFFERENT DURATIONS

Ideally, unit hydrographs are derived from simple isolated storms and if the duration of the various storms do not differ very much, say within a band width of ± 20% *D*, they would all be grouped under one average duration of *D* h. If in practical applications unit hydrographs of different duration are needed they are best derived from field data. Lack of adequate data normally precludes development of unit hydrographs covering a wide range of durations for a given catchment. Under such conditions a *D*-h unit hydrograph is used to develop unit hydrographs of differing durations, *nD*. Two methods are available for this purpose.

1. Method of superposition, and
2. the *S*-curve.

These are discussed below.

Method of Superposition

If a *D*-h unit hydrograph is available, and it is desired to develop a unit hydrograph of *nD* h, where *n* is an integer, it is easily accomplished by superposing *n* unit hydrographs with each graph separated from the previous one by *D* h. Figure 6.14 shows three 4-h

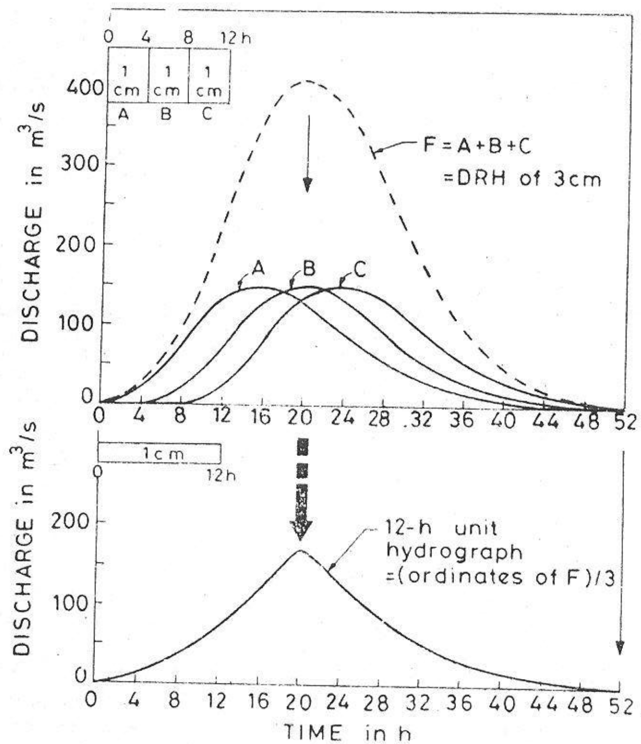

Fig. 6.14 Construction of a 12-h unit hydrograph from a 4-h unit hydrograph

unit hydrographs A, B and C. Curve B begins 4 h after A and C begins 4 h, after B. Thus the combination of these three curves is a DRH of 3 cm due to an ER of 12 h duration. If the ordinates of this DRH are now divided by 3, one obtains a 12-h unit hydrograph. The calculations are easy if performed in a tabular form (Table 6.6).

EXAMPLE 6.8 *Given the ordinates of a 4-h unit hydrograph (columns 1 and 2 in Table 6.6) derive the ordinates of a 12-h unit hydrograph.*

The calculations are performed in a tabular form in Table 6.6 In this

Column 3 = ordinates of 4-h UH lagged by 4 h

Column 4 = ordinates of 4-h UH lagged by 8 h

Column 5 = ordinates of DRH representing 3 cm ER in 12 h

Column 6 = ordinates of 12-h UH = (column 5)/3

The 12-h unit hydrograph is shown in Fig. 6.14.

TABLE 6.6 CALCULATION OF A 12-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH —*Example 6.8*

Time (h)	Ordinates of 4-h UH (m^3/s) A	B Lagged by 4 h	C Lagged by 8 h	DRH (Col. 2+3+4)	Ordinate of 12-h UH (m^3/s) (Col. 5)/3
1	2	3	4	5	6
0	0	—	—	0	0
4	20	0	—	20	6.7
8	80	20	0	100	33.3
12	130	80	20	230	76.7
16	150	130	80	360	120.0
20	130	150	130	410	136.7
24	90	130	150	370	123.3
28	52	90	130	272	90.7
32	27	52	90	169	56.3
36	15	27	52	94	31.3
40	5	15	27	47	15.7
44	0	5	15	20	6.7
48		0	5	5	1.7
52			0	0	0

The S-Curve

If it is desired to develop a unit hydrograph of duration mD, where m is a fraction, the method of superposition cannot be used. A different technique known as the *S*-curve method is adopted in such cases, and this method is applicable for rational values of m.

The *S-curve*, also known as *S-hydrograph* is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of D-h unit hydrographs spaced D-h apart. Figure 6.15 shows such a series of D-h hydrograph arranged with their starting points D-h apart. At any given time the ordinates of the various curves occurring at that time coordinate are summed up to obtain ordinates of the S-curve. A smooth curve through these ordinates result in an S-shaped curve called S-curve.

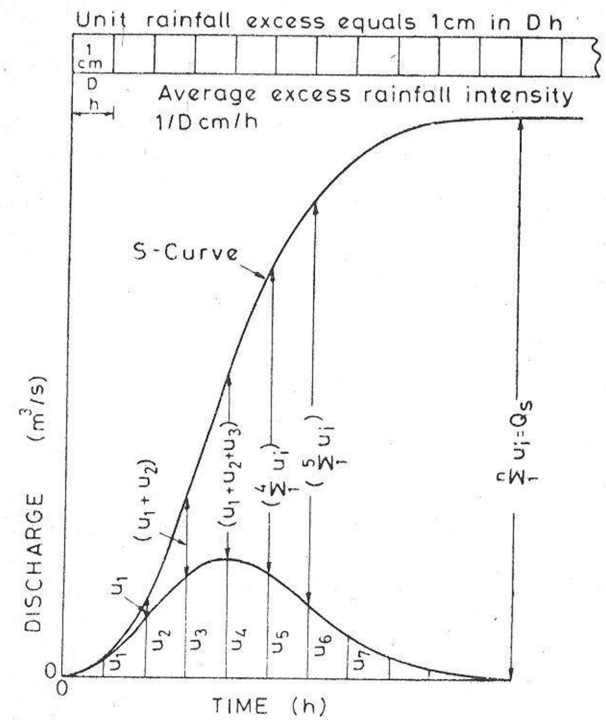

Fig. 6.15 *S*-curve

This S-curve is due to a D-h unit hydrograph. It has an initial steep portion and reaches a maximum equilibrium discharge at a time equal to the time base of the first unit hydrograph. The average intensity of ER producing the S-curve is $1/D$ cm/h and the equilibrium discharge,

$$Q_s = \left(\frac{A}{D} \times 10^4\right) \text{ m}^3\text{/h},$$

where A = area of the catchment in km² and D = duration in hours of ER of the unit hydrograph used in deriving the S-curve. Alternatively

$$Q_s = 2.778 \frac{A}{D} \text{ m}^3\text{/s} \quad (6.7)$$

where A is in km² and D is in h. The quantity Q_s represents the maximum rate at which an ER intensity of $1/D$ cm/h can drain out of a catchment of area A. In actual construction of an S-curve, it is found that the curve oscillates in the top portion at around the equilibrium value due to magnification and accumulation of small errors in the hydrograph. When it occurs, an average smooth curve is drawn such that it reaches a value Q_s at the time base of the unit hydrograph.

Consider two D-h S-curves A and B displaced by T h (Fig. 6.16). If the ordinates of B are subtracted from that of A, the resulting curve is a DRH produced by a rainfall excess of duration T h and magnitude $\left(\frac{1}{D} \times T\right)$ cm. Hence if the ordinate differences of A and B, i.e. $(S_A - S_B)$ are divided by T/D, the resulting ordinates denote a hydrograph due to an ER of 1 cm and of duration T h, i.e. a T-h unit hydrograph. The derivation of a T-h unit hydrograph as above can be achieved either by graphical means or by arithmetic computations in a tabular form as indicated in Example 6.9.

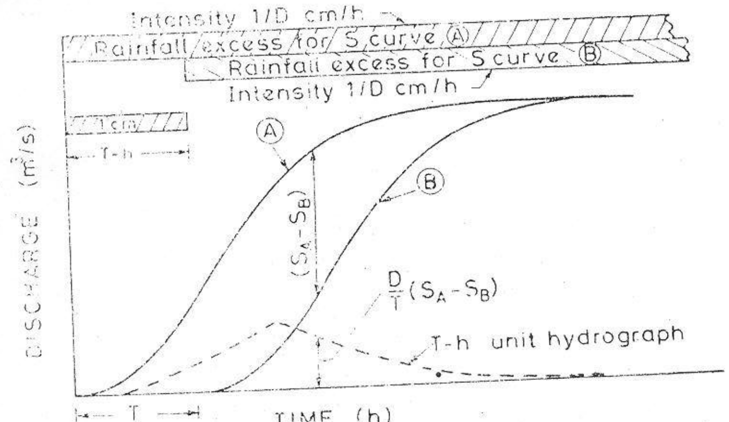

Fig. 6.16 Derivation of a T-h unit hydrograph by S-curve lagging method

EXAMPLE 6.9 *Solve Example 6.8 by the S-curve method.*

The computations are shown in Table 6.7. Column 2 shows the ordinates of the 4-h unit hydrograph. Column 3 gives the S-curve additions and column 4 the S-curve ordinates. The sequence of additions are shown by arrows. At $t = 4$ h, ordinate of the 4-h UH = ordinate of the S-curve. This value becomes the S-curve addition at $t = 2 \times 4 = 8$ h. At this $t = 8$ h, the ordinate of UH (80) + S-curve addition (20) = S-curve ordinate (100). The S-curve addition at $t = 3 \times 4 = 12$ h is 100, and so on. Column 5 shows the S-curve lagged by 12 h. Column 6 gives the subtraction of lagged S-curve (column 5) from the S-curve (column 4). Ordinates shown in column 6 are divided by $T/D = 12/4 = 3$ to obtain the ordinates of the 12-h unit hydrograph shown in column 7.

TABLE 6.7 DETERMINATION OF A 12-h UNIT HYDROGRAPH BY S-CURVE METHOD — *Example 6.9*

Time (h)	Ordinate of 4-h UH (m³/s)	S-curve addition (m³/s)	S-curve ordinate (m³/s) (Col. 2 + Col. 3)	S-curve lagged by 12 h (m³/s)	(Col. 4 − Col. 5)	Col. 6 (12/4) = 12-h UH ordinates (m³/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
4	20	0	20	—	20	6.7
8	80	20	100	—	100	33.3
12	130	100	230	0	230	76.7
16	150	230	380	20	360	120.0
20	130	380	510	100	410	136.7
24	90	510	600	230	370	123.3
28	52	600	652	380	272	90.7
32	27	652	679	510	169	56.3
36	15	679	694	600	94	31.3
40	5	694	699	652	47	15.7
44	0	699	699	679	20	6.7
48		699	699	694	5	1.7
54			699	699	0	0

EXAMPLE 6.10 *Ordinates of a 4-h unit hydrograph are given. Using this derive the ordinates of a 2-h unit hydrograph for the same catchment.*

Time (h)	0	4	8	12	16	20	24	28	32	36	40	44
Ordinate of 4-h UH (m³/s)	0	20	80	130	150	130	90	52	27	15	5	0

In this case the time interval of the ordinates of the given unit hydrograph should be at least 2 h. As the given ordinates are at 4-h intervals, the unit-hydrograph is plotted and its ordinates at 2-h intervals determined. The ordinates are shown in column 2 of Table 6.8. The S-curve additions and S-curve ordinates are shown in columns 3 and 4 respectively. First, the S-curve ordinates corresponding to the time intervals equal to successive

TABLE 6.8 DETERMINATION OF 2-h UNIT HYDROGRAPH FROM A 4-h UNIT HYDROGRAPH —*Example 6.10*

Time (h)	Ordinate of 4-h UH (m^3/s)	S-curve addition (m^3/s)	S-curve ordinate Col. (2) +(3) (m^3/s)	S-curve Lagged by 2 (h)	Col. (4) −(5)	2-h UH ordinate $\frac{\text{Col. (6)}}{(2/4)}$ (m^3/s)
1	2	3	4	5	6	7
0	0	—	0	—	0	0
2	8		8	0	8	16
4	20	0	20	8	12	24
6	43	8	51	20	31	62
8	80	20	100	51	49	98
10	110	51	161	100	61	122
12	130	100	230	161	69	138
14	146	161	307	230	77	154
16	150	230	380	307	73	146
18	142	307	449	380	69	138
20	130	380	510	449	61	122
22	112	449	561	510	51	102
24	90	510	600	561	39	78
26	70	561	631	600	31	62
28	52	600	652	631	21	42
30	38	631	669	652	17	34
32	27	652	679	669	10	20
34	20	669	689	679	10	(20)15
36	15	679	694	689	5	(10)10
38	10	689	699	694	5	(10)6
40	5	694	699	699	(0)	(0)3
42	2	699	701	699	(2)	(4)0
44	0	699	699	701	(−2)	(−4)0

Final adjusted values are given in col. 7.
Unadjusted values are given in parentheses.

durations of the given unit hydrograph (in this case at 0, 4, 8, 12,...h) are determined by following the method of Example 6.8. Next, the ordinates at intermediate intervals (viz. at t = 2, 6, 10, 14...h) are determined by having another series of S-curve additions. The sequence of these are shown by distinctive arrows in Table 6.8. To obtain a 2-h unit hydrograph the *S*-curve is lagged by 2 h (column 5) and this is subtracted from column 4 and the results listed in column 6. The ordinates in column 6 are now divided by $T/D = 2/4 = 0.5$, to obtain the required 2-h unit hydrograph ordinates, shown in column 7.

The errors in interpolation of unit hydrograph ordinates often result in oscillation of S-curve at the equilibrium value. This results in the derived *T*-h unit hydrograph having an abnormal sequence of discharges (sometimes even negative values) at the tail end. This is adjusted by fairing the S-curve and also the resulting *T*-h unit-hydrograph by smooth curves. For example, in the present example the 2-h unit hydrograph ordinates at time > 36-h are rather abnormal. These values are shown in parentheses. The adjusted values are entered in column 7.

6.9 USE AND LIMITATIONS OF UNIT HYDROGRAPH

As the unit hydrographs establish a relationship between the ERH and DRH for a catchment, they are of immense value in the study of the hydrology of a catchment. They are of great use in (i) the development of flood hydrographs for extreme rainfall magnitudes for use in the design of hydraulic structures, (ii) extention of flood-flow records based on rainfall records and (iii) development of flood forecasting and warning systems based on rainfall.

Unit hydrographs assume uniform distribution of rainfall over the catchment. Also, the intensity is assumed constant for the duration of the rainfall excess. In practice, these two conditions are never strictly satisfied. Nonuniform areal distribution and variation in intensity within a storm are very common. Under such conditions unit hydrographs can still be used if the areal distribution is consistent between different storms. However, the size of the catchment imposes an upper limit on the applicability of the unit hydrograph. This is because in very large basins the centre of the storm can vary from storm to storm and each can give different DRHs under otherwise identical situations. It is generally felt that about 5000 km^2 is the upper limit for unit-hydrograph use. Flood hydrographs in very large basins can be studied by dividing them into a number of smaller subbasins and developing DRHs by the unit-hydrograph method. These DRHs can then be routed through their respective channels to obtain the composite DRH at the basin outlet.

There is a lower limit also for the application of unit hydrographs. This limit is usually taken as about 200 ha. At this level of area, a number of factors affect the rainfall–runoff relationship and the unit hydrograph is not accurate enough for the prediction of DRH.

Other limitations to the use of unit hydrographs are:

1. Precipitation must be from rainfall only. Snow-melt runoff cannot be satisfactorily represented by unit hydrograph.
2. The catchment should not have unusually large storages in terms of tanks, ponds, large flood-bank storages, etc. which affect the linear relationship between storage and discharge.
3. If the precipitation is decidedly nonuniform, unit hydrographs cannot be expected to give good results.

In the use of unit hydrographs very accurate reproduction of results should not be expected. Variations in the hydrograph base of as much as ± 20% and in the peak discharge by ± 10% are normally considered acceptable.

6.10 DURATION OF THE UNIT HYDROGRAPH

The choice of the duration of the unit hydrograph depends on the rainfall records. If recording raingauge data, are available any convenient time depending on the size of the basin can be used. The choice is not much if only daily rainfall records are available. A rough guide for the choice of duration D is that it should not exceed the least of (i) the time of rise, (ii) the basin lag and (iii) the time of concentration. A value of D equal to about 1/4 of the basin lag is about the best choice. Generally, for basins with areas more than 1200 km² a duration D = 12 hrs is preferred.

6.11 DISTRIBUTION GRAPH

The distribution graph introduced by Bernard (1935) is a variation of the unit hydrograph. It is basically a D-h unit hydrograph with ordinates showing the percentage of the surface runoff occurring in successive periods of equal time intervals of D h. The duration of the rainfall excess (D h) is taken as the unit interval and distribution-graph ordinates are indicated at successive such unit intervals. Figure 6.17 shows a typical 4-h distribution graph. Note the ordinates plotted at 4-h intervals and the total area under the distribution graph adds up to 100%. The use of the distribution graph to generate a DRH for a known ERH is exactly the same as that of a unit hydrograph (Example 6.11). Distribution graphs are useful in comparing the runoff characteristics of different catchments.

EXAMPLE 6.11 *A catchment of 200 hectares area has rainfalls of 7.5 cm, 2.0 cm and 5.0 cm in three consecutive days. The average ϕ index can be assumed to be 2.5 cm/day. Distribution-graph percentages of the surface runoff which extended over 6 days for every rainfall of 1-day duration are 5, 15, 40, 25, 10 and 5. Determine the ordinates of the discharge hydrograph by neglecting the base flow.*

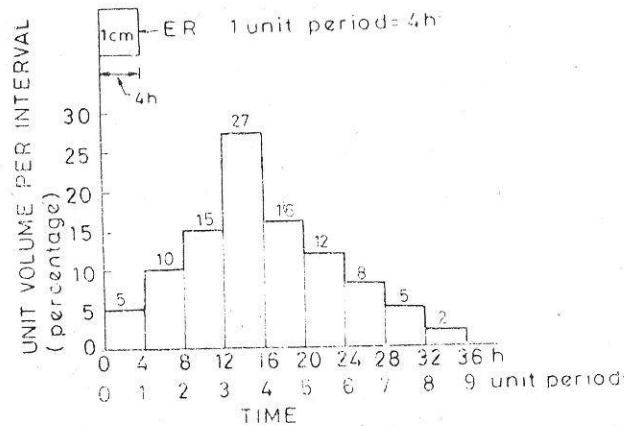

Fig. 6.17 Four-hour distribution graph

The calculations are performed in a tabular form in Table 6.9.

6.12 SYNTHETIC UNIT HYDROGRAPH

To develop unit dydrographs to a catchment, detailed information about the rainfall and the resulting flood hydrograph are needed. However, such information would be available only at a few locations and in a majority of catchments, especially those which are at remote locations, the data would normally be very scanty. In order to construct unit hydrographs for such areas, empirical equations of regional validity which relate the salient hydrograph characteristics to the basin characteristics are available. Unit hydrographs derived from such relationships are known as synthetic-unit hydrographs. A number of methods for developing synthetic-unit hydrographs are reported in literature. It should, however, be remembered that these methods being based on empirical correlations are applicable only to the specific regions in which they were developed and should not be considered as general relationships for use in all regions.

Snyder's Method

Snyder (1938), based on a study of a large number of catchments in the Appalachian Highlands of eastern United States developed a set of empirical equations for synthetic-unit hydrographs in those areas. These equations are in use in the USA, and with some modifications in many

Table 6.9 Calculation of DRH Using Distribution Graph—*Example 6.11*

Time interval (days)	Rainfall (cm)	Infiltration loss (cm)	Effective rainfall (cm)	Average distribution ratio (per cent)	Distributed runoff (cms) for rainfall excess of			Runoff	
					5 cm	0	2.5 cm	cm	m^3/s $\times 10^{-2}$
0–1	7.5	2.5	5.0	5	0.250	0		0.250	5.79
1–2	2.0	2.5	0	15	0.750	0	0	0.750	17.36
2–3	5.0	2.5	2.5	40	2.000	0	0.125	2.125	49.19
3–4				25	1.250	0	0.375	1.625	37.62
4–5				10	0.500	0	1.000	1.500	34.72
5–6				5	0.250	0	0.625	0.875	20.25
6–7				0	0	0	0.250	0.250	5.79
7–8						0	0.125	0.125	2.89
8–9							0	0	0

[Runoff of 1 cm in 1 day $= \dfrac{200 \times 100 \times 100}{86400 \times 100}$ m^3/s for 1 day $= 0.23148\ m^3/s$ for 1 day]

(The runoff ordinates are plotted at the mid-points of the respective time intervals to obtain the DRH)

other countries, and constitute what is known as Snyder's synthetic-unit hydrograph.

The most important characteristic of a basin affecting a hydrograph due to a given storm is *basin lag*. Actually basin lag (also known as *lag time*) is the time difference between the centroids of the input (rainfall excess) and the output (surface runoff), i.e. T_L indicated in Sec. 6.1. Physically, it represents the mean time of travel of water particles from all parts of the catchment to the outlet during a given storm. Its value is determined essentially on the physical features of the catchment, such as size, length, stream density and vegetation. For its determination, however, only a few important catchment characteristics are considered. For simplicity, Snyder has used a somewhat different definition of basin lag (denoted by t_p) in his methodology. This t_p is practically of the same order of magnitude as T_L and in this section the term basin lag is used to denote Snyder's t_p.

The first of the Snyder's equation relates the basin lag t_p, defined as the time interval from the mid-point of the unit rainfall excess to the peak of the unit hydrograph (Fig. 6.18), to the basin characteristics as

$$t_p = C_t \,(L\, L_{ca})^{0.3} \qquad (6.8)$$

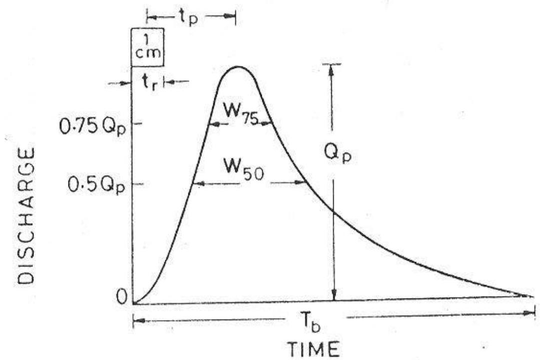

Fig. 6.18 Elements of a synthetic unit hydrograph

where t_p is in hours

L = basin length measured along the water course from the basin divide to the gauging station in km

L_{ca} = distance along the main water course from the gauging station to a point opposite the watershed centroid in km

C_t = a regional constant representing watershed slope and storage

The value of C_t in Snyder's study ranged from 1.35 to 1.65. However, studies by many investigators have shown that C_t depends upon the region under study and wide variations with the value of C_t ranging from 0.3 to 6.0 have been reported.

Linsley et al.[5] found that the basin lag t_p is better correlated with the catchment parameter $\left(\frac{LL_{ca}}{\sqrt{S}}\right)$ where S = basin slope. Hence, a modified form of Eq. (6.8) was suggested by them as

$$t_p = C_{tL}\left(\frac{LL_{ca}}{\sqrt{S}}\right)^n \quad (6.9)$$

where C_{tL} and n are basin constants. For the basins in the USA studied by them n was found to be equal to 0.38 and the values of C_{tL} were 1.715 for mountainous drainage areas, 1.03 for foot-hill drainage areas and 0.50 for valley drainage areas.

Snyder adopted a standard duration t_r hours of effective rainfall given by

$$t_r = \frac{t_p}{5.5} \quad (6.10)$$

The peak discharge Q_{ps} (m³/s) of a unit hydrograph of *standard duration* t_r h is given by Snyder as

$$Q_{ps} = \frac{2.78\ C_p\ A}{t_p} \quad (6.11)$$

where A = catchment area in km² and C_p = a regional constant. This equation is based on the assumption that the peak discharge is proportional to the average discharge of $\left(\frac{1\text{ cm} \times \text{catchment area}}{\text{duration of rainfall excess}}\right)$. The values of the coefficient C_p range from 0.56 to 0.69 for Snyder's study areas and is considered as an indication of the retention and storage capacity of the watershed. Like C_t, the values of C_p also vary quite considerably depending on the characteristics of the region and values of C_p in the range 0.31 to 0.93 have been reported.

If a non-standard rainfall duration t_R h is adopted, instead of the standard value t_r, to derive a unit hydrograph the value of the basin lag is affected. The modified basin lag is given by

$$t'_p = t_p + \frac{t_R - t_r}{4}$$

$$= \frac{21}{22}\ t_p + \frac{t_R}{4} \quad (6.12)$$

where t'_p = basin lag in hours for an effective duration of t_R h and t_p is as given by Eq. (6.8) or (6.9). The value of t'_p must be used instead of t_p in Eq. (6.11). Thus the peak discharge for a nonstandard ER of duration t_R is in m³/s

$$Q_p = 2.78\ C_p\ A/t'_p \quad (6.11a)$$

Note that when $t_R = t_r$,

$$Q_p = Q_{ps}$$

The time base of a unit hydrograph (Fig. 6.18) is given by Snyder as

$$t_b = 3 + \frac{t'_p}{8}\text{ days} = (72 + 3t'_p)\text{ h} \quad (6.13)$$

where t_b = time base. While Eq. (6.13) gives reasonable estimates of t_b for large catchments, it may give excessively large values of the time base for small catchments. Taylor and Schwartz[1] recommend

$$t_b = 5\left(t'_p + \frac{t_R}{2}\right) \quad (6.14)$$

with t_b (given in h) taken as the next larger integer value divisible by t_R i.e. t_b is about five times the time-to-peak.

To assist in the sketching of unit hydrographs, the widths of unit hydrographs at 50 and 75% of the peak (Fig. 6.18) have been found for US catchments by the US Army Corps of Engineers. These widths (in time units) are correlated to the peak discharge intensity and are given by

$$W_{50} = \frac{5.87}{q^{1.08}} \quad (6.15)$$

and

$$W_{75} = W_{50}/1.75 \quad (6.16)$$

where

W_{50} = width of unit hydrograph in h at 50% peak discharge

W_{75} = width of unit hydrograph in h at 75% peak discharge

$q = Q_p/A$ = peak discharge per unit catchment are in m³/s/km²

Since the coefficients C_t and C_p vary from region to region, in practical applications it is advisable that the value of these coefficients are determined from known unit hydrographs of a meteorologically homogeneous catchment and then used in the basin under study. This way Snyder's equations are of use in scaling the hydrograph information from one catchment to another similar catchment.

EXAMPLE 6.12 *Two catchments A and B are considered meteorologically similar. Their catchment characteristics are given below.*

Catchment A	Catchment B
L = 30 km	L = 45 km
L_{ca} = 15 km	L_{ca} = 25 km
A = 250 km²	A = 400 km²

For catchment A, a 2-h unit hydrograph was developed and was found to have a peak discharge of 50 m³/s. The time to peak from the beginning of the rainfall excess in this unit hydrograph was 9.0 h. Using Snyder's method, develop a unit hydrograph for catchment B.

For Catchment A:

$$t_R = 2.0 \text{ h}$$

Time to peak from beginning of ER $= \frac{t_R}{2} + t_p' = 9.0$ h

$$\therefore \quad t_p' = 8.0 \text{ h}$$

From Eq. (6.12),

$$t_p' = \frac{21}{22} t_p + \frac{t_R}{4}$$

$$= \frac{21}{22} t_p + 0.5 = 8.0$$

$$t_p = \frac{7.5 \times 22}{21} = 7.857 \text{ h}$$

From Eq. (6.8),

$$t_p = C_t (L L_{ca})^{0.3}$$

$$7.857 = C_t (30 \times 15)^{0.3}$$

$$C_t = 1.257$$

From Eq. (6.11a),

$$Q_p = 2.78 \; C_p \, A/t_p'$$

$$50 = 2.78 \times C_p \times 250/8.0$$

$$C_p = 0.576$$

For Catchment B: Using the values of $C_t = 1.257$ and $C_p = 0.576$ in catchment *B*, the parameters of the synthetic-unit hydrograph for catchment *B* are determined.

From Eq. (6.8),

$$t_p = 1.257 (45 \times 25)^{0.3} = 10.34 \text{ h}$$

By Eq. (6.10),

$$t_r = \frac{10.34}{5.5} = 1.88 \text{ h}$$

Using $t_R = 2.0$ h, i.e. for a 2-h unit hydrograph, by Eq. (6.12),

$$t_p' = 10.34 \times \frac{21}{22} + \frac{2.0}{4} = 10.37 \text{ h}$$

By Eq. (6.11a),

$$Q_p = \frac{2.78 \times 0.576 \times 400}{10.37}$$

$$= 61.77 \text{ m}^3/\text{s}, \quad \text{say } 62 \text{ m}^3/\text{s}$$

From Eq. (6.15),

$$W_{50} = \frac{5.87}{(62/400)^{1.08}} = 44 \text{ h}$$

By Eq. (6.16),

$$W_{75} = \frac{44}{1.75} = 25 \text{ h}$$

Time base : From Eq. (6.13), $t_b = 72 + (3 \times 10.37) = 103$ h

From Eq. (6.14), $t_b = 5\,(10.37 + 10) \approx 58$ h

Considering the values of W_{50} and W_{75} and noting that the area of catchment *B* is rather small, $t_b \approx 58$ h is more appropriate in this case.

Finalizing of Synthetic-Unit Hydrograph

After obtaining the values of Q_p, t_R, t_p', W_{75}, W_{50} and t_b from Snyder's equations, a tentative unit hydrograph is sketched. An S-curve is then developed and plotted. As the ordinates of the unit hydrograph are tentative, the S-curve thus obtained will have kinks. These are then smoothened and a logical pattern of the S-curve is sketched. Using this S-curve the t_R-h unit hydrograph is then derived back. Further, the area under the unit hydrograph is checked to see that it represents 1 cm of runoff. The procedure of adjustment through the S-curve is repeated till satisfactory results are obtained. It should be noted that out of the various parameters of the synthetic-unit hydrograph the least accurate will be the time base t_b and this can be changed to meet other requirements.

Dimensionless-Unit Hydrograph

Dimensionless-unit hydrographs based on a study of a large number of unit hydrographs are recommended by various agencies to facilitate construction of synthetic-unit hydrographs. A typical dimensionless unit hydrograph developed by the US Soil Conservation Service (SCS) is shown in Fig. 6.19. In this the ordinate is the discharge expressed as a ratio to the peak discharge (Q/Q_p) and the abscissa is the time expressed as a ratio of time to peak (t/t_{pk}). In terms of Snyder's notation, $t_{pk} = \left(\frac{t_R}{2} + t_p'\right)$. By definition, $Q/Q_p = 1.0$ when $t/t_{pk} = 1.0$. The coordinates of the SCS dimensionless unit hydrograph are given in Table 6.10 for use in developing a synthetic unit-hydrograph in place of Snyder's equations (6.13) through (6.16).

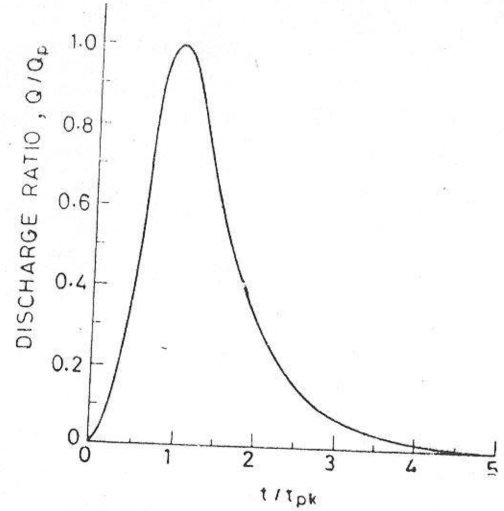

Fig. 6.19 Dimensionless SCS unit hydrograph

TABLE 6.10 COORDINATES OF SCS DIMENSIONLESS UNIT HYDROGRAPH[4]

t/t_{pk}	Q/Q_p	t/t_{pk}	Q/Q_p	t/t_{pk}	Q/Q_p
0	0	1.1	0.98	2.8	0.098
0.1	0.015	1.2	0.92	3.0	0.075
0.2	0.075	1.3	0.84	3.5	0.036
0.3	0.16	1.4	0.75	4.0	0.018
0.4	0.28	1.5	0.66	4.5	0.009
0.5	0.43	1.6	0.56	5.0	0.004
0.6	0.60	1.8	0.42		
0.7	0.77	2.0	0.32		
0.8	0.89	2.2	0.24		
0.9	0.97	2.4	0.18		
1.0	1.00	2.6	0.13		

The Indian Practice

The CWC (India) after a study of a large number of catchments in India of varying sizes in the range 25 to 500 km² has recommended the following relations for developing synthetic-unit hydrographs.

The peak discharge of a D-h unit hydrograph Q_{pd} in m³/s is

$$Q_{pd} = 4.44\ A^{3/4} \quad \text{for } S_m > 0.0028 \qquad (6.17)$$

and

$$Q_{pd} = 222\ A^{3/4}\ S_m^{2/3} \quad \text{for } S_m < 0.0028 \qquad (6.18)$$

where A = catchment area in km² and S_m = weighted mean slope given by

$$S_m = [L_{ca}/(L_1/\sqrt{S_1}+L_2/\sqrt{S_2}+\ldots + L_n/\sqrt{S_n})]^2 \qquad (6.19)$$

in which L_{ca} = distance along the river from the gauging station to a point opposite the centre of gravity of the area.

$L_1, L_2, \ldots L_n$ = length of main channel having slopes S_1, S_2, S_n respectively, obtained from topographic maps.

The lag time in hours (i.e. time interval from the mid-point of the rainfall excess to the peak) of a 1-h unit hydrograph, t_{p1} is given by

$$t_{p1} = \frac{3.95}{[Q_{pd}/A]^{0.9}} \qquad (6.20)$$

For design purposes the duration of rainfall excess is taken as

$$D = 1.1\ t_{p1}\ \text{h} \qquad (6.21)$$

Equations (6.17) through (6.21) enable one to determine the duration and peak discharge of a design-unit hydrograph. The time to peak has to be determined separately by using Eq. (6.8) or (6.9).

6.13 INSTANTANEOUS UNIT HYDROGRAPH (IUH)

The unit-hydrograph concept discussed in the preceding sections considered a D-h unit hydrograph. For a given catchment a number of unit hydrographs of different durations are possible. The shape of these different unit hydrographs depend upon the value of D. Figure 6.20 shows a typical variation of the shape of unit hydrographs for different values of D. As D is reduced, the intensity of rainfall excess being equal to $1/D$ increases and the unit hydrograph becomes more skewed. A finite unit hydrograph is indicated as the duration $D \to 0$. The limiting case of a unit hydrograph of zero duration is known as *instantaneous unit hydrograph* (IUH). Thus IUH is a fictitious, conceptual unit hydrograph which

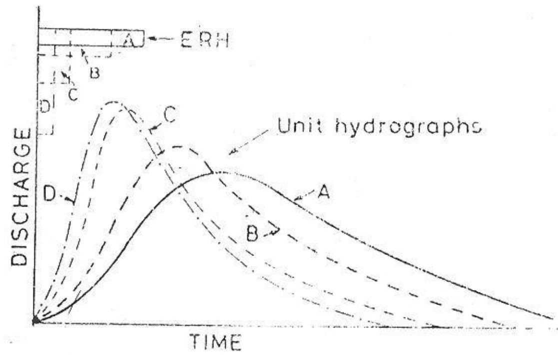

Fig. 6.20 Unit hydrographs of different durations

represents the surface runoff from the catchment due to an instantaneous precipitation of the rainfall excess volume of 1 cm. IUH is designed as $u(t)$ or sometimes as $u(0, t)$. It is a single-peaked hydrograph with a finite base width and its important properties can be listed as below:

1. $0 \leqslant u(t) \leqslant$ a positive value, for $t > 0$;
2. $u(t) = 0$ for $t \leqslant 0$;
3. $u(t) \rightarrow 0$ as $t \rightarrow \infty$;
4. $\int_0^\infty u(t)\, dt =$ unit depth over the catchment; and
5. time to the peak < time to the centroid of the curve.

Consider an effective rainfall $I(\tau)$ of duration t_0 applied to a catchment as in Fig. 6.21. Each infinitesimal element of this ERH will operate on the IUH to produce a DRH whose discharge at time t is given by

$$Q(t) = \int_0^{t'} u(t-\tau)\, I(\tau)\, d\tau \qquad (6.22)$$

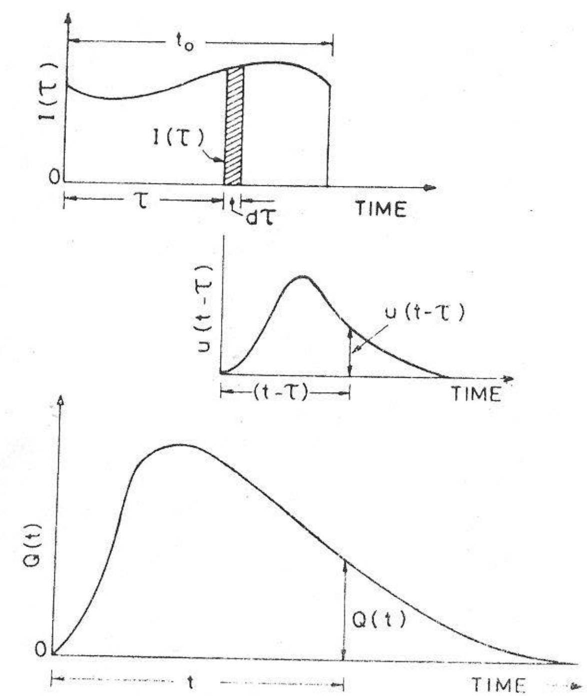

Fig. 6.21 Convolution of $I(\tau)$ and IUH

where $\quad t' = t$ when $t < t_0$

and $\quad t' = t_0$ when $t \geqslant t_0$

Equation (6.22) is called the *convolution integral* or *Duhamel integral*. The integral of Eq. (6.22) is essentially the same as the arithmetical computation of Eq. (6.5).

The main advantage of IUH is that it is independent of the duration of ERH and thus has one parameter less than a *D*-h unit hydrograph. This fact and the definition of IUH make it eminently suitable for theoretical analysis of rainfall excess–runoff relationship of a catchment. For a given catchment IUH, being independent of rainfall characteristics, is indicative of the catchment storage characteristics.

Derivation of IUH

Consider an S-curve, designated as S_1, derived from a *D*-h unit hydrograph. In this the intensity of rainfall excess, $i = 1/D$ cm/h. Let S_2 be another S-curve of intensity i cm/h. If S_2 is separated from S_1 by a time interval dt and the ordinates are subtracted, a DRH due to a rainfall excess of duration dt h and magnitude $i\,dt = dt/D$ h is obtained. A unit hydrograph of dt h is obtained from this by dividing the above DRH by $i\,dt$. Thus the dt-h unit hydrograph will have ordinates equal to $\left(\frac{S_2 - S_1}{i\,dt}\right)$. As dt is made smaller and smaller, i.e. as $dt \rightarrow 0$, an IUH results. Thus for an IUH, the ordinate at any time t is

$$u(t) = \lim_{dt \rightarrow 0} \left(\frac{S_2 - S_1}{i\,dt}\right) = \frac{1}{i}\frac{ds}{dt} \qquad (6.23)$$

If: $i = 1$, then $u(t) = dS'/dt$, where S' represents a S-curve of intensity 1 cm/h. Thus the ordinate of an IUH at any time t is the slope of the S-curve of intensity 1 cm/h (i.e. S-curve derived from a unit hydrograph of 1-h duration) at the corresponding time. Equation (6.23) can be used in deriving IUH approximately.

IUHs can be derived in many other ways, notably by (i) harmonic analysis (ii) Laplace transform and (iii) conceptual models. Details of these methods are beyond the scope of this book and can be obtained from Ref. 2.

Derivation of D-h Unit Hydrograph from IUH

For simple geometric forms of IUH, Eq. (6.22) can be used to derive a *D*-h unit hydrograph. For complex-shaped IUHs the arithmetic computation techniques used in deriving unit hydrographs of different durations (Sec. 6.7) can be adopted. For example, if two IUHs are lagged by D h and their corresponding ordinates are summed up and divided by two, the resulting hydrograph will be a *D* h unit hydrograph. A *D*-h unit hydrograph will have a peak lesser than that of the corresponding IUH and the time base of the *D*-h unit hydrograph will be larger than that of IUH by *D* h.

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PROBLEMS

6.1 The flood hydrograph of a small stream is given below. Analyse the recession limb of the hydrograph and determine the recession coefficients. Neglect inter-flow.

Time (days)	Discharge (m^3/s)	Time (days)	Discharge (m^3/s)
0	0.0	3.5	2.5
0.5	70.0	4.0	1.9
1.0	38.0	5.0	1.4
1.5	19.0	6.0	1.2
2.0	9.0	7.0	1.1
2.5	5.5		
3.0	3.5		

6.2 Given below are observed flows from a storm of 6-h duration on a stream with a catchment area of 500 km^2.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Observed flow (m^3/s)	0	100	250	200	150	100	70	50	35	25	15	5	0

Assuming the base flow to be zero, derive the ordinates of a 6-h unit hydrograph.

6.3 The following are the ordinates of the hydrograph of flow from a catchment area of 770 km^2 due to a 6-h rainfall. Derive the ordinates of a 6-h unit hydrograph. Make suitable assumptions regarding the base flow.

Time from beginning of storm (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m^3/s)	40	65	215	360	400	350	270	205	145	100	70	50	42

6.4 Analysis of the surface runoff records of a 1-day storm over a catchment yielded the following data:

Time (days)	0	1	2	3	4	5	6	7	8	9
Discharge (m^3/s)	20	63	151	133	90	63	44	29	20	20
Estimated base flow (m^3/s)	20	22	25	28	28	26	23	21	20	20

Determine the 24-h distribution graph percentages. If the catchment area is 600 km^2 determine the depth of rainfall exess.

6.5 The ordinates of a hydrograph of surface runoff resulting from 4.5 cm of rainfall excess of duration 8 h in a catchment are as follows:

Time (h)	0	5	13	21	28	32	35	41	45	55
Discharge (m^3/s)	0	40	210	400	600	820	1150	1440	1510	1420
Time (h)	61	91	98	115	138					
Discharge (m^3/s)	1190	650	520	290	0					

Determine the ordinates of an 8-h unit hydrograph for this catchment.

6.6 The peak of a flood hydrograph due to a 6-h storm is 470 m^3/s. The mean depth of rainfall is 8.0 cm. Assume an average infiltration loss of 0.25 cm/h and a constant base flow of 15 m^3/s and estimate the peak discharge of a 6-h unit hydrograph for this catchment.

6.7 Given the following data about a catchment of area 1000 km^2, determine the peak discharge corresponding to a storm of 5 cm in 1 h.

Time (h)	0	1	2	3	4	5
Rainfall (cm)	0	2.5	0	0	0	0
Runoff (m^3/s)	300	300	1200	450	300	300

6.8 The ordinates of a 6-h unit hydrograph are given.

Time (h)	0	3	6	9	12	18	24	30	36	42	48	54	60	66
6-h UH ordinate (m^3/s)	0	150	250	450	600	800	700	600	450	320	200	100	50	0

A storm had three successive 6-h intervals of rainfall magnitude of 3.0, 5.0 and 4.0 cm respectively. Assuming a ϕ index of 0.20 cm/h and a base flow of 30 m^3/s, determine and plot the resulting hydrograph of flow.

6.9 The ordinates of a 6-h unit hydrograph are as given below:

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Ordinate of 6-h UH (m^3/s)	0	20	60	150	120	90	66	50	32	20	10	0

If two storms, each of 1-cm rainfall excess and 6-h duration occur in succession, calculate the resulting hydrograph of flow. Assume base flow to be uniform at 10 m^3/s.

6.10 Using the 6-h unit hydrograph of Prob. 6.9, derive a 12-h unit hydrograph for the catchment.

6.11 The ordinates of a 2-hr unit hydrograph are given:

Time (h)	0	2	4	6	8	10	12	14	16	18	20	22
2-h UH ordinate (m^3/s)	0	25	100	160	190	170	110	70	30	20	6	0

Determine the ordinates of an S-curve hydrograph and using this determine the ordinates of a 4-h unit hydrograph.

6.12 Using the ordinates of a 12-h unit hydrograph given below, compute the ordinates of a 6-h unit hydrograph.

Time (h)	Ordinates of 12-h unit hydrograph (m^3/s)	Time (h)	Ordinates of 12-h unit hydrograph (m^3/s)
0	0	78	71
6	10	84	58
12	37	90	46
18	76	96	35
24	111	102	25
30	136	108	17
36	150	114	12
42	153	120	8
48	146	126	6
54	130	132	3
60	114	138	2
66	99	144	0
72	84		

Note that the tail portion of the resulting 6-h UH needs fairing.

6.13 A 3-h unit hydrograph for a basin has the following ordinates. Using the S-curve method, determine the 9-h unit-hydrograph ordinates.

Time (h)	0	3	6	9	12	15	18	21	24	27	30
Discharge (m^3/s)	0	12	75	132	180	210	183	156	135	144	96
Time (h)	33	36	39	42	45	48	51	54	57	60	
Discharge (m^3/s)	87	66	54	42	33	24	18	12	6	0	

6.14 Using the 6-h unit hydrograph given in Prob. 6.9 derive the flood hydrograph due to a storm given below.

Time from beginning of storm (h)	0	6	12	18
Accumulated rainfall (cm)	0	4	5	10

The ϕ index for the storm can be assumed to be 0.167 cm/h. Assume base flow to be 20 m^3/s constant throughout.

6.15 The 6-h unit hydrograph of a catchment is in the form of a triangle with the peak of 100 m^3/s occurring at 24 h from the start. The base is 72 h.

(a) What is the area of the catchment represented by this unit hydrograph?

(b) Calculate the flood hydrograph due to a storm of rainfall excess of 2 cm during the first 6 h and 4 cm during the second 6 h interval. The base flow can be assumed to be 25 m^3/s constant throughout.

6.16 A 6-h unit hydrograph for a catchment of area 1000 km^2 can be approximated as a triangle with base of 69 h. Calculate the peak discharge of this unit hydrograph.

6.17 A 4-h distribution graph has the following ordinates:

Unit periods (4-h units)	1	2	3	4	5	6
Distribution (percentage)	5	20	40	20	10	5

A catchment with an area of 50 km^2 has rainfalls of 3.5, 2.2 and 1.8 cm in three consecutive 4-h periods. Assuming an average ϕ index of 0.25 cm/h, determine the resulting direct runoff hydrograph.

6.18 A 6-h unit hydrograph of a catchment of area 259.2 km^2 is triangular in shape with a base of 48 h. The peak occurs at 12 h from the start. Derive the coordinates of a 6-h distribution graph for this catchment.

6.19 The following table gives the ordinates of a direct-runoff hydrograph resulting from two successive 3-h durations of rainfall-excess values of 2 and 4 cm respectively. Derive a 3-h unit hydrograph for the catchment.

Time (h)	0	3	6	9	12	15	18	21	24	27	30
Direct runoff (m^3/s)	0	120	480	660	460	260	160	100	50	20	0

6.20 Characteristics of two catchments M and N measured from a map are given below:

Item	Catchment M	Catchment N
L_{ca}	76 km	52 km
L	148 km	106 km
A	2718 km²	1400 km²

For a 6-h unit hydrograph in catchment M, the peak discharge is at 37 h from the start of the rainfall excess and its value is 200 m³/s. Assuming catchment M and N are meteorologically similar, determine the elements of a 6-h synthetic unit hydrograph for catchment N by using Snyder's method.

6.21 A basin has 400 sq. km of area, L = 35 km and L_{ca} = 10 km. Assuming C_t = 1.5 and C_p = 0.70 develop a 3-h synthetic-unit hydrograph for this basin using Snyder's method.

6.22 Using the peak discharge and time to peak of the unit hydrograph derived in Prob. 6.20, develop the full unit hydrograph by using the SCS dimensionless-unit hydrograph.

6.23 The IUH of a catchment is a triangle with a base of 36 h and a peak of 20 m³/s at 8 h from the start. Derive a 2-h unit hydrograph for this catchment.

QUESTIONS

6.1 The recession limb of a flood hydrograph can be expressed with positive values of coefficients,

as Q_t/Q_0 = (a) Ke^{at} (b) $a\ K_r^{-at}$ (c) a^{-at} (d) e^{-at^2}

6.2 For a given storm, other factors remaining same,
- (a) basins having low drainage density give smaller peaks in flood hydrographs
- (b) basins with larger drainage densities give smaller flood peaks
- (c) low drainage density basins give shorter time bases of hydrographs
- (d) the flood peak is independent of the drainage density.

6.3 Base-flow separation is performed
- (a) on a unit hydrograph to get the direct-runoff hydrograph
- (b) on a flood hydrograph to obtain the magnitude of effective rainfall
- (c) on flood hydrographs to obtain the rainfall hydrograph
- (d) on hydrographs of effluent streams only.

6.4 A direct-runoff hydrograph due to a storm was found to be triangular in shape with a peak of 150 m³/s, time from start of effective storm to peak of 24 h and a total time base of 72 h. The duration of the storm in this case was
- (a) < 24 h
- (b) between 24 to 72 h
- (c) 72 h
- (d) > 72 h.

6.5 A unit hydrograph has
- (a) one unit of peak discharge
- (b) one unit of rainfall duration
- (c) one unit of direct runoff
- (d) one unit of the time base of direct runoff.

6.6 The basic assumptions of the unit-hydrograph theory are
- (a) nonlinear response and time invariance
- (b) time invariance and linear response
- (c) linear response and linear time variance
- (d) nonlinear time variance and linear response.

6.7 A storm hydrograph was due to 3 h of effective rainfall. It contained 6 cm of direct runoff. The ordinates of DRH of this storm
- (a) when divided by 3 give the ordinates of a 6-h unit hydrograph
- (b) when divided by 6 give the ordinates of a 3-h unit hydrograph
- (c) when divided by 3 give the ordinates of a 3-h unit hydrograph
- (d) when divided by 6 give the ordinates of a 6-h unit hydrograph.

6.8 A triangular DRH due to a storm has a time base of 80 hrs and a peak flow of 50 m³/s occurring at 20 hrs from the start. If the catchment area is 144 km², the rainfall excess in the storm was
- (a) 20 cm
- (b) 7.2 cm
- (c) 5 cm
- (d) none of these.

6.9 A 12-hr unit hydrograph of a catchment is triangular in shape with a base width of 144 hrs and a peak discharge value of 23 m³/s. This unit hydrograph refers to a catchment of area
- (a) 756 km²
- (b) 596 km²
- (c) 1000 km²
- (d) none of these.

6.10 A 6-h unit hydrograph of a catchment is triangular in shape with a base width of 64 h and peak ordinate of 20 m³/s. If a 0.5 cm rainfall excess occurs in 6 h in that catchment, the resulting surface-runoff hydrograph will have
- (a) a base of 128 h
- (b) a base of 32 h
- (c) a peak of 40 m³/s
- (d) a peak of 10 m³/s.

6.11 A 90 km² catchment has a 4-h unit hydrograph which can be approximated as a triangle. If the peak ordinate of this unit hydrograph is 10 m³/s, the time base is
- (a) 120 h
- (b) 64 h
- (c) 50 h
- (d) none of these.

6.12 A triangular DRH due to a 6-h storm in a catchment has a time base of 100 h and a peak flow of 40 m³/s. The catchment area is 180 km². The 6-h unit hydrograph of this catchment will have a peak flow in m³/s of
- (a) 10
- (b) 20
- (c) 30
- (d) none of these.

6.13 The curve u_m is a 6-h unit hydrograph for a catchment having 1 mm of direct runoff and u_c is a 6-h unit hydrograph for the same catchment and having 1 cm of surface runoff.
- (a) ordinates of u_m are 1/10 the corresponding ordinates of u_c
- (b) base of u_m is 1/10 the base of u_c
- (c) ordinates of u_m are 10 times the corresponding ordinates of u_c
- (d) base of u_m is 10 times that of u_c.

6.14 If a 4-h unit hydrograph of a catchment has a peak ordinate of 60 m³/s, the peak ordinate of an 8-h unit hydrograph for the same catchment will be
- (a) > 60 m³/s
- (b) = 60 m³/s
- (c) < 60 m³/s
- (d) data inadequate.

6.15 A catchment with an area of 756 km² has a 6-h unit hydrograph which is a triangle with a base of 70 h. The peak discharge of DRH due to 5 cm of rainfall excess in 6 h from that catchment is in m³/s
- (a) 535
- (b) 60
- (c) 756
- (d) 300.

6.16 For a catchment with an area of 360 km² the equilibrium discharge of an S-curve obtained by 4-hr unit hydrograph summation is in m³/s.
- (a) 250
- (b) 90
- (c) 90 × 10⁴
- (d) none of these.

6.17 In an S-curve derived by a unit hydrograph U of D-h duration
- (a) the equilibrium discharge is independent of D

(b) the time base of U represents the time at which the S-curve attains its maximum value
(c) the equilibrium discharge of an S-curve is independent of the catchment area of the basin
(d) the ordinate is in units of volume.

6.18 An instantaneous unit hydrograph is a direct runoff hydrograph
(a) of 1 cm magnitude due to a rainfall excess of 1-h duration
(b) that occurs instantaneously due to a unit rainfall excess of duration D h
(c) of unit rainfall excess precipitating instantaneously over the catchment
(d) occurring at any instant in a long storm.

6.19 Instantaneous unit hydrograph is a hydrograph of
(a) unit duration and infinitely small rainfall excess
(b) infinitely small duration and of unit rainfall excess
(c) zero effective precipitation
(d) zero frequency.

7 FLOODS

7.1 INTRODUCTION

A flood is an unusually high stage in a river—normally the level at which the river overflows its banks and inundates the adjoining area. The damages caused by floods in terms of loss of life, property and economic loss due to disruption of economic activity are all too well-known. Crores of rupees are spent every year in flood control and flood forecasting. The hydrograph of extreme floods and stages corresponding to flood peaks provide valuable data for purposes of hydrologic design. Further, of the various characteristics of the flood hydrograph, probably the most important and widely used parameter is the flood peak. At a given location in a stream, flood peaks vary from year to year and their magnitude constitutes a hydrologic series which enable one to assign a frequency to a given flood-peak value. In the design of practically all hydraulic structures the peak flow that can be expected with an assigned frequency (say 1 in 100 years) is of primary importance to adequately proportion the structure to accommodate its effect. The design of bridges, culvert waterways and spillways for dams and estimation of scour at a hydraulic structure are some examples wherein flood-peak values are required.

To estimate the magnitude of a flood peak the following alternative methods are available:

1. Rational method,
2. empirical method,
3. unit-hydrograph technique, and
4. flood-frequency studies.

The use of a particular method depends upon (i) the desired objective, (ii) the available data and (iii) the importance of the project. Further the *rational formula* is only applicable to small-size (<50 km^2) catchments and the unit-hydrograph method is normally restricted to moderate-size catchments with areas less than 5000 km^2.

7.2 RATIONAL METHOD

Consider a rainfall of uniform intensity and very long duration occurring over a basin. The runoff rate gradually increases from zero to a constant value as indicated in Fig. 7.1. The runoff increases as more and more flow from remote areas of the catchment reach the outlet. Designating the time taken for a drop of water from the farthest part of the catchment to

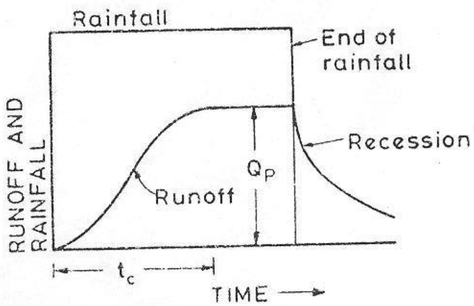

Fig. 7.1 Runoff hydrograph due to uniform rainfall

reach the outlet as t_c = time of concentration, it is obvious that if the rainfall continues beyond t_c, the runoff will be constant and at the peak value. The peak value of the runoff is given by

$$Q_p = C\,A\,i;\ \text{for } t \geqslant t_c \tag{7.1}$$

where C = coefficient of runoff = (runoff/rainfall), A = area of the catchment and i = intensity of rainfall. This is the basic equation of the *rational method*. Using the commonly used units, Eq. (7.1) is written for field application as

$$Q_p = \frac{1}{3.6}\,C\,(i_{t_c,p})\,A \tag{7.2}$$

where Q_p = peak discharge (m^3/s)

C = coefficient of runoff

$i_{t_c,p}$ = the mean intensity of precipitation (mm/h) for a duration equal to t_c and an exceedence probability P

A = drainage area in km^2

The use of this method to compute Q_p requires three parameters: t_c, ($i_{t_c\,p}$) and C.

Time of Concentration (t_c)

There are a number of empirical equations available for the estimation of the time of concentration. Two of these are described below.

US Practice

For small drainage basins, the time of concentration is approximately equal to the lag time of the peak flow. Thus

$$t_c = t_p \text{ of Eq. (6.9)} = C_{tL}\left(\frac{LL_{ca}}{\sqrt{S}}\right)^n \tag{7.3}$$

where t_c = time of concentration in hours, C_{tL}, L, L_{ca}, n and S have the the same meaning as in Eq. (6.9).

Kirpich Equation 1940

$$t_c = 0.01947\,L^{0.77}\,S^{-0.385} \tag{7.4}$$

where t_c = time of concentration (minutes)

L = maximum length of travel of water (m) and

S = slope of the catchment = $\Delta H/L$ in which

ΔH = difference in elevation between the most remote point on the catchment and the outlet.

Rainfall Intensity ($i_{t_c,p}$)

The rainfall intensity corresponding to a duration t_c and the desired probability of exceedence P (i.e. return period $T = 1/P$) is found from the rainfall–frequency duration relationship for the given catchment area (Chap. 2). This will usually be a relationship of the form

$$i_{t_c\,p} = \frac{K\,T^x}{(t_c + a)^m} \tag{7.5}$$

in which K, a, x and m are constants. Published rainfall maps exhibiting this form of relationship are consulted to evaluate $i_{t_c\,p}$. In USA the peak discharges for purposes of urban area drainage are calculated by using P = 0.05 to 0.1.

Runoff Coefficient (C)

The coefficient C represents the integrated effect of the catchment losses and hence depends upon the nature of the surface, surface slope and rainfall intensity. The effect of rainfall intensity is not considered in the available tables of values of C. Some typical values of C are indicated in Table 7.1.

The rational formula is found to be suitable for peak-flow prediction in small catchments up to 50 km^2 in area. It finds considerable application in urban drainage designs and in the design of small culverts and bridges.

TABLE 7.1 VALUE OF THE COEFFICIENT C IN EQ. (7.2)

Type of area	Value of C
A. *Urban area* (P = 0.05 to 0.10)	
Lawns: Sandy-soil, flat, 2%	0.05–0.10
Sandy soil, steep, 7%	0.15–0.20
Heavy soil, average, 2-7%	0.18–0.22
Residential areas:	
Single family areas	0.30–0.50
Multi units, attached	0.60–0.75
Industrial: Light	0.50–0.80
Heavy	0.60–0.90
Streets	0.70–0.95
B. *Agricultural Area*	
Flat: Tight clay, cultivated	0.50
woodland	0.40
Sandy loam, cultivated	0.20
woodland	0.10
Hilly: Tight clay, cultivated	0.70
woodland	0.60
Sandy loam, cultivated	0.40
woodland	0.30

It should be noted that the word "rational" is rather a misnomer as the method involves the determination of parameters t_c and C in a subjective manner. Detailed description and the practice followed in using the rational method in various countries are given in detail in Ref. 6.

EXAMPLE 7.1 *An urban area has a runoff coefficient of 0.30 and an area of 0.85 km². The slope of the catchment is 0.006 and the maximum length of travel of water is 950 m. The maximum depth of rainfall with a 25-year return period is as below:*

Duration (min)	5	10	20	30	40	60
Depth of rainfall (mm)	17	26	40	50	57	62

If a culvert for drainage at the outlet of this area is to be designed for a return period of 25 years, estimate the required peak-flow rate.

The time of concentration is obtained by the Kirpich formula [Eq. (7.4)] as

$$t_c = 0.01947 \times (950)^{0.77} \times (0.006)^{-0.385}$$
$$= 27.4 \text{ min}$$

By interpolation,

Maximum depth of rainfall for 27.4-min duration

$$= \frac{(50-40)}{10} \times 7.4 + 40 = 47.4 \text{ mm}$$

$$\text{Average intensity} = i_{t_c,p} = \frac{47.4}{27.4} \times 60 = 103.8 \text{ mm/h}$$

By Eq. (7.2), $$Q_p = \frac{0.30 \times 103.8 \times 0.85}{3.6} = 7.35 \text{ m}^3/\text{s}$$

7.3 EMPIRICAL FORMULAE

The empirical formulae used for the estimation of the flood peak are essentially regional formulae based on statistical correlation of the observed peak and important catchment properties. To simplify the form of the equation, only a few of the many parameters affecting the flood peak are used. For example, almost all formulae use the catchment area as a parameter affecting the flood peak and most of them neglect the flood frequency as a parameter. In view of these, the empirical formula are applicable only in the region from which they were developed and when applied to other areas they can at best give approximate values.

Flood-Peak-Area Relationships

By far the simplest of the empirical relationships are those which relate the flood peak to the drainage area. The maximum flood discharge Q_p from a catchment area A is given by these formulae as

$$Q_p = f(A)$$

While there are a vast number of formulae of this kind proposed for various parts of the world, only a few popular formulae used in various parts of India are given below,

Dickens Formula (1865)

$$Q_p = C_D A^{3/4} \quad (7.6)$$

where Q_p = maximum flood discharge (m³/s)

A = catchment area (km²)

C_D = Dickens constant with value between 6 to 30

The following are some guidelines in selecting the value of C_D:

	Value of C_D
North-Indian plains	6
North-Indian hilly regions	11–14
Central India	14–28
Coastal Andhra and Orissa	22–28

For actual use the local experience will be of aid in the proper selection of C_D. Dickens formula is used in the central and northern parts of the country.

Ryves Formula (1884)

$$Q_p = C_R A^{2/3} \qquad (7.7)$$

where Q_p = maximum flood discharge (m^3/s)

A = catchment area (km^2)

and C_R = Ryves coefficient

This formula originally developed for the Tamil Nadu region, is in use in Tamil Nadu and parts of Karnataka and Andhra Pradesh. The values of C_R recommended by Ryves for use are:

$C_R = 6.8$ for areas within 80 km from the east coast

$= 8.5$ for areas which are 80-160 km from the east coast

$= 10.2$ for limited areas near hills

However, various major reservoir projects built in Tamil Nadu since 1950 have adopted considerably much higher values of C_R than the above.

Inglis Formula (1930)

This formula is based on flood data of catchments in Western Ghats in Maharashtra. The flood peak Q_p in m^3/s is expressed as

$$Q_p = \frac{124\,A}{\sqrt{A + 10.4}} \qquad (7.8)$$

where A is the catchment area in km^2.

Equation (7.8) with small modifications in the constant in the numerator (124) is in use Maharashtra for designs in small catchments.

Other Formulae

There are many such empirical formulae developed in various parts of the world. References 3 and 4 list many such formulae suggested for use in various parts of India as well as of the world.

There are some empirical formulae which relate the peak discharge to the basin area and also include the flood frequency. Fuller's formula (1914) derived for catchments in USA is a typical one of this kind and is given by

$$Q_{Tp} = C_f A^{0.8} (1 + 0.8 \log T) \qquad (7.9)$$

where Q_{Tp} = maximum 24-h flood with a frequency of T years in m^3/s, A = catchment area in km^2, C_f = a constant with values between 0.18 to 1.88.

Envelope Curves

In regions having same climatological characteristics, if the available flood data are meagre, the enveloping curve technique can be used to develop a relationship between the maximum flood flow and drainage area. In this method the available flood-peak data from a large number of catchments which do not significantly differ from each other in terms of meteorological and topographical characteristics are collected. The data are then plotted on a log-log paper as flood peak vs catchment area. This would result in a plot in which the data would be scattered. If an enveloping curve that would encompass all the plotted data point is drawn, it can be used to obtain maximum peak discharges for any given area. Envelop curves thus obtained are very useful in getting quick rough estimations of peak values. If equations are fitted to these enveloping curves, they provide empirical flood formulae of the type, $Q = f(A)$.

Kanwarsain and Karpov (1967) have presented enveloping curves representing the relationship between the peak-flood flow and catchment area for Indian conditions. Two curves, one for the south Indian rivers and the other for north Indian and central Indian rivers, are developed (Fig. 7.2). These two curves are based on data covering large catchment areas, in the range 10^3 to 10^6 km^2.

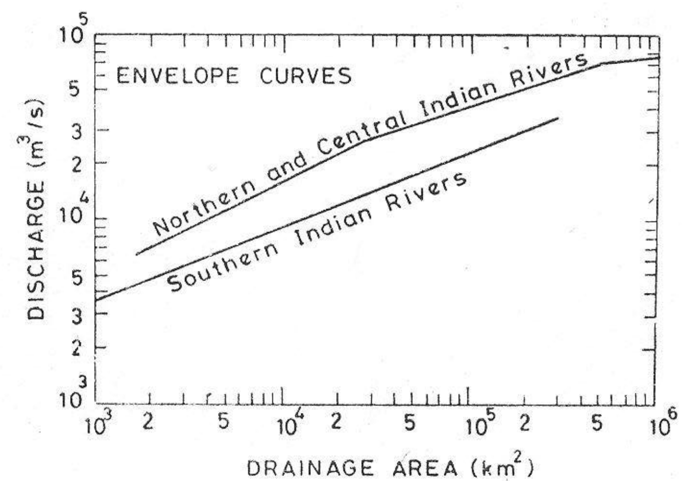

Fig. 7.2 Enveloping curves for Indian rivers

Based on the maximum recorded floods throughout the world, Baird and McIllwraith (1951) have correlated the maximum flood discharge Q_{mp} in m^3/s with catchment area A in km^2 as

$$Q_{mp} = \frac{3025\,A}{(278 + A)^{0.78}} \qquad (7.10)$$

EXAMPLE 7.2 *Estimate the maximum flood flow for the following catchments by using an appropriate empirical formula:*

1. $A_1 = 40.5$ *sq. km. for western Ghat area, Maharashtra*
2. $A_2 = 40.5$ km^2 *in Gangetic plain*
3. $A_3 = 40.5$ km^2 *in the Cauvery delta, Tlmil Nadu*
4. *What is the peak discharge for* $A = 40.5$ km^2 *by maximum world-flood experience.*

1. For this catchment, the Inglis formula is recommended.
By the Inglis formula [Eq. (7.8)],

$$Q_p = \frac{124 \times 40.5}{\sqrt{40.5+10.4}} = 704 \text{ m}^3/\text{s}$$

2. In this case Dickens formula [Eq. (7.6)] with $C_D = 6.0$ is recommended. Hence

$$Q_p = 6.0 \times (40.5)^{0.75} = 96.3 \text{ m}^3/\text{s}$$

3. In this case Ryves formula [Eq. (7.7) with $C_R = 6.8$ is preferred, and this gives

$$Q_p = 6.8\ (40.5)^{2/3} = 80.2 \text{ m}^3/\text{s}$$

4. By Eq. (7.10) for maximum peak discharge based on world experience,

$$Q_{mp} = \frac{3025 \times 40.5}{(278+40.5)^{0.78}} = 1367 \text{ m}^3/\text{s}.$$

7.4 UNIT-HYDROGRAPH

The unit-hydrograph technique described in the previous chapter can be used to predict the peak-flood hydrograph if the rainfall producing the flood, infiltration characteristics of the catchment and the appropriate unit hydrograph are available. For design purposes, extreme rainfall situations are used to obtain the design storm, viz., the hyetograph of the rainfall excess causing extreme floods. The known or derived unit hydrograph of the catchment is then operated upon by the design storm to generate the desired flood hydrograph. Details about this use of unit hydrograph are given in Sec. 7.9.

7.5 FLOOD-FREQUENCY STUDIES

Hydrologic processes such as floods are exceedingly complex natural events. They are resultants of a number of component parameters and are therefore very difficult to model analytically. For example, the floods in a catchment depend upon the characteristics of the catchment, rainfall and antecedent conditions, each one of these factors in turn depend upon a host of constituent parameters. This makes the estimation of the flood peak a very complex problem leading to many different approaches. The empirical formulae and unit-hydrograph methods presented in the previous sections are some of them. Another approach to the prediction of flood flows, and also applicable to other hydrologic processes such as rainfall etc. is the statistical method of frequency analysis.

The values of the annual maximum flood from a given catchment area for large number of successive years constitute a hydrologic data series called the *annual series*. The data are then arranged in decreasing order of magnitude and the probability P of each event being equalled to or exceeded (plotting position) is calculated by the plotting-position formula

$$P = \frac{m}{N+1} \tag{7.11}$$

where m = order number of the event and N = total number of events in the data. The recurrence interval, T (also called the return period or frequency) is calculated as

$$T = 1/P \tag{7.12}$$

The relationship between T and the probability of occurrence of various events is the same as described in Sec. 2.11. Thus, for example, the probability of occurrence of the event r times in n successive years is given by

$$P_{rn} = {}^nC_r\ P^r\ q^{n-r}$$

where $\quad q = 1 - P$

Consider, for example, a list of flood magnitudes of a river arranged in descending order as shown in Table 7.2. The length of the record is 50 years.

TABLE 7.2 CALCULATION OF FREQUENCY T

Order No. m	Flood magnitude Q (m³/s)	T in years $= 51/m$
1	160	51.00
2	135	25.50
3	128	17.00
4	116	12.75
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
49	65	1.04
50	63	1.02

The last column shows the return period T of various flood magnitude, Q. A plot of Q vs T yields the probability distribution. For small return periods (i.e. for interpolation) or where limited extrapolation is required, a simple best-fitting curve through plotted points can be used as the probability distribution. A logarithmic scale for T is often advantageous. However, when larger extrapolations of T are involved, theoretical probability distributions have to be used. In frequency analysis of floods the usual problem is to predict extreme flood events. Towards this, specific extreme-value distributions are assumed and the required statistical parameters calculated from the available data. Using these the flood magnitude for a specific return period is estimated.

Chow (1951) has shown that most frequency-distribution functions applicable in hydrologic studies can be expressed by the following equation known as the *general equation of hydrologic frequency analysis*:

$$x_T = \bar{x} + K\sigma \qquad (7.13)$$

where x_T=value of the variate X of a random hydrologic series with a return period T, $\bar{x}$=mean of the variate, σ=standard deviation of the variate, K=frequency factor which depends upon the return period, T and the assumed frequency distribution. Some of the commonly used frequency distribution functions for the predication of extreme flood values are:

1. Gumbel's extreme-value distribution,
2. log-Pearson Type III distribution, and
3. log normal distribution.

Only the first two distributions are dealt with in this book with emphasis on application. Further details and theoretical basis of these and other methods are available in Refs 1, 3, 6 and 7.

7.6 GUMBEL'S METHOD

This extreme value distribution was introduced by Gumbel (1941) and is commonly known as Gumbel's distribution. It is one of the most widely used probability-distribution functions for extreme values in hydrologic and meteorologic studies for prediction of flood peaks, maximum rainfalls, maximum wind speed, etc.

Gumbel defined a flood as the largest of the 365 daily flows and the annual series of flood flows constitute a series of largest values of flows. According to his theory of extreme events, the probability of occurrence of an event equal to or larger than a value x_0 is

$$P(X \geqslant x_0) = 1 - e^{-e^{-y}} \qquad (7.14)$$

in which y is a dimensionless variable given by

$$y = \alpha(x - a)$$

$$a = \bar{x} - 0.45005\,\sigma_x$$

$$\alpha = 1.2825/\sigma_x$$

Thus

$$y = \frac{1.2825(x - \bar{x})}{\sigma_x} + 0.577 \qquad (7.15)$$

where $\bar{x}$ = mean and σ_x = standard deviation of the variate X. In practice it is the value of X for a given P that is required and as such Eq. (7.14) is transposed as

$$y_P = -\ln[-\ln(1-P)] \qquad (7.16)$$

Noting that the return period $T = 1/P$ and designating

y_T = the value of y, commonly called the reduced variate, for a given T

$$y_T = -\left[\ln.\ln\frac{T}{T-1}\right] \qquad (7.17)$$

or

$$y_T = -\left[0.834 + 2.303\log\log\frac{T}{T-1}\right] \qquad (7.17\text{-a})$$

Now rearranging Eq. (7.15), the value of the variate X with a return period T is

$$x_T = \bar{x} + K\sigma_x \qquad (7.18)$$

where

$$K = \frac{(y_T - 0.577)}{1.2825} \qquad (7.19)$$

Note that Eq. (7.19) is of the same form as the general equation of hydrologic-frequency analysis Eq. (7.13). Further, Eqs. (7.18) and (7.19) constitute the basic Gumbel's equations and are applicable to an infinite sample size (i.e. $N \to \infty$).

Since practical annual data series of extreme events such as floods, maximum rainfall depths, etc., all have finite lengths of record, Eq. (7.19) is modified to account of finite N as given below for practical use.

Gumbel's Equation for Practical Use

Equation (7.18) giving the value of the variate X with a recurrence interval T is used as

$$x_T = \bar{x} + K\sigma_{n-1} \qquad (7.20)$$

where σ_{n-1} = standard deviation of the sample

$$= \sqrt{\frac{\Sigma(x-\bar{x})^2}{N-1}}$$

K = frequency factor expressed as

$$K = \frac{y_T - \bar{y}_n}{S_n} \qquad (7.21)$$

in which y_T = reduced variate, a function of T and is given by

$$y_T = -\left[\ln\cdot\ln\frac{T}{T-1}\right] \qquad (7.22)$$

or $$y_T = -\left[0.834 + 2.303 \log \log \frac{T}{T-1}\right]$$

$\bar{y}_n$ = reduced mean, a function of sample size N and is given in Table 7.3; for $N \to \infty$, $\bar{y}_n \to 0.577$

s_n = reduced standard deviation, a function of sample size N and is given in Table 7.4; for $N \to \infty$, $s_n \to 1.2825$

These equations are used under the following procedure to estimate the flood magnitude corresponding to a given return based on an annual flood series.

1. Assemble the discharge data and note the sample size N. Here the annual flood value is the variate X. Find $\bar{x}$ and σ_{n-1} for the given data.
2. Using Tables 7.3 and 7.4 determine $\bar{y}_n$ and s_n appropriate to given N.
3. Find y_T for a given T by Eq. (7.22).
4. Find K by Eq. (7.21).
5. Determine the required x_T by Eq. (7.20).

The method is illustrated in Example 7.3.

To verify whether the given data follow the assumed Gumbel's distribution, the following procedure may be adopted. The values of x_T for some return periods $T < N$ are calculated by using Gumbel's formula and plotted as x_T vs T on a convenient paper such as a semi-log, log–log or Gumbel probability paper. The use of Gumbel probability paper results in a straight line for x_T vs T plot. Gumbel's distribution has the property which gives $T = 2.33$ years for the average of the annual series when N is very large. Thus the value of a flood with $T = 2.33$ years is called the *mean annual flood*. In graphical plots this gives a mandatory point through which the line showing variation of x_T with T must pass. For the given data, values of return periods (plotting positions) for various recorded values, x of the variate are obtained by the relation $T = (N+1)/m$ and plotted on the graph described above. A good fit of observed data with the theoretical variation line indicates the applicability of Gumbel's distribution to the given data series (Fig. 7.3). By extrapolation of the straight line x_T vs T, values of x_T for $T > N$ can be determined easily (Example 7.3).

Gumbel Probability Paper

The Gumbel probability paper is an aid for convenient graphical representation of Gumbel's distribution. It consists of an abscissa specially marked for various convenient values of the return period T. To construct the T-scale on the abscissa, first construct an arithmetic scale of y_T values, say from −2 to +3, as in Fig. 7.3. For selected values of T, say 2, 10, 50, 100, 500 and 1000, find the values of y_T by Eq. (7.22) and mark off those positions on the abscissa. The T-scale is now ready for use (Fig. 7.3).

TABLE 7.3 REDUCED MEAN $\bar{y}_n$ IN GUMBEL'S EXTREME VALUE DISTRIBUTION

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.4952	0.4996	0.5035	0.5070	0.5100	0.5128	0.5157	0.5181	0.5202	0.5220
20	0.5236	0.5252	0.5268	0.5283	0.5296	0.5309	0.5320	0.5332	0.5343	0.5353
30	0.5362	0.5371	0.5380	0.5388	0.5396	0.5402	0.5410	0.5418	0.5424	0.5430
40	0.5436	0.5442	0.5448	0.5453	0.5458	0.5463	0.5468	0.5473	0.5477	0.5481
50	0.5485	0.5489	0.5493	0.5497	0.5501	0.5504	0.5508	0.5511	0.5515	0.5518
60	0.5521	0.5524	0.5527	0.5530	0.5533	0.5535	0.5538	0.5540	0.5543	0.5545
70	0.5548	0.5550	0.5552	0.5555	0.5557	0.5559	0.5561	0.5563	0.5565	0.5567
80	0.5569	0.5570	0.5572	0.5574	0.5576	0.5578	0.5580	0.5581	0.5583	0.5585
90	0.5586	0.5587	0.5589	0.5591	0.5592	0.5593	0.5595	0.5596	0.5598	0.5599
100	0.5600									

TABLE 7.4 REDUCED STANDARD DEVIATION s_n IN GUMBEL'S EXTREME VALUE DISTRIBUTION

N = sample size

N	0	1	2	3	4	5	6	7	8	9
10	0.9496	0.9676	0.9833	0.9971	1.0095	1.0206	1.0316	1.0411	1.0493	1.0565
20	1.0628	1.0696	1.0754	1.0811	1.0864	1.0915	1.0961	1.1004	1.1047	1.1086
30	1.1124	1.1159	1.1193	1.1226	1.1255	1.1285	1.1313	1.1339	1.1363	1.1388
40	1.1413	1.1436	1.1458	1.1480	1.1499	1.1519	1.1538	1.1557	1.1574	1.1590
50	1.1607	1.1623	1.1638	1.1658	1.1667	1.1681	1.1696	1.1708	1.1721	1.1734
60	1.1747	1.1759	1.1770	1.1782	1.1793	1.1803	1.1814	1.1824	1.1834	1.1844
70	1.1854	1.1863	1.1873	1.1881	1.1890	1.1898	1.1906	1.1915	1.1923	1.1930
80	1.1938	1.1945	1.1953	1.1959	1.1967	1.1973	1.1980	1.1987	1.1994	1.2001
90	1.2007	1.2013	1.2020	1.2026	1.2032	1.2038	1.2044	1.2049	1.2055	1.2060
100	1.2065									

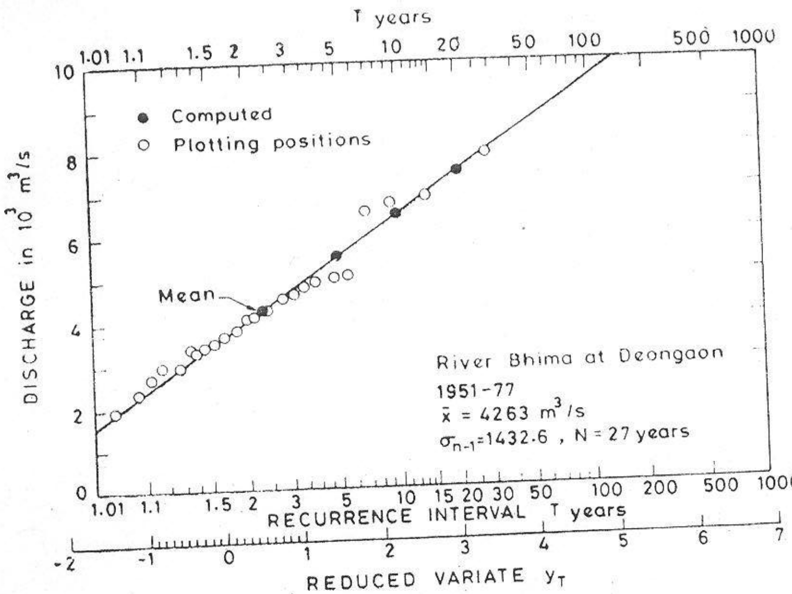

Fig. 7.3 Flood probability analysis by Gumbel's distribution

The ordinate of a Gumbel paper on which the value of the variate, x_T (flood discharge, maximum rainfall depth, etc.) are plotted may have either an arithmetic scale or logarithmic scale. Since by Eqs (7.18) and (7.19) x_T varies linearly with y_T, a Gumbel distribution will plot as a straight line on a Gumbel probability paper. This property can be used advantageously for graphical extrapolation, wherever necessary.

EXAMPLE 7.3 *Maximum recorded floods in the river Bhima at Deorgaon, a tributary of the river Krishna, for the period 1951 to 1977 is given below. Verify whether the Gumbel extreme-value distribution fit the recorded values. Estimate the flood discharge with recurrence interval of (i) 100 years and (ii) 150 years by graphical extrapolation.*

Year	1951	1952	1953	1954	1955	1956	1957	1958	1959
Max. flood (m³/s)	2947	3521	2399	4124	3496	2947	5060	4903	3757

Year	1960	1961	1962	1963	1964	1965	1966	1967	1968
Max. flood (m³/s)	4798	4290	4652	5050	6900	4366	3380	7826	3320

Year	1969	1970	1971	1972	1973	1974	1975	1976	1977
Max. flood (m³/s)	6599	3700	4175	2988	2709	3873	4593	6761	1971

The flood discharge values are arranged in descending order and the plotting position recurrence interval T_P for each discharge is obtained as

$$T_P = \frac{N+1}{m} = \frac{28}{m}$$

where m = order number. The discharge magnitude Q are plotted against the corresponding T_P on a Gumbel extreme probability paper (Fig. 7.3).

The statistics $\bar{x}$ and σ_{n-1} for the series are next calculated and are shown in Table 7.5. Using these the discharge x_T for some chosen recurrence interval is calculated by using Gumbel's formulae [Eqs. (7.22), (7.21) and (7.20)].

TABLE 7.5 CALCULATION OF T_P FOR OBSERVED DATA—*Example 7.3*

Order number	Flood discharge x (m³/s)	T_p (years)	Order number	Flood discharge x (m³/s)	T_p (years)
1	7826	28.00	15	3873	1.87
2	6900	14.00	16	3757	1.75
3	6761	9.33	17	3700	1.65
4	6599	7.00	18	3521	1.56
5	5060	5.60	19	3496	1.47
6	5050	4.67	20	3380	1.40
7	4903	4.00	21	3320	1.33
8	4798	3.50	22	2988	1.27
9	4652	3.11	23	2947	—
10	4593	2.80	24	2947	1.17
11	4366	2.55	25	2709	1.12
12	4290	2.33	26	2399	1.08
13	4175	2.15	27	1971	1.04
14	4124	2.00			

N = 27 years, $\bar{x}$ = 4263 m³/s, σ_{n-1} = 1432.6 m³/s

From Tables 7.3 and 7.4, for N = 27, $\bar{y}_n$ = 0.5332 and S_n = 1.1004. Choosing T = 10 years, by Eq. (7.22),

$$y_T = -[\ln \cdot \ln (10/9)] = 2.25037$$

$$K = \frac{2.25037 - 0.5332}{1.1004} = 1.56$$

$$x_T = 4263 + (1.56 \quad 1432.6)$$

$$= 6499 \text{ m}^3/\text{s}$$

Similarly, values of x_T are calculated for two more T values as shown below.

T (years)	x_T [obtained by Eq. (7.20)] (m^3/s)
5.0	5522
10.0	6499
20.0	7436

These values are shown in Fig. 7.3. It is seen that due to the property of the Gumbel's extreme probability paper these points lie on a straight line. A straight line is drawn through these points. It is seen that the observed data fit well with the theoretical Gumbel's extreme-value distribution.

[*Note*: In view of the linear relationship of the theoretical x_T and T on a Gumbel probability paper it is enough if only two values of T and the corresponding x_T are calculated. However, if Gumbel's probability paper is not available, a semi-log plot with log scale for T will have to be used and a large set of (x_T, T) values are needed to identify the theoretical curve.]

By extrapolation of the theoretical x_T vs T relationship, from Fig. 7.3,

At $T = 100$ years, $x_T = 9600\ m^3/s$

At $T = 150$ years, $x_T = 10{,}700\ m^3/s$

[By using Eq. (7.20) to (7.22), $x_{100} = 9558\ m^3/s$ and $x_{150} = 10088\ m^3/s$.]

EXAMPLE 7.4 *Flood-frequency computations for the river Chambal at Gandhisagar dam, by using Gumbel's method, yielded the following results:*

Return period T (years)	Peak flood (m^3/s)
50	40,809
100	46,300

Estimate the flood magnitude in this river with a return period of 500 years.

By Eq. (7.20),

$$x_{100} = \bar{x} = K_{100}\,\sigma_{n-1}$$

$$x_{50} = \bar{x} + K_{50}\,\sigma_{n-1}$$

$$(K_{100} - K_{50})\,\sigma_{n-1} = x_{100} - x_{50} = 46300 - 40809 = 5491$$

But $$K_T = \frac{y_T}{S_n} - \frac{\bar{y}_n}{S_n}$$

where S_n and $\bar{y}_n$ are constants for the given data series.

$$\therefore \quad (y_{100} - y_{50})\,\frac{\sigma_{n-1}}{S_n} = 5491$$

By Eq. (7.22),

$$y_{100} = -[\ln \cdot \ln (100/99)] = 4.60015.$$

$$y_{50} = -[\ln \cdot \ln (50/49)] = 3.90194$$

$$\frac{\sigma_{n-1}}{S_n} = \frac{5491}{(4.60015 - 3.90194)} = 7864$$

For $T = 500$ years, by Eq. (7.22),

$$y_{500} = -[\ln \cdot \ln (500/499)] = 6.21361$$

$$(y_{500} - y_{100})\,\frac{\sigma_{n-1}}{S_n} = x_{500} - x_{100}$$

$$(6.21361 - 4.60015) \times 7864 = x_{500} - 46300$$

$$x_{500} = 58988, \quad \text{say } 59{,}000\ m^3/s$$

Confidence Limits

Since the value of the variate for a given return period, x_T determined by Gumbel's method can have errors due to the limited sample data used, an estimate of the confidence limits of the estimate is desirable. The confidence interval indicates the limits about the calculated value between which the true value can be said to lie with a specific probability based on sampling errors only.

For a confidence probability c, the confidence interval of the variate x_T is bounded by values x_1 and x_2 given by[5]

$$x_{1/2} = x_T \pm f(c)\, S_e \qquad (7.23)$$

where $f(c)$ = function of the confidence probability c determined by using the table of normal variates as

c in per cent	50	68	80	90	95	99
$f(c)$	0.674	1.00	1.282	1.645	1.96	2.58

$$S_e = \text{probable error} = b\,\frac{\sigma_{n-1}}{\sqrt{N}} \qquad (7.23a)$$

$$b = \sqrt{1 + 1.3\,K + 1.1\,K^2}$$

K = frequency factor given by Eq. (7.21)

σ_{n-1} = standard deviation of the sample

N = sample size

It is seen that for a given sample and T, 80% confidence limits are twice as large as the 50% limits and 95% limits are thrice as large as 50% limits.

EXAMPLE 7.5 *Data covering a period of 92 years for the river Ganga at Raiwala yielded the mean and standard derivation of the annual flood series as 6437 and 2951 m³/s respectively. Using Gumbel's method estimate the flood discharge with a return period of 500 years. What are the (a) 95% and (b) 80% confidence limits for this estimate.*

From Table 7.3 for $N = 92$ years, $\bar{y}_n = 0.5589$

and $s_n = 1.2020$ from Table 7.4.

$$y_{500} = -[\ln \cdot \ln (500/499)]$$
$$= 6.21361$$
$$K_{500} = \frac{6.21361 - 0.5589}{1.2020} = 4.7044$$
$$x_{500} = 6437 + 4.7044 \times 2951 = 20320 \text{ m}^3/\text{s}$$

From Eq. (7.33 a)

$$b = \sqrt{1 + 1.3\,(4.7044) + 1.1\,(4.7044)^2}$$
$$= 5.61$$
$$s_e = \text{probable error } 5.61 \times \frac{2951}{\sqrt{92}} = 1726$$

(*a*) For 95% confidence probability $f(c) = 1.96$ and by Eq. (7.23)

$$x_{1/2} = 20320 \pm (1.96 \times 1726)$$
$$x_1 = 23703 \text{ m}^3/\text{s} \quad \text{and} \quad x_2 = 16937 \text{ m}^3/\text{s}$$

Thus the estimated discharge of 20320 m³/s has a 95% probability of lying between 23700 and 16940 m³/s.

(*b*) For 80% confidence probability, $f(c) = 1.282$ and by Eq. (7.23)

$$x_{1/2} = 20320 \pm (1.282 \times 1726)$$
$$x_1 = 22533 \text{ m}^3/\text{s} \quad \text{and} \quad x_2 = 18107 \text{ m}^3/\text{s}$$

The estimated discharge of 20320 m³/s has a 80% probability of lying between 22530 and 18110 m³/s.

For the data of Example 7.5, the values of x_T for different values of T are calculated and shown plotted on a Gumbel probability paper in Fig. 7.4. This variation is marked as "fitted line" in the figure. Also shown in this plot are the 95 and 80% confidence limits for various values of T. It is seen that as the confidence probability increases, the confidence interval also increases. Further, an increase in the return period T causes the confidence band to spread. Theoretical work by Alexeev (1961) has shown that for Gumbel's distribution the coefficient of skew $C_s \to 1.14$ for very low values of N. Thus the Gumbel's distribution will give erroneous results if the sample has a value of C_s very much different from 1.14.

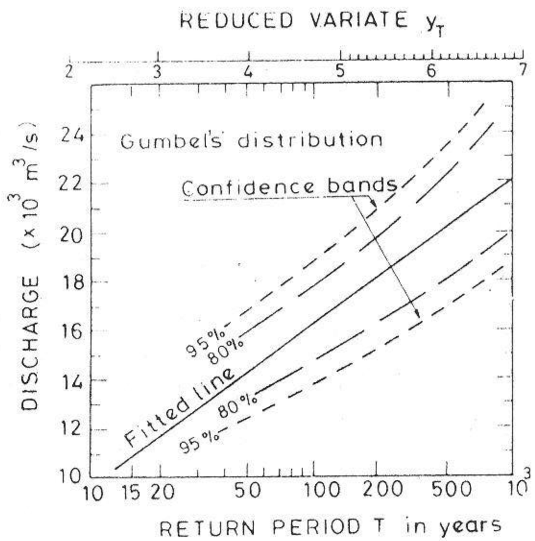

Fig. 7.4 Confidence bands for Gumbel's distribution—Example 7.5

7.7 LOG-PEARSON TYPE III DISTRIBUTION

This distribution is extensively used in USA for projects sponsored by the US Government. In this the variate is first transformed into logarithmic form (base 10) and the transformed data is then analysed. If X is the variate of a random hydrologic series, then the series of Z variates where

$$z = \log x \tag{7.24}$$

are first obtained. For this z series, for any recurrence interval T, Eq. (7.13) gives

$$z_T = \bar{z} + K_z\,\sigma_z \tag{7.25}$$

where K_s = a frequency factor which is a function of recurrence interval T and the coefficient of skew C_s,

σ_z = standard deviation of the Z variate sample

$$\sqrt{\Sigma (z - \bar{z})^2 / N - 1)} \tag{7.25 a}$$

and C_s = coefficient of skew of variate Z

$$= \frac{N\,\Sigma (z - \bar{z})^3}{(N-1)(N-2)(\sigma_z)^3} \tag{7.25 b}$$

$\bar{z}$ = mean of the z values

N = sample size = number of years of record

The variation of $K_z = f(C_s, T)$ is given in Table 7.6.

TABLE 7.6 $K_z = f(C_s, T)$ FOR USE IN LOG-PEARSON TYPE III DISTRIBUTION

Coefficient of skew, C_s	Recurrence interval T in years: 2	10	25	50	100	200	1000
3.0	−0.396	1.180	2.278	3.152	4.051	4.970	7.250
2.5	−0.360	1.250	2.262	3.048	3.845	4.652	6.600
2.2	−0.330	1.284	2.240	2.970	3.705	4.444	6.200
2.0	−0.307	1.302	2.219	2.912	3.605	4.298	5.910
1.8	−0.282	1.318	2.193	2.848	3.499	4.147	5.660
1.6	−0.254	1.329	2.163	2.780	3.388	3.990	5.390
1.4	−0.225	1.337	2.128	2.706	3.271	3.828	5.110
1.2	−0.195	1.340	2.087	2.626	3.149	3.661	4.820
1.0	−0.164	1.340	2.043	2.542	3.022	3.489	4.540
0.9	−0.148	1.339	2.018	2.498	2.957	3.401	4.395
0.8	−0.132	1.336	1.998	2.453	2.891	3.312	4.250
0.7	−0.116	1.333	1.967	2.407	2.824	3.223	4.105
0.6	−0.099	1.328	1.939	2.359	2.755	3.132	3.960
0.5	−0.083	1.323	1.910	2.311	2.686	3.041	3.815
0.4	−0.066	1.317	1.880	2.261	2.615	2.949	3.670
0.3	−0.050	1.309	1.849	2.211	2.544	2.856	3.525
0.2	−0.033	1.301	1.818	2.159	2.472	2.763	3.380
0.1	−0.017	1.292	1.785	2.107	2.400	2.670	3.235
0.0	0.000	1.282	1.751	2.054	2.326	2.576	3.090
−0.1	0.017	1.270	1.716	2.000	2.252	2.482	2.950
−0.2	0.033	1.258	1.680	1.945	2.178	2.388	2.810
−0.3	0.050	1.245	1.643	1.890	2.104	2.294	2.675
−0.4	0.066	1.231	1.606	1.834	2.029	2.201	2.540
−0.5	0.083	1.216	1.567	1.777	1.955	2.108	2.400
−0.6	0.099	1.200	1.528	1.720	1.880	2.016	2.275
−0.7	0.116	1.183	1.488	1.663	1.806	1.926	2.150
−0.8	0.132	1.166	1.448	1.606	1.733	1.837	2.035
−0.9	0.148	1.147	1.407	1.549	1.660	1.749	1.910
−1.0	0.164	1.128	1.366	1.492	1.588	1.664	1.880
−1.4	0.225	1.041	1.198	1.270	1.318	1.351	1.465
−1.8	0.282	0.945	1.035	1.069	1.087	1.097	1.130
−2.2	0.330	0.844	0.888	0.900	0.905	0.907	0.910
−3.0	0.396	0.660	0.666	0.666	0.667	0.667	0.668

After finding z_T by Eq. (7.25), the corresponding value of x_T is obtained by Eq. (7.24) as

$$x_T = \text{antilog}\,(z_T) \tag{7.26}$$

Sometimes, the coefficient of skew C_s is adjusted to account for the size of the sample by using the following relation proposed by Hazen (1930)

$$\hat{C}_s = C_s\left(1 + \frac{8.5}{N}\right) \tag{7.27}$$

where $\hat{C}_s$ = adjusted coefficient of skew. However the standard procedure for use log-Pearson Type III distribution adopted by U.S. Water Resources Council does not include this adjustment for skew.

When the skew is zero, i.e. $C_s = 0$, the log-Pearson Type III distribution reduces to *log normal distribution*. The log-normal distribution plots as a straight line on logarithmic probability paper.

EXAMPLE 7.6 *For the annual flood series data given in Example 7.3, estimate the flood discharge for a return period of (a) 100 years and (b) 200 years by using log-Pearson Type III distribution.*

The variate $z = \log x$ is first calculated for all the discharges. Then the statistics $\bar{z}$, σ_z and C_s are calculated as shown in Table 7.7.

$$\sigma_z = 0.1427 \qquad C_s = \frac{27 \times 0.0030}{(26)\,(0.1427)^3\,(25)}$$

$$\bar{z} = 3.6071 \qquad\qquad = 0.043$$

TABLE 7.7 VARIATE z—*Example 7.6*

Year	Flood x (m³/s)	$z = \log x$	Year	Flood x (m³/s)	$z = \log x$
1951	2947	3.4694	1965	4366	3.6401
1952	3521	3.5467	1966	3380	3.5289
1953	2399	3.3800	1967	7826	3.8935
1954	4124	3.6153	1968	3320	3.5211
1955	3496	3.5436	1969	6599	3.8195
1956	2947	3.4694	1970	3700	3.5682
1957	5060	3.7042	1971	4175	3.6207
1958	4903	3.6905	1972	2988	3.4754
1959	3751	3.5748	1973	2709	3.4328
1960	4798	3.6811	1974	3873	3.5880
1961	4290	3.6325	1975	4593	3.6621
1962	4652	3.6676	1976	6761	3.8300
1963	5050	3.7033	1977	1971	3.2947
1964	6900	3.8388			

The flood discharge for a given T is calculated as below. Values of K for given T and $C_s = 0.043$ are read from Table 7.6.

T (yrs)	K_z	$K_z\ \sigma_z$	z_T [Eq. (7·25)]	x_T = antilog z_T (m^3/s)
100	2.358	0.3365	3.9436	8782
200	2.616	0.3733	3.9804	9559

7.8 PARTIAL-DURATION SERIES

In the annual hydrologic data series of floods, only one maximum value of flood per year is selected as the data point. It is likely that in some catchments there are more than one independent floods in a year and many of these may be of appreciably high magnitude. To enable all the large flood peaks to be considered for analysis, a flood magnitude larger than an arbitrary selected base value are included in the analysis. Such a data series is called *partial-duration series.*

In using the partial-duration series, it is necessary to establish that all events considered are independent. Hence the partial-duration series is adopted mostly for rainfall analysis where the conditions of independency of events are easy to establish. Its use in flood studies is rather rare. The recurrence interval of an event obtained by annual series (T_A) and by the partial duration series (T_P) are related by

$$T_P = \frac{1}{\ln T_A - \ln (T_A - 1)} \tag{7.28}$$

From this it can be seen that the difference between T_A and T_P is significant for $T_A < 10$ years and that for $T_A > 20$, the difference is negligibly small.

7.9 REGIONAL FLOOD FREQUENCY ANALYSIS

When the available data at a catchment is too short to conduct frequency analysis, a regional analysis is adopted. In this a hydrologically homogeneous region from the statistically point of view is considered. Available long-time data from neighbouring catchments are tested for homogeneity and a group of stations satisfying the test are identified. This group of stations constitutes a region and all the station data of this region are pooled and analysed as a group to find the frequency characteristics of the region. The mean annual flood Q_{ma}, which corresponds to a recurrence interval of 2.33 years is used for nondimensionalising the results. The variation of Q_{ma} with drainage area and the variation of Q_T/Q_{ma} with T, where Q_T is the discharge for any T are the basic plots prepared in this analysis. Details of the method are available in Ref. 1.

7.10 LIMITATIONS OF FREQUENCY STUDIES

The flood-frequency analysis described in the previous sections is a direct means of estimating the desired flood based upon the available flood-flow data of the catchment. The results of the frequency analysis depend upon the length of data. The minimum number of years of record required to obtain satisfactory estimates depends upon the variability of data and hence on the physical and climatological characteristics of the basin. Generally a minimun of 30 years of data is considered as essential. Smaller lengths of records are also used when it is unavoidable. However, frequency analysis should not be adopted if the length of records is less than 10 years.

Flood-frequency studies are most reliable in climates that are uniform from year to year. In such cases a relatively short record gives a reliable picture of the frequency distribution. With increasing lengths of flood records, it affords a viable alternative method of flood-flow estimation in most cases.

7.11 DESIGN FLOOD

In the design of hydraulic structures it is not practical from economic considerations to provide for the safety of the structure and the system at the maximum-possible flood in the catchment. Small structures such as culverts and storm drainages can be designed for less severe floods as the consequences of a higher-than-design flood may not be very serious. It can cause temporary inconvenience like the disruption of traffic and very rarely severe property damage and loss of life. On the other hand, storage structures such as dams demand greater attention to the magnitude of floods used in the design. The failure of these structures causes large loss of life and great property damage on the down-stream of the structure. From this it is apparent that the type, importance of the structure and economic development of the surrounding area dictate the design criteria for choosing the flood magnitude. This section highlights the procedures adopted in selecting the flood magnitude for the design of some hydraulic structures.

The following definitions are first noted:

Design Flood

Flood adopted for the design of a structure.

Spillway Design Flood

Design flood used for the specific purpose of designing the spillway of a storage structure. This term is frequently used to denote the maximum discharge that can be passed in a hydraulic structure without any damage or serious threat to the stability of the structure.

Standard Project Flood (SPF)

The flood that would result from a severe combination of meteorological and hydrological factors that are reasonably applicable to the region. Extremely rare combinations of factors are excluded.

Probable Maximum Flood (PMF)

The extreme flood that is physically possible in a region as a result of severemost combinations, including rare combinations of meteorological and hydrological factors.

The PMF is used in situations where the failure of the structure would result in loss of life and catastrophic damage and as such complete security from potential floods is sought. On the other hand, SPF is often used where the failure of a structure would cause less severe damages. Typically, the SPF is about 40 to 60% of the PMF for the same drainage basin. The criteria used for selecting the design flood for various hydraulic structures vary from one country to another. Table 7.8 gives a brief summary of the guidelines adopted by CWC, India to select design floods.

TABLE 7.8 GUIDELINES FOR SELECTING DESIGN FLOODS

S. No.	Structure	Recommended design flood
1.	Spillways for major and medium projects with storages more than 60 Mm^3	(a) PMF determined by unit hydrograph and probable maximum precipitation (PMP) (b) If (a) is not applicable or possible flood-frequency method with T = 1000 years
2.	Permanent barrage and minor dams with capacity less than 60 Mm^3	(a) SPF determined by unit hydrograph and standard project storm (SPS) which is usually the largest recorded storm in the region (b) Flood with a return period of 100 years (a) Or (b) whichever gives higher value.
3.	Pickup weirs	Flood with a return period of 100 or 50 years depending on the importance of the project.
4.	Aqueducts	
	(a) Waterway	Flood with T = 50 years
	(b) Foundations and free board	Flood with T = 100 years
5.	Project with very scanty or inadequate data	Empirical formulae

7.12 DESIGN STORM

To estimate the design flood for a project by the use of a unit hydrograph, one needs the design storm. This can be the storm-producing probable maximum precipitation (PMP) for deriving PMF or a standard project storm (SPS) for SPF calculations. The computations are performed by experienced hydrometeorologists by using meteorological data. Various methods ranging from highly sophisticated hydrometeorological methods to simple analysis of past rainfall data are in use depending on the availability of reliable relevant data and expertise.

The following is a brief outline of a procedure followed in India:

1. The duration of the critical rainfall is first selected. This will be the basin lag if the flood peak is of interest. If the flood volume is of prime interest, the duration of the longest storm experienced in the basin is selected.
2. Past major storms in the region which conceivably could have occurred in the basin under study are selected. DAD analysis is performed and the enveloping curve representing maximum depth-duration relation for the study basin obtained.
3. Rainfall depths for convenient time intervals (e.g. 6 h) are scaled from the enveloping curve. These increments are to be arranged to get a critical sequence which produces the maximum flood peak when applied to the relevant unit hydrograph of the basin.

The critical sequence of rainfall increments can be obtained by trial and error. Alternatively, increments of precipitation are first arranged in a table of relevant unit hydrograph ordinates such that (i) the maximum rainfall increment is against the maximum unit hydrograph ordinate, (ii) the second highest rainfall increment is against the second largest unit hydrograph ordinate, and so on, and (iii) the sequence of rainfall increments arranged above is now reversed, with the last item first and first item last. The new sequence gives the design storm (Example 7.7).

4. The design storm is then combined with hydrologic abstractions most conducive to high runoff, viz. low initial loss and lowest infiltration rate to get the hyetograph of rainfall excess to operate upon the unit hydrograph.

Further details about the above procedure and other methods for computing design storms are available in Ref. 6. Reference 2 gives details of the estimation of the design flood peak by unit hydrographs for small drainage basins of areas from 25–500 km^2.

EXAMPLE 7.7: *The ordinates of cumulative rainfall from the enveloping maximum depth–duration curve for a basin are given below. Also given are the ordinates of a 6-h unit hydrograph. Design the critical sequence of rainfall excesses by taking the ϕ index to be 0.15 cm/h.*

Time from start (h)	0	6	12	18	24	30	36	42	48	54	60
Cumulative rainfall (cm)	0	16	24.1	30	34	37	39	40.5	41.3		
6-h UH ordinate (m^3/s)	0	20	54	98	126	146	154	152	138	122	106

Time from start (h)	66	72	78	84	90	96	102	108	114	129	132
6-h UH ordinate (m^3/s)	92	79	64	52	40	30	20	14	10	6	0

The critical storm and rainfall excesses are calculated in a tabular form in Table 7.8.

TABLE 7.8 CALCULATION OF CRITICAL STORM—*Example 7.8*

Time (h)	Cumulative rainfall (cm)	6-h incremental rainfall (cm)	Ordinate of 6-h UH (m^3/s)	First arrangement of rainfall increment	Design sequence of rainfall increment	Infiltration loss (cm)	Rainfall excess of design storm (cm)
1	2	3	4	5	6	7	8
0	0		0		0	0	0
6	15.0	15.0	20		1.5	0.9	0.6
12	24.1	9.1	54		2.0	0.9	1.1
18	30.0	5.9	98	0.8	4.0	0.9	3.1
24	34.0	4.0	126	3.0	9.1	0.9	8.1
30	37.0	3.0	146	5.9	15.0	0.9	14.1
36	39.0	2.0	154	15.0	5.9	0.9	5.0
42	40.5	1.5	152	9.1	3.0	0.9	2.1
48	41.3	0.8	138	4.0	0.8	0.9	0
54			122	2.0			
60			106	1.5			
66			92				
72			79				
78			64				
84			52				
90			40				
96			30				
102			20				
108			14				
114			10				
120			6				
132			0				

1. (Column 6 is reversed sequence of column 5)
2. Infiltration loss = 0.15 cm/h = 0.9 cm/6 h

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PROBLEMS

7.1 A catchment of area 120 ha has a time of concentration of 30 min and runoff coefficient of 0.3. If a storm of duration 45 min results in 3.0 cm of rain over the catchment, estimate the resulting peak flow rate.

7.2 Information on the 50-year storm is given below.

Duration (minutes)	15	30	45	60	180
Rainfall (mm)	40	60	75	100	120

A culvert has to drain 200 ha of land with a maximum length of travel of 1.25 km. The general slope of the catchment is 0.001 and its runoff coefficient is 0.20. Estimate the peak flow by the rational method for designing the culvert for a 50-year flood.

7.3 A basin is divided by 1-h *isochrones* into four sub-areas of size 200, 250, 350 and 170 hectares from the upstream end of the outlet respectively. A rainfall event of 5-h duration with intensities of 1.7 cm/h for the first 2 h and 1.25 cm/h for the next 3 occurs uniformly over the basin. Assuming a constant runoff coefficient of 0.5, estimate the peak rate of runoff.

(*Note*: An *isochrone* is a line on the catchment map joining points having equal time of travel of surface runoff. See Sec. 8.8)

7.4 A flood of 4000 m^3/s in a certain river has a return period of 40 years. (a) What is its probability of exceedence? (b) What is the probability that a flood of 4000 m^3/s or greater magnitude may occur in the next 20 years? (c) What is the probability of occurrence of a flood of magnitude less than 4000 m^3/s?

7.5 Complete the following:

(a) Probability of a 10-year flood occurring at least once in the next 5 years is———.

(b) Probability that a flood of magnitude equal to or greater than the 20-year flood will not occur in the next 20 years is———

(c) Probability of a flood equal to or greater than a 50-year flood occurring next year is———————.

(d) Probability of a flood equal to or greater than a 50-year flood occurring three times in the next 10 years is———————.

(e) Probability of a flood equal to or greater than a 50-year flood occurring at least once in next 50 years is———————.

7.6 What return period must a designer use in the design of a culvert if he is willing to accept a 10% risk that a flood will occur in the next 10 years?

[*Note*: The probability of an event occurring at least once in N successive years is known as *risk*.]

7.7 A table showing the variation of the frequency factor K in the Gumbel's extreme value distribution with the sample size N and return period T is often given in books. The following is an incomplete listing of K for T=1000 years. Complete the table.

Sample size, N	25	30	35	40	45	50	55	60	65	70
Value of $K(T, N)$ for T= 1000 years	5.842	5.727	—	5.576	—	5.478	—	—	—	5.359

7.8 The following table gives the observed annual flood values in the River Bhagirathi at Tehri. Estimate the flood peaks with return periods of 50, 100 and 1000 years by using:

(a) Gumbel's extreme value distribution, and

(b) log-Pearson type III distribution

Year	1963	1964	1965	1966	1967	1968	1969
Flood discharge (m³/s)	3210	4000	1250	3300	2480	1780	1860
Year	1970	1971	1972	1973	1974	1975	
Flood discharge (m³/s)	4130	3110	2320	2480	3405	1820	

7.9 A hydraulic structure on a stream has been designed for a discharge of 350 m³/s. If the available flood data on the stream is for 20 years and the mean and standard deviation for annual flood series are 121 and 60 m³/s respectively, calculate the return period for the design flood by using Gumbel's method.

7.10 The flood data of a river was analysed for the prediction of extreme values by log-Pearson Type III distribution. Using the variate $Z=\log Q$, *where* Q=flood discharge in the river, it was found that $\bar{z}=2.510$, $\sigma_z=0.162$ and coefficient of skew $C_s=0.70$. Estimate the flood discharges with return periods of 50, 100, 200 and 1000 years in this river.

7.11 The following data give flood-data statistics of two rivers in UP:

S. No.	River	Length of records (years)	Mean annual flood (m³/s)	σ_{n-1}
1	Ganga at Raiwala	92	6437	2951
2	Yamuna at Tajewala	54	5627	3360

(a) Estimate the 100- and 1000-year floods for these two rivers by using Gumbel's method.

(b) What are the 95% confidential intervals for the predicted values?

7.12 For a river, the estimated flood peaks for two return periods by the use of Gumbel's method are as follows:

Return period (years)	Peak flood (m³/s)
100	435
50	395

What flood discharge in this river will have a return period of 1000 years.

7.13 Using 30 years data and Gumbel's method the flood magnitudes, for return periods of 100 and 50 years for a river are found to be 1200 and 1060 m³/s respectively.

(a) Determine the mean and standard deviation of the data use and

(b) estimate the magnitude of a flood with a return period of 500 years.

7.14 The ordinates of a mass curve of rainfall from a severe storm in a catchment is given. Ordinates of a 12-h unit hydrograph applicable to the catchment are also given. Using the given mass curve, develop a design storm to estimate the design flood for the catchment. Taking the ϕ index as 0.15 cm/h, estimate the resulting flood hydrograph. Assume the base flow to be 50 m³/s.

Time (h)	0	12	24	36	48	60	72	84	96	108	120	132
Cumulative rainfall (cm)	0	10.2	30.5	34.0	36.0							
12-h UH ordinate (m³/s)	0	32	96	130	126	98	75	50	30	15	7	0

QUESTIONS

7.1 A culvert is designed for a peak flow Q_p on the basis of the rational formula. If a storm of the same intensity as used in the design but of duration twice larger occurs, the resulting peak discharge will be

(a) Q_p (b) $2\,Q_p$ (c) $Q_p/2$ (d) Q_p^2.

7.2 A catchment of area 90 ha has a runoff coefficient of 0.4. A storm of duration larger than the time of concentration of the catchment and of intensity 4.5 cm/h creates a peak discharge rate of

(a) 11.3 m^3/s (b) 0.45 m^3/s (c) 450 m^3/s (d) 4.5 m^3/s.

7.3 Of the various empirical formulae relating the flood peak to the catchment area,

(a) Ryves formula is used all over the country
(b) Dickens formula is used all over the country
(c) Inglis formula is in use in a large part of north India
(d) Dickens formula is in use in a large part of north India.

7.4 For an annual flood series arranged in decreasing order of magnitude, the return period for a magnitude listed at position m in a total of N entries is

(a) m/N (b) $m/(N+1)$ (c) $(N+1)/m$ (d) $N/(m+1)$.

7.5 The general equation for hydrological frequency analysis states that x_T=value of a variate with a return period of T years is given by x_T=

(a) $\bar{x} - K\sigma$ (b) $\bar{x} + K\sigma$ (c) $K\sigma$ (d) $\bar{x} / K\sigma$.

7.6 For a return period of 100 years the Gumbel's reduced variate y_T is

(a) −4.600 (b) 4.600 (c) 0.517 (d) 1.2835.

7.7 An annual flood series contains 100 years of flood data. For a return period of 200 years the Gumbel's reduced variate is

(a) 5.296 (b) 4.600 (c) 1.2835 (d) 0.5170.

7.8 To use Gumbel's method to estimate the magnitude of a flood with a return period of T years, the following data pertaining to annual flood series are sufficient:

(a) mean and standard deviation
(b) mean, standard deviation and length of record
(c) standard deviation and length of record
(d) mean, standard deviation and coefficient of skew.

7.9 To estimate the flood magnitude with a return period T by the log-Pearson Type III method, the following data pertaining to annual flood series are sufficient:

(a) mean, standard deviation and coefficient of skew of discharge data;
(b) mean and standard deviation of the log of discharges and the number of years of record;
(c) mean, standard deviation and coefficient of skew of the log of discharges;
(d) mean and standard deviation of the log of discharges.

7.10 The recurrence interval of an event is T_A in annual series and T_P in partial-duration series.

(a) T_A is always smaller than T_P;
(b) Difference between T_A and T_P is negligible for $T_A < 5$ years.
(c) Difference between T_A and T_P is negligible for $T_A > 10$ years.
(d) Difference between T_A and T_P is significant for $T_A > 100$ years.

7.11 The term *mean annual flood* denotes

(a) mean of floods in partial-duration series
(b) mean annual flow
(c) a flood with a recurrence interval of 2.33 years
(d) a flood with a recurrence interval of $N/2$ years, where N=number of years of record.

7.12 The confidence interval for the estimate of a value x_T by Gumbel's method depends on

(a) the confidence probability only
(b) the return period only
(c) the confidence probability and return period
(d) the confidence probability and number of years of record.

7.13 The use of the unit hydrograph for estimating floods is limited to catchments of size less than

(a) 5000 km^2 (b) 500 km^2 (c) 10^6 km^2 (d) no upper limit.

7.14 The probable maximum flood is

(a) the standard project flood of an extremely large river
(b) a flood adopted in the design of all kinds of spillways
(c) a flood adopted in all hydraulic structures
(d) an extremely large but physically possible flood in the region.

8

FLOOD ROUTING

8.1 INTRODUCTION

The flood hydrograph discussed in Chap. 6 is in fact a wave. The stage and discharge hydrographs represent the passage of waves of river depth and discharge respectively. As this wave moves down the river, the shape of the wave gets modified due to various factors, such as channel storage, resistance, lateral addition or withdrawal of flows, etc. When a flood wave passes through a reservoir, its peak is attenuated and the time base is enlarged due to the effect of storage. Flood waves passing down a river have their peaks attenuated due to friction if there is no lateral inflow. The addition of lateral inflows can cause a reduction of attenuation or even amplification of a flood wave. The study of the basic aspects of these changes in a flood wave passing through a channel system forms the subject matter of this chapter.

Flood routing is the technique of determining the flood hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections. The hydrologic analysis of problems such as flood forecasting, flood protection, reservoir design and spillway design invariably include flood routing. In these applications two broad categories of routing can be recognised. These are:

1. Reservoir routing, and
2. channel routing.

In reservoir routing the effect of a flood wave entering a reservoir is studied. Knowing the volume-elevation characteristic of the reservoir and the outflow-elevation relationship for the spillways and other outlet structures in the reservoir, the effect of a flood wave entering the reservoir is studied to predict the variations of reservoir elevation and outflow discharge with time. This form of reservoir routing is essential (i) in the design of the capacity of spillways and other reservoir outlet structures and (ii) in the location and sizing of the capacity of reservoirs to meet specific requirements.

In channel routing the changes in the shape of a hydrograph as it travels down a channel is studied. By considering a channel reach and an input hydrograph at the upstream end, this form of routing aims to predict the flood hydrograph at various sections of the reach. Information on the flood-peak attenuation and the duration of high-water levels obtained by channel routing is of utmost importance in flood-forecasting operations and flood-protection works.

A variety of routing methods are available and they can be broadly classified into two categories as: (i) hydrologic routing and (ii) hydraulic routing. Hydrologic-routing methods employ essentially the equation of continuity. Hydraulic methods, on the other hand, employ the continuity equation together with the equation of motion of unsteady flow. The basic differential equations used in the hydraulic routing, known as St. Venant equations afford a better description of unsteady flow than hydrologic methods.

8.2 BASIC EQUATIONS

The passage of a flood hydrograph through a reservoir or a channel reach is an unsteady-flow phenomenon. It is classified in open-channel hydraulics as gradually varied unsteady flow. The equation of continuity used in all hydrologic routing as the primary equation states that the difference between the inflow and outflow rate is equal to the rate of change of storage, i.e.

$$I - Q = \frac{dS}{dt} \tag{8.1}$$

where I = inflow rate, Q = outflow rate and S = storage. Alternatively, in a small time interval Δt the difference between the total inflow volume and total outflow volume in a reach is equal to the change in storage in that reach

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S \tag{8.2}$$

where $\bar{I}$ = average inflow in time Δt, $\bar{Q}$ = average outflow in time Δt and ΔS = change in storage. By taking $\bar{I} = (I_1 + I_2)/2$, $\bar{Q} = (Q_1 + Q_2)/2$ and $\Delta S = S_2 - S_1$, with suffixes 1 and 2 to denote the beginning and end of the time interval Δt, Eq. (8.2) is written as

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1 \tag{8.3}$$

The time interval Δt should be sufficiently short so that the inflow and outflow hydrographs can be assumed to be straight line in that time interval. Further, Δt must be shorter than the time of transit of the flood wave through the reach.

In the differential form the equation of continuity for unsteady flow in a reach with no lateral flow is given by

$$\frac{\partial Q}{\partial x} + T\frac{\partial y}{\partial t} = 0 \qquad (8.4)$$

where T = top width of the section and y = depth of flow.

The equation of motion for a flood wave is derived from the application of the momentum equation as

$$\frac{\partial y}{\partial x} + \frac{V}{g}\frac{\partial V}{\partial x} + \frac{1}{g}\frac{\partial V}{\partial t} = S_0 - S_f \qquad (8.5)$$

where V = velocity of flow at any section, S_0 = channel bed slope and S_f = slope of the energy line. The continuity equation [Eq. (8.4)] and the equation of motion [Eq. (8.5)] are believed to have been first developed by A.J.C. Barrè de Saint Venant (1871) and are commonly known as St Venant equations. Hydraulic-flood routing involves the numerical solution of St Venant equations through the use of digital computers. Details about these equations, such as their derivations and various forms are available in Ref. 7.

8.3 HYDROLOGIC STORAGE ROUTING

A flood wave $I(t)$ enters a reservoir provided with an outlet such as a spillway. The outflow is a function of the reservoir elevation only, i.e. $Q = Q(h)$. The storage in the reservoir is a function of the reservoir elevation, $S = S(h)$. Further, due to the passage of the flood wave through the reservoir, the water level in the reservoir changes with time, $h = h(t)$ and hence the storage and discharge change with time (Fig. 8.1). It is required to find the variation of S, h and Q with time, i.e. find $S = S(t)$, $Q = Q(t)$ and $h = h(t)$ given $I = I(t)$.

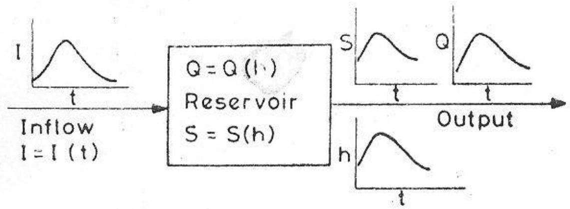


Fig. 8.1 Storage routing (Schematic)

If an uncontrolled spillway is provided in a reservoir typically

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L_e H^{3/2}$$

where H = head over the spillway, L_e = effective length of the spillway crest and C_d = coefficient of discharge. Similarly, for other forms of outlets, such as gated spillways, sluice gates, etc. other relations for $Q(h)$ will be available.

For reservoir routing, the following data have to be known:

1. Storage volume vs elevation for the reservoir;
2. water-surface elevation vs outflow and hence storage vs outflow discharge;
3. inflow hydrograph, $I = I(t)$; and
4. initial values of S, I and Q at time $t = 0$.

There are a variety of methods available for routing of floods through a reservoir. All of them use Eq. (8.2) but in various rearranged manners. Two commonly used methods are described below.

Modified Pul's Method

Equation (8.3) is rearranged as

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1\,\Delta t}{2}\right) = \left(S_2 + \frac{Q_2\,\Delta t}{2}\right) \qquad (8.6)$$

At the starting of flood routing, the initial storage and outflow discharges are known. In Eq. (8.6) all the terms in the left-hand side are known at the beginning of a time step Δt. Hence the value of the function $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ at the end of the time step is calculated by Eq. (8.6). Since the relation $S = S(h)$ and $Q = Q(h)$ are known, $\left(S + \frac{Q\,\Delta t}{2}\right)_2$ will enable one to determine the reservoir elevation and hence the discharge at the end of the time step. The procedure is repeated to cover the full inflow hydrograph.

For practical use in hand computation, the following semigraphical method is very convenient.

1. From the known storage-elevation and discharge-elevation data, prepare a curve of $\left(S + \frac{Q\,\Delta t}{2}\right)$ vs elevation (Fig. 8.2). Here Δt is any chosen interval, approximately 20 to 40% of the time of rise of the inflow hydrograph.
2. On the same, plot prepare a curve of outflow discharge vs elevation (Fig. 8.2).
3. The storage, elevation and outflow discharge at the starting of routing are known. For the first time interval Δt, $\left(\frac{I_1 + I_2}{2}\right)\Delta t$ and $\left(S_1 - \frac{Q_1\,\Delta t}{2}\right)$ are known and hence by Eq. (8.6) the term $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ is determined.

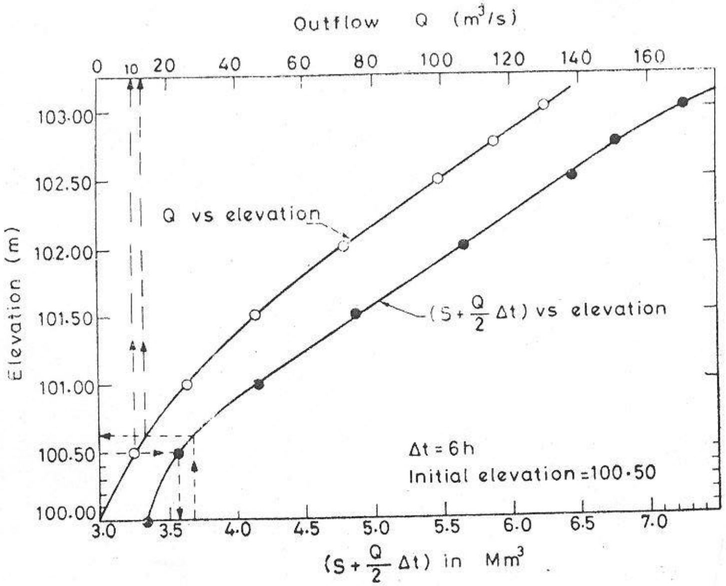


Fig. 8.2 Modified Pul's method of storage routing

4. The water-surface elevation corresponding to $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ is found by using the plot of step (1). The outflow discharge Q_2 at the end of the time step Δt is found from plot of step (2).
5. Deducting $Q_2\,\Delta t$ from $\left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$ gives $\left(S - \frac{Q\,\Delta t}{2}\right)_1$ for the beginning of the next time step.
6. The procedure is repeated till the entire inflow hydrograph is routed.

EXAMPLE 8.1: *A reservoir has the following elevation, discharge and storage relationships:*

Elevation	Storage (10^6 m³)	Outflow discharge (m³/s)
100.00	3.350	0
100.50	3.472	10
101.00	3.880	26
101.50	4.383	46
102.00	4.882	72
102.50	5.370	100
102.75	5.527	116
103.00	5.856	130

When the reservoir level was at 100.50, the following flood hydrograph entered the reservoir.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Discharge (m³/s)	10	20	55	80	73	58	46	36	55	20	15	13	11

Route the flood and obtain (i) the outflow hydrograph and (ii) the reservoir elevation vs time curve during the passage of the flood wave.

A time interval $\Delta t = 6$h is chosen. From the available data the elevation–discharge–$\left(S + \frac{Q\,\Delta t}{2}\right)$ table is prepared.

$$\Delta t = 6 \times 60 \times 60 = 0.0216 \times 10^6 \text{ s}$$

Elevation (m)	100.00	100.50	101.00	101.50	102.00	102.50	102.75	103.00
Discharge Q (m³/s)	0	10	26	46	72	100	116	130
$\left(S + \frac{Q\,\Delta t}{2}\right)$ (Mm³)	3.35	3.58	4.16	4.88	5.66	6.45	6.78	7.26

A graph of Q vs elevation and $\left(S + \frac{Q\,\Delta t}{2}\right)$ vs elevation is prepared (Fig. 8.2). At the start of routing, elevation-100.50, $Q = 10.0$ m³/s, and $\left(S - \frac{Q\,\Delta t}{2}\right) = 3.362$ Mm³. Starting from this value of $\left(S - \frac{Q\,\Delta t}{2}\right)$, Eq. (8.6) is used to get $\left(S + \frac{Q\,\Delta t}{2}\right)$ at the end of first time step of 6 h as

$$\left(S + \frac{Q\,\Delta t}{2}\right)_2 = (I_1 + I_2)\,\frac{\Delta t}{2} + \left(S - \frac{Q\,\Delta t}{2}\right)_1$$

$$= (10 + 20) \times \frac{0.0216}{2} + (3.362) = 3.686 \text{ Mm}^3$$

Looking up in Fig. 8.2, the water-surface elevation corresponding to to $\left(S + \frac{Q\,\Delta t}{2}\right) = 3.686$ Mm³ is 100.62 and the corresponding outflow discharge Q is 13 m³/s. For the next step, the initial value of $\left(S - \frac{Q\,\Delta t}{2}\right) = \left(S + \frac{Q\,\Delta t}{2}\right)$ of the previous step $- Q.\,\Delta t = (3.686 - 13 \times 0.0216) = 3.405$ Mm³.

The process is repeated for the entire duration of the inflow hydrograph in a tabular form as shown in Table 8.1.

TABLE 8.1 FLOOD ROUTING THROUGH A RESERVOIR—EXAMPLE 8.1

$\Delta t = 6$ h $= 0.0216$ Ms, $\bar{I} = (I_1 + I_2)/2$

Time (h)	Inflow I (m³/s)	$\bar{I}$ (m³/s)	$\bar{I}.\Delta t$ (Mm³)	$S - \frac{\Delta t.Q}{2}$ (Mm³)	$S + \frac{\Delta t.Q}{2}$ (Mm³)	Elevation m	Q (m³/s)
1	2	3	4	5	6	7	8
0	10	15.00	0.324	3.362	3.636	100.50	10
6	20	37.50	0.810	3.405	4.215	100.62	13
12	55	67.50	1.458	3.632	5.090	101.04	27
18	80	76.50	1.652	3.945	5.597	101.64	53
24	73	65.50	1.415	4.107	5.522	101.96	69
30	58	52.00	1.123	4.096	5.219	101.91	66
36	46	41.00	0.886	3.988	4.874	101.72	57
42	36	31.75	0.686	3.902	4.588	101.48	45
48	27.5	23.75	0.513	3.789	4.302	101.30	37
54	20	17.50	0.378	3.676	4.054	100.10	29
60	15	14.00	0.302	3.557	3.859	100.93	23
66	13	12.00	0.259	3.470	3.729	100.77	18
72	11			3.427		100.65	14

Using the data in columns 1, 8 and 7, the outflow hydrograph (Fig. 8.3) and a graph showing the variation of reservoir elevation with time (Fig. 8.4) are prepared.

Sometimes a graph of $\left(S - \frac{Q\,\Delta t}{2}\right)$ vs elevation prepared from known data is plotted in Fig. 8.2 to aid in calculating the items in column 5. Note that the calculations are sequential in nature and any error at any stage is carried forward. The accuracy of the method depends upon the value of Δt; smaller values of Δt give greater accuracy.

Goodrich Method

Another popular method of hydrologic reservoir routing, known as Goodrich method utilizes Eq. (8.3) rearranged as

$$I_1 + I_2 - Q_1 - Q_2 = \frac{2S_2}{\Delta t} - \frac{2S_1}{\Delta t}$$

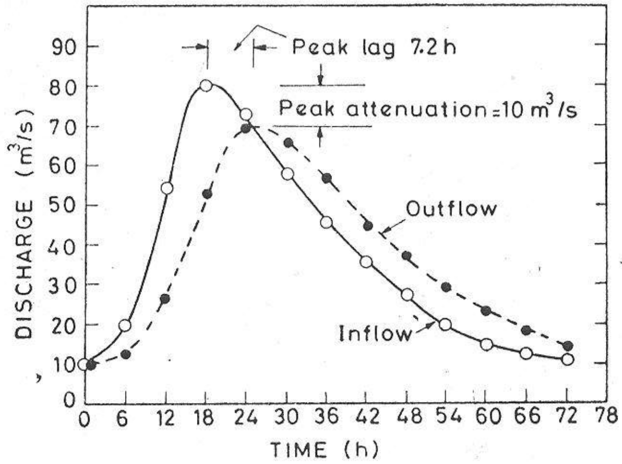


Fig. 8.3 Variation of inflow and outflow discharges—Example 8.1

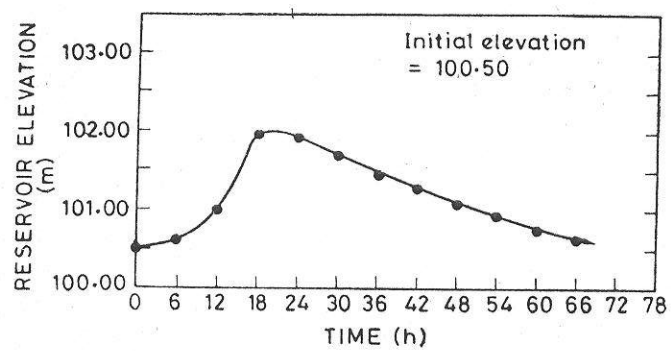


Fig. 8.4 Variation of reservoir elevation with time—Example 8.1

where suffixes 1 and 2 stand for the values at the beginning and end of a time step Δt respectively. Collecting the known and initial values together,

$$(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right) \tag{8.7}$$

For a given time step, the left-hand side of Eq. 8.7 is known and the term $\left(\frac{2S}{\Delta t} + Q\right)_2$ is determined by using Eq. (8.7). From the known storage-elevation-discharge data, the function $\left(\frac{2S}{\Delta t} + Q\right)_2$ is known as a function of elevation. Hence the discharge, elevation and storage at the end of the time step are obtained. For the next time step,

$$\left[\left(\frac{2S}{\Delta t} + Q\right)_2 - 2Q_2\right] \text{ of the previous time step}$$

$$= \left(\frac{2S}{\Delta t} - Q\right)_1 \text{ for use as the initial values}$$

The procedure is illustrated in Example 8.2.

EXAMPLE 8.2: *Route the following flood hydrograph through the reservoir of Example 8.1 by the Goodrich method:*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	10	30	85	140	125	96	75	60	46	35	25	20

The initial conditions are: when t = 0, the reservoir elevation is 100.60 m,

A time increment Δt = 6 h = 0.0216 Ms is chosen. Using the known storage–elevation–discharge data, the following table is prepared.

Elevation (m)	100.00	100.50	101.00	101.50	102.00	102.50	102.75	103.00
Outflow Q (m^3/s)	0	10	26	46	72	100	116	130
$\left(\frac{2S}{\Delta t} + Q\right)$ (m^3/s)	310.2	331.5	385.3	451.8	524.0	597.2	627.8	672.2

A graph depicting Q vs elevation and $\left(\frac{2S}{\Delta t} + Q\right)$ vs elevation is prepared from this data (Fig. 8.5).

At t = 0, Elevation = 100.60, from Fig. 8.5, Q = 12 m^3/s and

$$\left(\frac{2S}{\Delta t} + Q\right) = 340 \text{ m}^3/\text{s}.$$

$$\left(\frac{2S}{\Delta t} - Q\right)_1 = 340 - 24 = 316 \text{ m}^3/\text{s}$$

For the first time interval of 6 h,

$$I_1 = 10, \quad I_2 = 30, \quad Q_1 = 12, \text{ and}$$

$$\left(\frac{2S}{\Delta t} + Q\right)_2 = (10 + 30) + 316 = 356 \text{ m}^3/\text{s}$$

From Fig. 8.5 the reservoir elevation for this $\left(\frac{2S}{\Delta t} + Q\right)_2$ is 100.74.

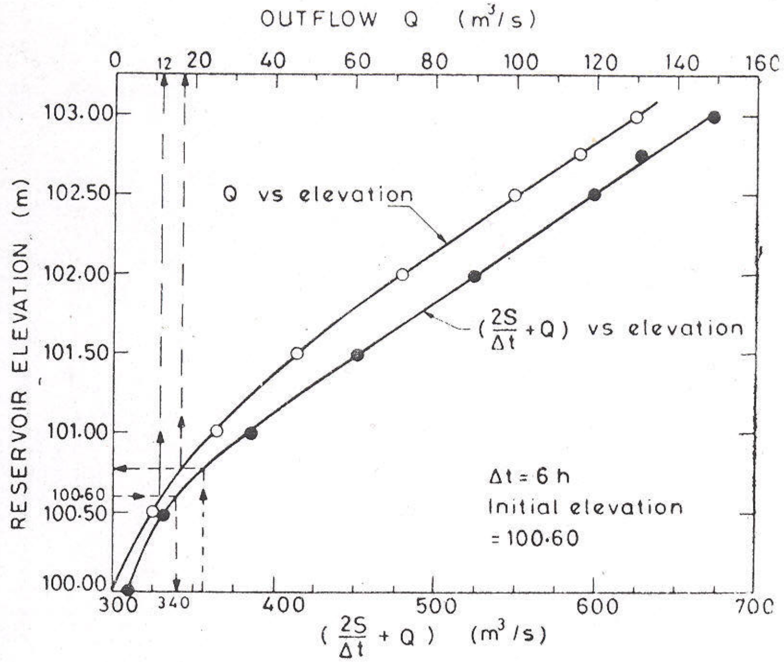


Fig. 8.5 Goodrich method of storage routing

For the next time increment

$$\left(\frac{2S}{\Delta t} - Q\right)_1 = 356 - 2 \times 12 = 322 \text{ m}^3/\text{s}$$

The procedure is repeated in a tabular form (Table 8.2) till the entire flood is routed.

Using the data in columns 1, 8 and 7, the outflow hydrograph and a graph showing the variation of reservoir elevation with time (Fig. 8.6) are plotted.

In this method also, the accuracy depends upon the value of Δt chosen; smaller values of Δt give greater accuracy.

Other Methods

In addition to the above two methods, there are a large number of other methods which depend on different combinations of the parameters of the basic continuity equation [Eq. (8.3)]. Graphical and analog methods are also available for reservoir routing. References 3 and 5 can be consulted for details on some of these methods.

Table 8.2 Reservoir Routing—Goodrich Method—Example 8.2

$\Delta t = 6.0$ h $= 0.0216$ Ms

Time (h)	I (m³/s)	(I_1+I_2)	$\left(\frac{2S}{\Delta t} - Q\right)$ (m³/s)	$\left(\frac{2S}{\Delta t} + Q\right)$ (m³/s)	Elevation (m)	Discharge Q (m³/s)
1	2	3	4	5	6	7
0	10			(340)	100.60	12
		40	316	356		
6	30				100.74	17
		115	322	437		
12	85				101.38	40
		225	357	582		
18	140				102.50	95
		265	392	657		
24	125				102.92	127
		221	403	624		
30	96				102.70	112
		171	400	571		
36	75				102.32	90
		135	391	526		
42	60				102.02	73
		106	380	486		
48	46				101.74	57
		81	372	453		
54	35				101.51	46
		60	361	421		
60	25				101.28	37
		45	347	392		
66	20				101.02	27
			335			

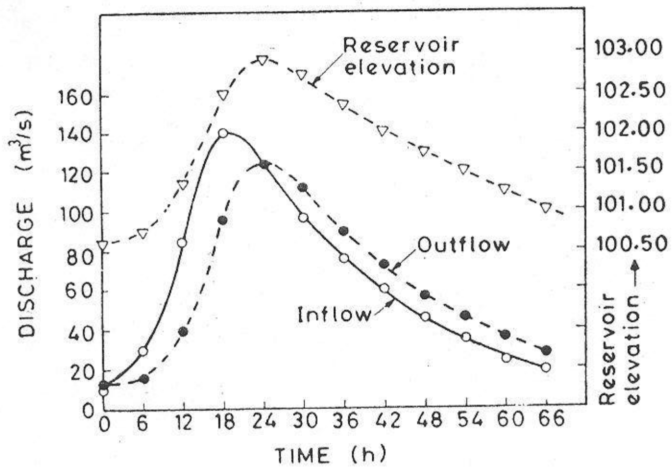


Fig. 8.6 Results of reservoir routing—Example 8.2

8.4 ATTENUATION

Figures 8.3 and 8.6 show the typical result of routing a flood hydrograph through a reservoir. Owing to the storage effect, the peak of the outflow hydrograph will be smaller than that of the inflow hydrograph. This reduction in the peak value is called *attenuation*. Further, the peak of the outflow occurs after the peak of the inflow; the time difference between the two peaks is known as *lag*. The attenuation and lag of a flood hydrograph at a reservoir are two very important aspects of a reservoir operating under a flood-control criteria. By judicious management of the initial reservoir level at the time of arrival of a critical flood, considerable attenuating of the floods can be achieved. The storage capacity of the reservoir and the characteristics of spillways and other outlets control the lag and attenuation of an inflow hydrograph.

In Figs. 8.3 and 8.6 in the rising part of the outflow curve where the inflow curve is higher than the outflow curve, the area between the two curves indicate the accumulation of flow as storage. In the falling part of the outflow curve, the outflow curve is higher than the inflow curve and the area between the two indicate depletion from the storage. When the outflow from a storage reservoir is uncontrolled, as in a freely operating spillway, the peak of the outflow hydrograph will occur at the point of intersection of the inflow and outflow curves (Fig. 8.3).

8.5 HYDROLOGIC CHANNEL ROUTING

In reservoir routing presented in the previous sections, the storage was a unique function of the outflow discharge, $S=f(Q)$. However, in channel routing the storage is a function of both outflow and inflow discharges and hence a different routing method is needed. The flow in a river during a flood belongs to the category of gradually varied unsteady flow. The water surface in a channel reach is not only not parallel to the channel bottom but also varies with time (Fig. 8.7). Considering a channel reach having a flood flow, the total volume in storage can be considered under two categories as:

1. Prism storage, and
2. wedge storage.

Prism Storage

It is the volume that would exist if uniform flow occurred at the downstream depth, i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge Storage

It is the wedge-like volume formed between the actual water surface profile and the top surface of the prism storage.

At a fixed depth at a downstream section of a river reach the prism storage is constant while the wedge storage changes from a positive value

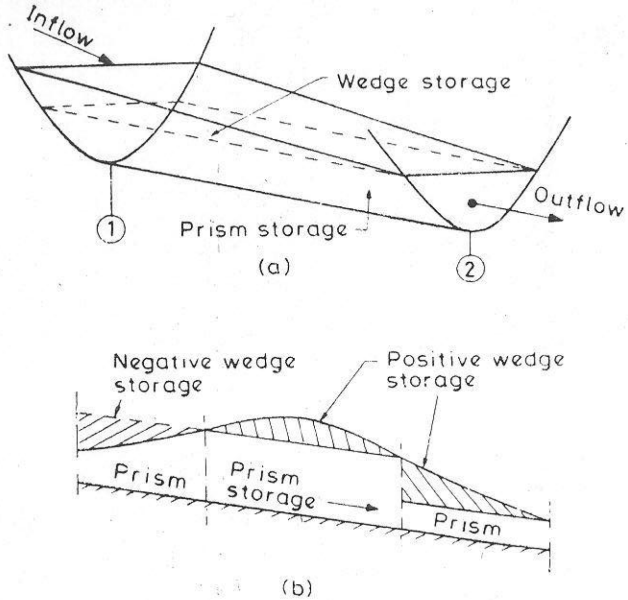


Fig. 8.7 Storage in a channel reach

at an advancing flood to a negative value during a receding flood. The prism storage S_p is similar to a reservoir and can be expressed as a function of the outflow discharge, $S_p = f(Q)$. The wedge storage S_w can be accounted for by expressing it as $S_w = f(I)$. The total storage in the channel reach can then be expressed as

$$S = K[x\, I^m + (1-x)\, Q^m] \qquad (8.8)$$

where K and x are coefficients and m = a constant exponent. It has been found that the value of m varies from 0.6 for rectangular channels to a value of about 1.0 for natural channels.

Muskingum Equation

Using $m = 1.0$, Eq. (8.8) reduces to a linear relationship for S in terms of I and Q as

$$S = K\,[x\, I + (1-x)\, Q] \qquad (8.9)$$

and this relationship is known as the *Muskingum equation*. In this the parameter x is known as *weighting factor*. When $x = 0$, obviously the storage is a function of discharge only and the Eq. (8.9) reduces to

$$S = K\,Q \qquad (8.10)$$

Such a storage is known as *linear storage* or *linear reservoir*. When $x = 0.5$ both the inflow and outflow are equally important in determining the storage.

The coefficient K is known as *storage-time constant* and has the dimensions of time. It is approximately equal to the time of travel of a flood wave through the channel reach.

Estimation of K and x

Figure 8.8 shows a typical inflow and outflow hydrograph through a channel reach. Note that the outflow peak does not occur at the point of intersection of the inflow and outflow hydrographs. Using the continuity equation [Eq. (8.3)],

$$(I_1 + I_2)\,\frac{\Delta t}{2} - (Q_1 + Q_2)\,\frac{\Delta t}{2} = \Delta S$$

the increment in storage at any time t and time element Δt can be calculated. Summation of the various incremental storage values enable one to find the channel storage S vs time relationship (Fig. 8.8).

If an inflow and outflow hydrograph set is available for a given reach, values of S at various time intervals can be determined by the above

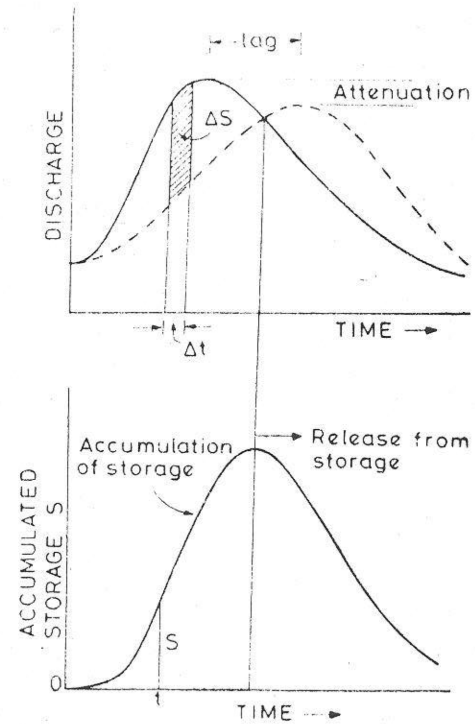


Fig. 8.8 Hydrographs and storage in channel routing

technique. By choosing a trial value of x, values of S at any time t are plotted against the corresponding $[x\,I + (1-x)\,Q]$ values. If the value of x is chosen correctly, a straight-line relationship as given by Eq. (8.9) will result. However, if an incorrect value of x is used, the plotted points will trace a looping curve. By trial and error, a value of x is so chosen that the data very nearly describe a straight line (Fig. 8.9). The inverse slope of this straight line will give the value of K.

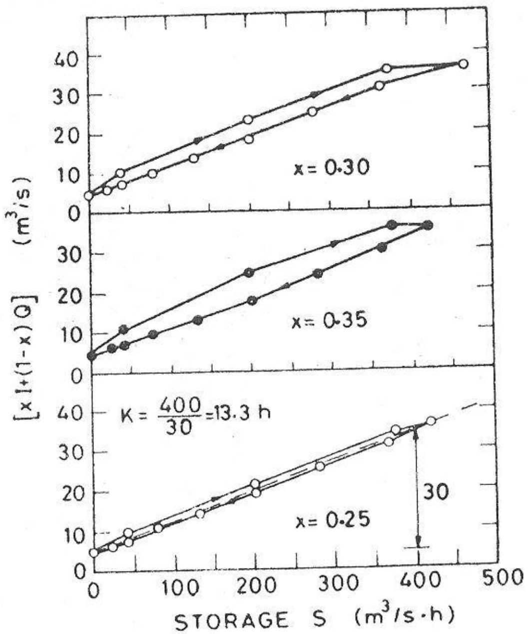


Fig. 8.9 Determination of K and x for a channel reach

Normally, for natural channels, the value of x lies between 0 to 0.3. For a given reach, the values of x and K are assumed to be constant.

EXAMPLE 8.3: *The following inflow and outflow hydrographs were observed in a river reach. Estimate the values of* K *and* x *applicable to this reach for use in the Muskinghum equation.*

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	5	20	50	50	32	22	15	10	7	5	5	5
Outflow (m^3/s)	5	6	12	29	38	35	29	23	17	13	9	7

Using a time increment $\Delta t = 6$ h, the calculations are performed in a tabular manner as in Table 8.3. The incremental storage ΔS and S are calculated in columns 6 and 7 respectively. It is advantageous to use the units $[(m^3/s).h]$ for storage terms.

TABLE 8.3 DETERMINATION OF K AND x—EXAMPLE 8.3

$\Delta t = 6$ h, Storage in (m^3/s). h

Time (h)	I (m^3/s)	Q (m^3/s)	$(I-Q)$	Average $(I-Q)$	ΔS = Col. 5 $\times \Delta t$ $\left(\frac{m^3}{s} h\right)$	$S = \Sigma \Delta S$ $\left(\frac{m^3}{s} h\right)$	$[xI + (1-x)\,Q]$ (m^3/s) $x = 0.35$	$x = 0.30$	$x = 0.25$
1	2	3	4	5	6	7	8	9	10
0	5	5	0			0	5.0	5.0	5.0
				7.0	42				
6	20	6	14			42	10.9	10.2	9.5
				26.0	156				
12	50	12	38			198	25.3	23.4	21.5
				29.5	177				
18	50	29	21			375	36.4	35.3	34.3
				7.5	45				
24	32	38	−6			420	35.9	36.2	36.5
				−9.5	−57				
30	22	35	−13			363	30.5	31.1	31.8
				−13.5	−81				
36	15	29	−14			282	24.1	24.8	25.5
				−13.5	−81				
42	10	23	−13			201	18.5	19.1	19.8
				−11.5	−69				
48	7	17	−10			132	13.5	14.0	14.5
				−9.0	−54				
54	5	13	−8			78	10.2	10.6	11.0
				−6.0	−36				
60	5	9	−4			42	7.6	7.8	8.0
				−3.0	−18				
66	5	7	−2			24	6.3	6.4	6.5

As a first trial $x = 0.35$ is selected and the value of $[x\,I+(1-x)\,Q]$ evaluated (column s) and plotted against S in Fig. 8.9. Since a looped curve is obtained, further trials are performed with $x=0.30$ and 0.25. It is seen from Fig. 8.9 that for $x = 0.25$ the data very nearly describe a straight line and as such $x = 0.25$ is taken as the appropriate value for the reach.

From Fig. 8.9, $K = 13.3$ h

Muskingum Method of Routing

For a given channel reach by selecting a routing interval Δt and using the Muskingum equation, the change in storage is

$$S_2 - S_1 = K\,[x\,(I_2 - I_1) + (1-x)\,(Q_2 - Q_1)] \tag{8.11}$$

where suffixes 1 and 2 refer to the conditions before and after the time interval Δt. The continuity equation for the reach is

$$S_2 - S_1 = \left(\frac{I_2 + I_1}{2}\right)\Delta t - \left(\frac{Q_2 + Q_1}{2}\right)\Delta t \tag{8.12}$$

From Eqs (8.11) and (8.12), Q_2 is evaluated as

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \quad (8.13)$$

where $C_0 = \frac{-Kx + 0.5 \Delta t}{K - Kx + 0.5 \Delta t}$

$$C_1 = \frac{Kx + 0.5 \Delta t}{K - Kx + 0.5 \Delta t}$$

$$C_2 = \frac{K - Kx - 0.5 \Delta t}{K - Kx + 0.5 \Delta t}$$

Note that $C_0 + C_1 + C_2 = 1$. Equation (8.13) provides a simple linear equation for channel routing. It has been found that best results the routing interval should be so chosen that $K > \Delta t > 2\,Kx$. If $\Delta t < 2\,Kx$, the coefficient C_0 will be negative.

EXAMPLE 8.4: *Route the following hydrograph through a river reach for which K = 12.0 h and x = 0.20. At the start of the inflow flood, the outflow discharge is 10 m³/s.*

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m³/s)	10	20	5)	60	55	45	35	27	20	15

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

TABLE 8.4 MUSKINGUM METHOD OF ROUTING—EXAMPLE 8.4

$\Delta t = 6$ h

Time (h)	I (m³/s)	$0.048\, I_2$	$0.429\, I_1$	$0.523\, Q_1$	Q (m³/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2·88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04

The calculations are shown in Table 8.4. For the first time interval, 0 to 6 h,

$I_1 = 10.0 \qquad C_1 I_1 = 4.29$

$I_2 = 20.0 \qquad C_0 I_2 = 0.96$

$Q_1 = 10.0 \qquad C_2 Q_1 = 5.23$

From Eq. (8.13), $\qquad Q_2 = 10.48$ m³/s

For the next time step, 6 to 12 h, $Q_1 = 10.48$ m³/s. The procedure is repeated for the duration of the inflow hydrograph. By plotting the inflow and outflow hydrographs the attenuation and peak lag are found to be 10 m³/s and 12 h respectively.

Alternative Form

Equations (8.11) and (8.12) can be combined in an alternative form of the routing equation as

$$Q_2 = Q_1 + B_1 (I_1 - Q_1) + B_2 (I_2 - I_1) \quad (8.14)$$

where $B_1 = \frac{\Delta t}{K(1-x) + 0.5 \Delta t}$

$$B_2 = \frac{0.5 \Delta t - Kx}{K(1-x) + 0.5 \Delta t}$$

The use of Eq. (8.14) is essentially the same as that of Eq. (8.13).

8.6 HYDRAULIC METHOD OF FLOOD ROUTING

The hydraulic method of flood routing is essentially a solution of the basic St Venant equations [Eq's (8.4) and (8.5)]. These equations are simultaneous, quasi-linear, first-order partial differential equations of the hyperbolic type and are not amenable to general analytical solutions. Only for highly simplified cases can one obtain the analytical solution of these equations. The development of modern, high-speed digital computers during the past two decades has given rise to the evolution of many sophisticated numerical techniques. The various numerical methods for solving St Venant equations can be broadly classified into two categories:

1. Approximate methods, and
2. complete numerical methods.

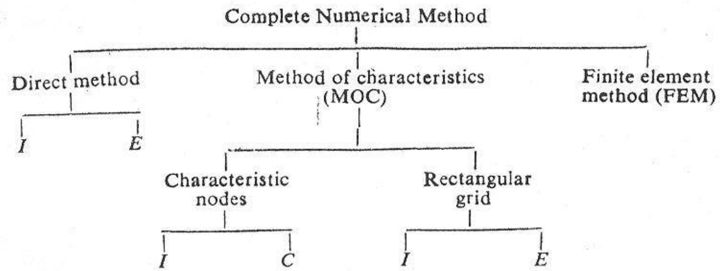
Approximate Methods

These are based on the equation of continuity only or on a drastically curtailed equation of motion. The hydrological method of storage routing and Muskingum channel routing discussed earlier belong to this category.

Other methods in this category are diffusion analogy and kinematic wave models.

Complete Numerical Methods

These are the essence of the hydraulic method of routing and are classified into many categories as below:



I = Implicit method, E = Explicit method

In the direct method, the partial derivatives are replaced by finite differences and the resulting algebraic equations are then solved. In the method of characteristics (MOC) St Venant equations are converted into a pair of ordinary differential equations (i.e. characteristic forms) and then solved by finite-difference techniques. In the finite element method (FEM) the system is divided into a number of elements and partial differential equations are integrated at the nodal points of the elements.

The numerical schemes are further classified into explicit and implicit methods. In the explicit method the algebraic equations are linear and the dependent variables are extracted explicitly at the end of each time step. In the implicit method the dependent variables occur implicitly and the equations are nonlinear. Each of these two methods have a host of finite-differencing schemes to choose from. Details of hydraulic flood routing and a bibliography of relevant literature are available in Refs 5, 6 and 7.

8.7 ROUTING IN CONCEPTUAL HYDROGRAPH DEVELOPMENT

Even though the routing of floods through a reservoir or channel discussed in the previous section were developed for field use, they have found another important use in the conceptual studies of hydrographs. The routing through a reservoir which gives attenuation and channel routing which gives translation to an input hydrograph are treated as two basic modifying operators. The following two fictitious items are used in the studies for development of synthetic hydrographs through conceptual models:

1. *Linear reservoir*: a reservoir in which the storage is directly proportional to the discharge, ($S = KQ$). This element is used to provide attenuation to a flood wave.
2. *Linear channel*: a fictitious channel in which the time required to translate a discharge Q through a given reach is constant. An inflow hydrograph passes through such a channel with only translation and no attenuation.

Conceptual modelling for IUH development has undergone rapid progress since the first work by Zoch (1937). Detailed reviews of various contributions to this field are available in Refs 2 and 3 and the details are beyond the scope of this book. However, one simple method, viz, Clark's method (1945) which utilizes the Muskingum method of routing through a linear reservoir is indicated below as a typical example of the use of routing in conceptual models.

8.8 CLARK'S METHOD FOR IUH

Clark's method, also known as *Time-area histogram* method aims at developing an IUH due to an instantaneous rainfall excess over a catchment. It is assumed that the rainfall excess first undergoes pure translation and then attenuation. The translation is achieved by a travel time-area histogram and the attenuation by routing the results of the above through a linear reservoir at the catchment outlet.

Time-Area Curve

Time here refers to the time of concentration. As defined earlier in Sec. 7.2, the time of concentration t_c is the time required for a unit volume of water from the farthest point of catchment to reach the outlet. It represents the maximum time of translation of the surface runoff of the catchment. In gauged areas the time interval between the end of the rainfall excess and the point of inflection of the resulting surface runoff (Fig. 8.10) provides a good way of estimating t_c from known rainfall-runoff data. In ungauged areas the empirical formulae Eq. (7.3) or (7.4) can be used to estimate t_c.

The total catchment area drains into the outlet in t_c hours. If points on the area having equal time of travel, (say t_1 h where $t_1 < t_c$), are considered and located on a map of the catchment, a line joining them is called an *Isochrone* (or *runoff isochrone*). Figure (8.11) shows a catchment being divided into $N (= 8)$ subareas by isochrones having an equal time interval. To assist in drawing isochrones, the longest water course is chosen and its profile plotted as elevation vs distance from the outlet; the distance is then divided into N parts and the elevations of the subparts measured on the profile transferred to the contour map of the catchment.

The inter-isochrone areas $A_1, A_2, \ldots, A_N$ are used to construct a travel time-area histogram (Fig. 8.12). If a rainfall excess of 1 cm occurs instantaneously and uniformly over the catchment area, this time-area

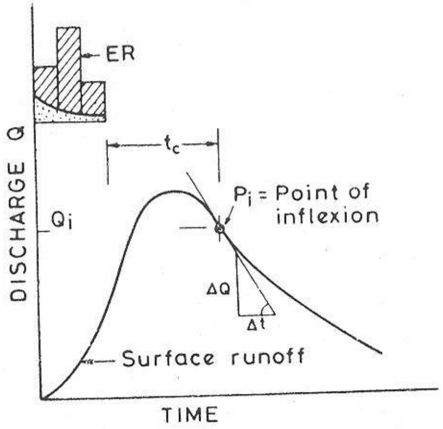


Fig. 8.10 Surface runoff of a catchment

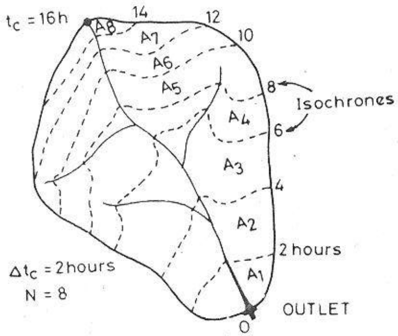


Fig. 8.11 Isochrones in a catchment

histogram represents the sequence in which the volume of rainfall will be moved out of the catchment and arrive at the outlet. In Fig. 8.12, a sub-area A_r km² represents a volume of A_r km. cm = $A_r \times 10^4$ m³/s moving out in time $\Delta t_c = t_c/N$ hrs. The hydrograph of outflow obtained by this figure while properly accounting for the sequence of arrival of flows, do not provide for the storage properties of the catchment. To overcome this deficiency, Clark assumed a linear reservoir to be hypothetically available at the outlet to provide the requisite attenuation.

Routing

The linear reservoir at the outlet is assumed to be described by $S = KQ$,

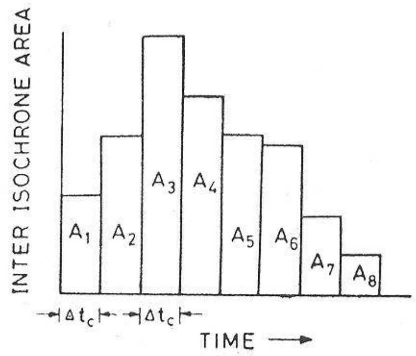


Fig. 8.12 Time-area histogram

where K is the storage time constant. The value of K can be estimated by considering the point of inflection P_i of a surface runoff hydrograph (Fig. 8.10). At this point the inflow into the channel has ceased and beyond this point the flow is entirely due to withdrawal from the channel storage. The continuity equation

$$I - Q = \frac{dS}{dt}$$

becomes $$-Q = \frac{dS}{dt} = -K\frac{dQ}{dt} \quad \text{(by Eq. 8.10)}$$

Hence $$K = -Q_i/(dQ/dt)_i \tag{8.15}$$

where suffix i refers to the point of inflection, (Fig. 8.10), and K can be estimated from a known surface runoff hydrograph of the catchment. The constant K can also be estimated from the data on the recession limb of a hydrograph (Sec. 6.3).

Knowing K of the linear reservoir, the inflows at various times are routed by the Muskingum method. Note that since a linear reservoir is used $x = 0$ in Eq. (8.9). The inflow rate between an inter-isochrone area A_r km² with a time interval Δt_c (h) is

$$I = \frac{A_r \times 10^4}{3600\,\Delta t_c} = 2.78\,\frac{A_r}{\Delta t_c}\ \text{m}^3/\text{s}$$

The Muskingum routing equation would now be by Eq. (8.13),

$$Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \tag{8.16}$$

where
$$C_0 = (0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$$
$$C_1 = (0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$$
$$C_2 = (K - 0.5\,\Delta t_c)/(K + 0.5\,\Delta t_c)$$

i.e. $C_0 = C_1$. Also since the inflows are derived from the histogram $I_1 = I_2$ for each interval. Thus Eq. (8.16) becomes

$$Q_2 = 2C_1 I_1 + C_2 Q_1 \quad (8.17)$$

Routing of the time-area histogram by Eq. (8.17) gives the ordinates of IUH for the catchment. Using this IUH any other D-h unit hydrograph can be derived.

EXAMPLE 8.5: *A drainage basin has the following characteristics: Area = 110 km², time of concentration = 18 h, storage constant = 12 h and inter-isochrone area distribution as below:*

Travel time t (h)	0-2	2-4	4-6	6-8	8-10	10-12	12-14	14-16	16-18
Inter-Iso-chrone area (km²)	3	9	20	22	16	18	10	8	4

Determine the IUH for this catchment.

$K = 12$ h, $\quad t_c = 18$ h, $\quad \Delta t_c = 2$ h

$$C_1 = \frac{0.5 \times 2}{12 + 0.5 \times 2} = 0.077$$

$$C_2 = \frac{12 - 0.5 \times 2}{12 + 0.5 \times 2} = 0.846$$

Equation (8.17) becomes $Q_2 = 0.154\, I_1 + 0.846\, Q_1$
= Ordinate of IUH

At $t = 0$, $Q_1 = 0$,

$I_1 = 2.78\, A_r/2 = 1.39\, A_r$ m³/s

The calculations are shown in Table 8.5.

8.9 FLOOD CONTROL

The term "flood control" is commonly used to denote all the measures adopted to reduce damages to life and property by floods. As there is always a possibility, however remote it may be, of an extremely large flood occurring in a river the complete control of the flood to a level of zero loss is neither physically possible nor economically feasible. The flood control measures that are in use can be classified as:

1. Structural methods:
   (i) Storage and detention reservoirs,
   (ii) levees (flood embankments),
   (iii) channel improvement,

TABLE 8.5 CALCULATIONS OF IUH—EXAMPLE 8.5

Time (h)	Area A_r (km²)	I (m³/s)	0.154 I_1	0.846 Q_1	Ordinate of IUH (m³/s)
1	2	3	4	5	6
0	0	0	0	0	0
2	3	4.17	0.64	0	0.64
4	9	12.51	1.93	0.54	2.47
6	20	27.80	4.28	2.09	6.37
8	22	30.58	4.71	5.39	10.10
10	16	22.24	3.42	8.54	11.96
12	18	25.02	3.85	10.12	13.97
14	10	13.90	2.14	11.82	13.96
16	8	11.12	1.71	11.81	13.52
18	4	5.56	0.86	11.44	12.30
20	0	0	0	10.40	10.40
22				8.80	8.80
24				7.45	7.45
26				6.30	6.30
28				5.30	5.30
				:	:
				so on	so on

   (iv) flood ways (new channels), and
   (v) soil conservation.
2. Non-structural methods:
   (i) Flood plain zoning, and
   (ii) flood warning and evacuation.

Storage Reservoirs

Storage reservoirs offer one of the most reliable and effective methods of flood control. Ideally, in this method, a part of the storage in the reservoir is kept apart to absorb the incoming flood. Further, the stored water is released in a controlled way over an extended time so that downstream channels do not get flooded. Figure 8.13 shows an ideal operating plan of a flood control reservoir. As most of the present-day storage reservoirs have multipurpose commitments, the manipulation of reservoir levels to satisfy many conflicting demands is a very difficult and complicated task. It so happens that many storage reservoirs while reducing the floods and flood damages do not always aim at achieving optimum benefits in the flood-control aspect. To achieve complete flood control in the entire length

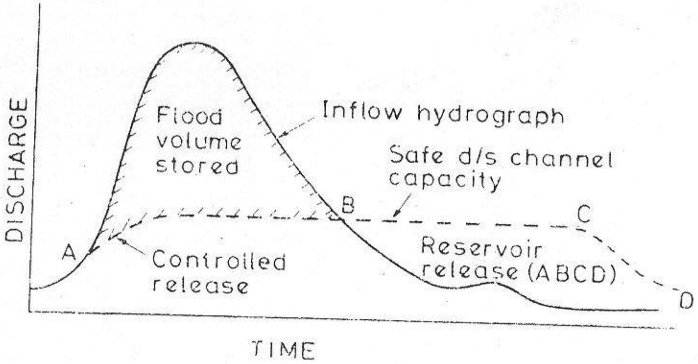


Fig. 8.13 Flood control operation of a reservoir

of a river, a large number of reservoirs at strategic locations in the catchment will be necessary.

The Hirakud and Damodar Valley Corporation (DVC) reservoirs are examples of major reservoirs in the country which have specific volumes earmarked for flood absorption.

Detention Reservoirs

A detention reservoir consists of an obstruction to a river with an uncontrolled outlet. These are essentially small structures and operate to reduce the flood peak by providing temporary storage and by restriction of the outflow rate. These structures are not common in India.

Levees

Levees, also known as *dikes* or *flood embankments* are earthen banks constructed parallel to the course of the river to confine it to a fixed course and limited cross-sectional width. The heights of levees will be higher than the design flood level with sufficient free board. The confinement of the river to a fixed path frees large tracts land from inundation and consequent damage (Fig. 8.14).

Levees are one of the oldest and most common methods of flood-protection works adopted in the world. Also, they are probably the cheapest of structural flood-control measures. While the protection offered by a levee against flood damage is obvious, what is not often appreciated is the potential damage in the event of a levee failure. The levees, being earth embankments require considerable care and maintenance. In the event of being overtopped, they fail and the damage caused can be enormous. In fact, the sense of protection offered by a levee encourages economic activity along the embankment and if the levee is overtopped the loss would be more than what would have been if there were no levees. Confinement of flood banks of a river by levees to a narrower space leads to higher flood levels for a given discharge. Further, if the bed levels of the river also

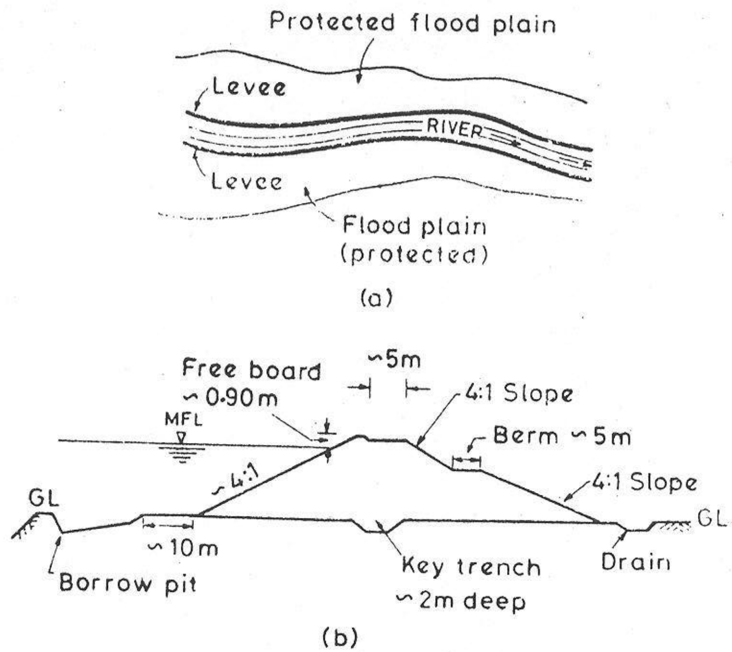


Fig. 8.14 **A typical levee: (a) Plan (schematic)**
(b) Cross-section

rises, as they do in agrading rivers, the top of the levees have to be raised at frequent time intervals to keep up its safety margin.

The design of a levee is a major task in which costs and economic benefits have to be considered. The cross-section of a levee will have to be designed like an earth dam for complete safety against all kinds of saturation and drawdown possibilities. In many instances, especially in locations where important structures and industries are to be protected, the water side face of levees are protected by stone or concrete revetment. Regular maintenance and contingency arrangements to fight floods are absolutely necessary to keep the levees functional.

Masonry structures used to confine the river in a manner similar to levees are known as *flood walls*. These are used to protect important structures against floods, especially where the land is at a premium.

Floodways

Floodways are natural channels into which a part of the flood will be diverted during high stages. A flood way can be a natural or man-made channel and its location is controlled essentially by the topography. Generally, wherever they are feasible, floodways offer an economical alternative to other structural flood-control measures. To reduce the level of the river Jhelum at Srinagar, a supplementary channel has been constructed to act as a floodway with a capacity of 300 m^3/s. This channel is located 5 km upstream of Srinagar city and has its outfall in lake Wullar.

Channel Improvement

The works under this category involve:

1. Widening or deepening of the channel to increase the cross-sectional area;
2. reduction of the channel roughness, by clearing of vegetation from the channel perimeter;
3. short circuiting of meander loops by cutoff channels, leading to increased slopes.

All these three methods are essentially short-term measures and require continued maintenance.

Soil Conservation

Soil-conservation measures in the catchment when properly planned and effected lead to an all-round improvement in the catchment characteristics affecting abstractions. Increased infiltration, greater evapotranspiration and reduced soil erosion are some of its easily identifiable results. It is believed that while small and medium floods are reduced by soil-conservation measures, the magnitude of extreme floods are unlikely to be affected by these measures.

8.10 FLOOD FORECASTING

Forecasting of floods in advance enables a warning to be given to the people likely to be affected and further enables civil-defence measures to be organized. It thus forms a very important and relatively inexpensive nonstructural flood-control measure. However, it must be realised that a flood warning is meaningful if it is given sufficiently in advance. Also, erroneous warnings will cause the populace to loose faith in the system. Thus the dual requirements of reliability and advance notice are the essential ingredients of a flood-forecasting system.

The flood forecasting techniques can be broadly divided into three categories:

1. Short-range forecasts,
2. medium-range forecasts, and
3. long-range forecasts.

Short-Range Forecasts

In this the river stages at successive stations on a river are correlated with hydrological parameters, such as precipitation over the local area, antecedent precipitation index, and variation of the stage at the upstream base point during the travel time of a flood. This gives an advance warning of 12–40 h for floods. The flood forecasting currently being used for the metropolitan city of Delhi is based on this technique.

Medium-Range Forecasts

In this rainfall–runoff relationships are used to predict flood levels with a warning of 2–5 days. Coaxial graphical correlations of runoff with rainfall and other parameters like the time of the year, storm duration and antecedent wetness have been developed to a high stage of refinement by the US Weather Bureau.

Long-Range Forecasts

Using radars and meteorological satellite data, advance information about critical storm-producing weather systems, their rain potential and time of occurrence of the event are predicted well in advance.

8.11 FLOOD CONTROL IN INDIA

In India the Himalayan rivers account for nearly 60% of the flood damage in the country. Floods in these rivers occur during monsoon months and usually in the months of August or September. The damages caused by floods are very difficult to estimate and a figure of Rs 10 billion (1000 crores) as the annual flood damage in the country gives the right order of magnitude. It is estimated that annually on an average 40 Mha of land are liable to flooding and of this about 12 Mha have some kind of flood-control measure. There are about 12,500 km of levees and about 25,600 km drainage channels affording protection from floods. About 60 to 80% of flood damages occur in the states of UP, Bihar, West Bengal, Assam and Orissa.

Flood forecasting is handled by CWC in close collaboration with the IMD, which lends meteorological data support. Nine flood Met offices with the aid of recording raingauges provide daily synoptic situations, actual rainfall amounts and rainfall forecasts to CWC. The CWC has 141 flood-forecasting stations situated on various basins to provide a forecasting service to a population of nearly 40 million.

A national programme for flood control was launched in 1954 and an amount of about 976 crores has been spent since then till the beginning of the Sixth Five-Year Plan. The Planning Commission has provided an outlay of 1045 crores in the sixth Five-Year Plan for flood control. These figures highlight the seriousness of the flood problem and the efforts made towards mitigating flood damages. The experience gained in the flood control measures in the country are embodied in the report of the Rashtriya Barh Ayog (RBA) (National Flood Commission) submitted in

March 1980. This report, containing ...
on all aspects of flood control forms the basis for the evolution of the present national policy on floods.

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PROBLEMS

8.1 The storage, elevation and outflow data of a reservoir are given below:

Elevation (m)	Storage $10^6 \times m^3$	Outflow discharge (m^3/s)
299.50	4.8	0
300.20	5.5	0
300.70	6.0	15
301.20	6.6	40
301.70	7.2	75
302.20	7.9	115
302.70	8.8	160

The spillway crest is at elevation 300.20. The following flood flow is expected into the reservoir.

Time (h)	0	3	6	9	12	15	18	21	24	27
Discharge (m^3/s)	10	20	52	60	53	43	32	22	16	10

If the ... route the flood to obtain (a) the outflow hydrograph and (b) the reservoir elevation vs time curve.

8.2 Solve Prob. 8.1 if the reservoir elevation at the start of the inflow hydrograph is at 301.50.

8.3 A small reservoir has the following storage elevation relationship.

Elevation (m)	55.00	58.00	60.00	61.00	62.00	63.00
Storage ($10^3 \times m^3$)	250	650	1000	1250	1500	1800

A spillway provided with its crest at elevation 60.00 has the discharge relationship $Q = 15H^{3/2}$, where H = head of water over the spillway crest. When the reservoir elevation is at 58.00; a flood hydrograph given below enters the reservoir. Route the flood and determine the maximum reservoir elevation, peak outflow and attenuation of the flood peak.

Time (hrs)	0	6	12	15	18	24	30	36	42
Inflow (m^3/s)	5	20	50	60	50	32	22	15	10

8.4 The storage–elevation–discharge characteristic of a reservoir is as follows:

Elevation (m)	100.00	100.50	101.00
Discharge (m^3/s)	12	18	25
Storage ($10^3 \times m^3$)	400	450	550

When the reservoir elevation is at 101.00, the inflow is at a constant rate of 10 m^3/s. Find the time taken for the water surface to drop to the elevation 100.00.

8.5 A small reservoir has a spillway crest at elevation 200.00. Above this elevation. the storage and outflow from the reservoir can be expressed as

Storage: $S = 36000 + 18000\,y$ (m^3)

Outflow: $Q = 10\,y$ (m^3/s)

where y = height of the reservoir level above the spillway crest in m.

Route an inflow flood hydrograph which can be approximated by a triangle as

$I = 0$ at $t = 0$ h

$I = 30\ m^3/s$ at $t = 6$ h (peak flow)

$I = 0$ at $t = 26$ h (end of inflow).

Assume the reservoir elevation as 200.00 at $t = 0$ h.

Use a time step of 2 h.

8.6 Observed values of inflow and outflow hydrographs at the ends of a reach in a river are given below. Determine the best values of K and x for use in the Muskingum method of flood routing.

March 1980. This report, containing a large number of recommendations on all aspects of flood control forms the basis for the evolution of the present national policy on floods.

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Time (h)	0	3	6	9	12	15	18	21	24	27
Discharge (m^3/s)	10	20	52	60	53	43	32	22	16	10

If the reservoir surface is at elevation 300.00 at the commencement of the inflow, route the flood to obtain (a) the outflow hydrograph and (b) the reservoir elevation vs time curve.

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When the reservoir elevation is at 101.00, the inflow is at a constant rate of 10 m^3/s. Find the time taken for the water surface to drop to the elevation 100.00.

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where y = height of the reservoir level above the spillway crest in m.

Route an inflow flood hydrograph which can be approximated by a triangle as

$I = 0$ at $t = 0$ h

$I = 30$ m^3/s at $t = 6$ h (peak flow)

$I = 0$ at $t = 26$ h (end of inflow).

Assume the reservoir elevation as 200.00 at $t = 0$ h.

Use a time step of 2 h.

8.6 Observed values of inflow and outflow hydrographs at the ends of a reach in a river are given below. Determine the best values of K and x for use in the Muskingum method of flood routing.

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66
Inflow (m^3/s)	20	80	210	240	215	170	130	90	60	40	28	16
outflow (m^3/s)	20	20	50	150	200	210	185	155	120	85	55	23

8.7 Route the following flood through a reach for which $K = 22$ h and $x = 0.25$. Plot the inflow and outflow hydrographs and determine the peak lag and attenuation. At $t = 0$ the outflow discharge is 40 m^3/s.

Time (h)	0	12	24	36	48	60	72	84	96	108	120	132	144
Inflow (m^3/s)	40	65	165	250	240	205	170	130	115	85	70	60	54

8.8 A stream has a uniform flow of 10 m^3/s. A flood in which the discharge increases linearly from 10 m^3/s to a peak of 70 m^3/s in 6 h and then decreases linearly to a value of 10 m^3/s in 24 h from the peak arrives at a reach. Route the flood through the reach in which $K = 10$ h and $x = 0$.

8.9 A drainage basin has area = 137 km^2, storage constant $K = 9.5$ h and time of concentration = 7 h. The following inter-isochrone area distribution data are available:

Time (h)	0-1	1-2	2-3	3-4	4-5	5-6	6-7
Inter-isochrone area (km^2)	10	38	20	45	32	10	2

Determine (a) the IUH and (b) the 1-h unit hydrograph for the catchment.

8.10 Solve Prob. 8.8 with $K = 10$ h and $x = 0.5$. Determine the peak lag and attenuation and compare with the corresponding values of Prob. 8.8.

8.11 Show that the reservoir routing equation for a linear reservoir is

$$\frac{dQ}{dt} + \alpha Q = \alpha I$$

where α is a constant. Obtain the outflow from such a reservoir due to an inflow $I = I_0 + \beta t$ occurring from $t = 0$ to t_0 with the boundary condition $Q = 0$ at $t = 0$.

QUESTIONS

8.1 The hydrologic flood-routing methods use
(a) equation of continuity only
(b) equation of motion only
(c) both momentum and continuity equations
(d) energy equation only

8.2 The hydraulic methods of flood routing use
(a) equation of continuity only
(b) equation of motion only
(c) both the equation of motion and equation of continuity
(d) energy equation only

8.3 The St Venant equations for unsteady open-channel flow are
(a) continuity and momentum equations
(b) momentum equation in two different forms
(c) momentum and energy equations
(d) energy equation.

8.4 In routing a flood through a reach the point of intersection of inflow and outflow hydrographs coincides with the peak of outflow hydrograph
(a) in all cases of flood routing
(b) when the inflow is into a reservoir with an uncontrolled outlet
(c) in channel routing only
(d) in all cases of reservoir routing.

8.5 Which of the following is a proper reservoir-routing equation ?

(a) $\frac{1}{2}(I_1 - I_2)\Delta t + \left(S_1 + \frac{Q_1 \Delta t}{2}\right) = \left(S_2 - \frac{Q_2 \Delta t}{2}\right)$

(b) $(I_1 + I_2)\Delta t + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right)$

(c) $\frac{1}{2}(I_1 + I_2)\Delta t + \left(S_2 - \frac{Q_2 \Delta t}{2}\right) = \left(S_1 + \frac{Q_1 \Delta t}{2}\right)$

(d) $(I_1 + I_2) + \left(\frac{2S_1}{\Delta t} - Q_1\right) = \left(\frac{2S_2}{\Delta t} + Q_2\right)$.

8.6 The Muskingum method of flood routing is a
(a) form of reservoir routing method
(b) hydraulic routing method
(c) complete numerical solution of St Venant equations
(d) hydrologic channel-routing method.

8.7 The Muskingum method of flood routing assumes the storage S is related to inflow rate I and outflow rate Q of a reach as $S =$
(a) $K[xI - (1-x)Q]$ (b) $K[xQ + (1-x)I]$
(c) $K[xI + (1-x)Q]$ (d) $Kx[I - (1-x)Q]$.

8.8 The Muskingum method of flood routing gives $Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1$. The coefficients in this equation will have values such that
(a) $C_0 + C_1 = C_2$ (b) $C_0 - C_1 - C_2 = 1$ (c) $C_0 + C_1 + C_2 = 0$
(d) $C_0 + C_1 + C_2 = 1$.

8.9 If the storage S, inflow rate I and outflow rate Q for a river reach is written as
$S = K[x I^n + (1-x) Q^n]$
(a) $n = 0$ represents storage routing through a reservoir
(b) $n = 1$ represents the Muskingum method
(c) $n = 0$ represents the Muskingum method
(d) $n = 0$ represents a linear channel.

8.10 A linear reservoir is one in which
(a) the volume varies linearly with elevation
(b) the storage varies linearly with the outflow rate
(c) the storage varies linearly with time
(d) the storage varies linearly with the inflow rate.

8.11 An isochrone is a line on the basin map
(a) joining raingauge stations with equal rainfall duration
(b) joining points having equal standard time
(c) connecting points having equal time of travel of the surface runoff to the catchment outlet
(d) that connects points of equal rainfall depth in a given time interval.

8.12 In the Muskingum method of channel routing if $x = 0.5$, it represents an outflow hydrograph
(a) that has reduced peak
(b) with an amplified peak
(c) that is exactly the same as the inflow hydrograph
(d) with a peak which is exactly half of the inflow peak,

9 GROUNDWATER

9.1 INTRODUCTION

In the previous chapters various aspects of surface hydrology that deal with surface runoff have been discussed. Study of subsurface flow is equally important since about 22% of the world's fresh water resources exist in the form of groundwater. Further, the subsurface water forms a critical input for the sustenance of life and vegetation in arid zones. Because of its importance as a significant source of water supply, various aspects of groundwater dealing with the exploration, development and utilization have been extensively studied by workers from different disciplines, such as geology, geophysics, geochemistry, agricultural engineering, fluid mechanics and civil engineering and excellent treatises are available[1, 2, 3, 4, 5, 6, 7, 8, 9]. This chapter confines itself to only an elementary treatment of the subject of groundwater as a part of engineering hydrology.

9.2 FORMS OF SUBSURFACE WATER

Water in the soil mantle is called *subsurface water* and is considered in two zones (Fig. 9.1):

1. Saturated zone, and
2. aeration zone.

Saturated Zone

This zone, also known as *groundwater zone* is the space in which all the pores of the soil are filled with water. The water table forms its upper limit and marks a free surface, i.e. a surface having atmospheric pressure.

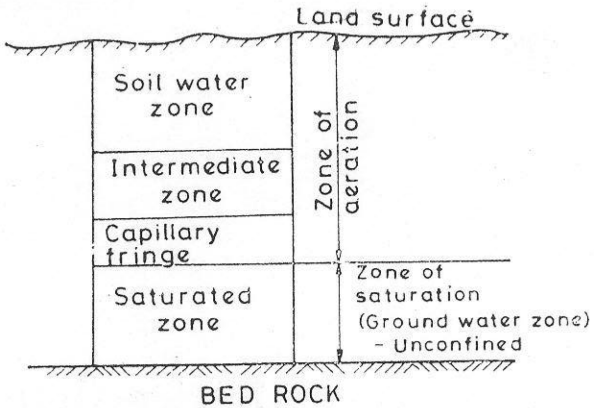

Fig. 9.1 Classification of subsurface water

Zone of Aeration

In this zone the soil pores are only partially saturated with water. The space between the land surface and the water table marks the extent of this zone. Further, the zone of aeration has three subzones:

Soil Water Zone

This lies close to the ground surface in the major root band of the vegetation from which the water is lost to the atmosphere by evapotranspiration.

Capillary Fringe

In this the water is held by capillary action. This zone extends from the water table upwards to the limit of the capillary rise.

Intermediate Zone

This lies between the soil water zone and the capillary fringe.

The thickness of the zone of aeration and its constituent subzones depend upon the soil texture and moisture content and vary from region to region. The soil moisture in the zone of aeration is of importance in agricultural practice and irrigation engineering. The present chapter is concerned only with the saturated zone.

All earth materials, from soils to rocks have pore spaces. Although these pores are completely saturated with water below the water table, from the groundwater utilization aspect only such material through which water moves easily and hence can be extracted with ease are significant. On this basis the saturated formations are classified into four categories:

1. Aquifer,
2. aquitard
3. aquiclude, and
4. aquifuge.

These are discussed below:

Aquifer

An aquifer is a saturated formation of earth material which not only stores water but yields it in sufficient quantity. Thus an aquifer transmits water relatively easily due to its high permeability. Unconsolidated deposits of sand and gravel form good aquifers.

Aquitard

It is a formation through which only seepage is possible and thus the yield is insignificant compared to an aquifer. It is partly permeable.

Aquiclude

It is a geological formation which is essentially impermeable to the flow of water. It may be considered as closed to water movement even though it may contain large amounts of water due to its high porosity. Clay is an example of an aquiclude.

Aquifuge

It is a geological formation which is neither porous nor permeable. There are no interconnected openings and hence it cannot transmit water. Massive compact rock without any fractures is an aquifuge.

The definitions of aquifer, aquitard and aquiclude as above are relative. A formation which may be considered as an aquifer at a place where water is at a premium (e.g. arid zones) may be classified as an aquitard or even aquiclude in an area where plenty of water is available.

The availability of groundwater from an aquifer at a place depends upon the rates of withdrawal and replenishment (recharge). Aquifers play the roles of both a transmission conduit and a storage. Aquifers are classified as unconfined aquifers and confined aquifers on the basis of their occurrence and field situation. An *unconfined aquifer* (also known as *water table aquifer*) is one in which a free surface, i.e. a water table exists (Fig. 9.2). Only the saturated zone of this aquifer is of importance in groundwater studies. Recharge of this aquifer takes place through infiltration of precipitation from the ground surface. A well driven into an unconfined aquifer will indicate a static water level corresponding to the water table level at that location.

A *confined aquifer*, also known as *artesian aquifer*, is an aquifer which is confined between two impervious beds such as aquicludes or aquifuges (Fig. 9.2). Recharge of this aquifer takes place only in the area where it is exposed at the ground surface. The water in the confined aquifer will be under pressure and hence the piezometric level will be much higher than

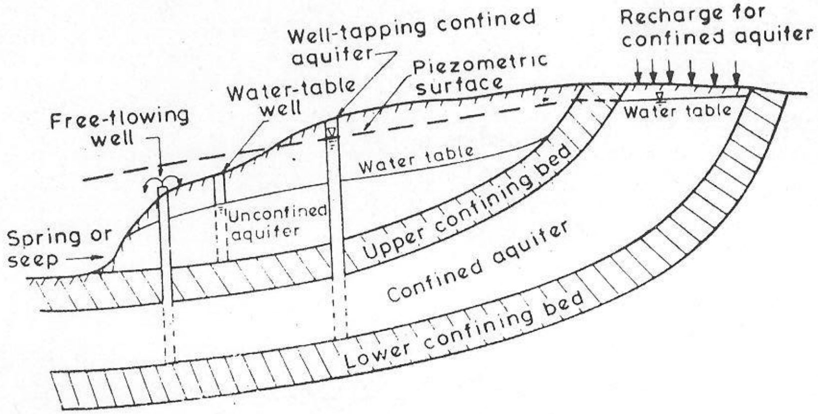

Fig. 9.2 Confined and unconfined acquifers

the top level of the aquifer. At some locations: the piezometric level can attain a level higher than the land surface and a well driven into the aquifer at such a location will flow freely without the aid of any pump. In fact, the term "artesian" is derived from the fact that a large number of such free-flow wells were found in Artois, a former province in north France. Instances of free-flowing wells having as much as a 50-m head at the ground surface are reported.

Water Table

A water table is the free water surface in an unconfined aquifer. The static level of a well penetrating an unconfined aquifer indicates the level of the water table at that point. The water table is constantly in motion adjusting its surface to achieve a balance between the recharge and outflow from the subsurface storage. Fluctuations in the water level in a dug well during various seasons of the year, lowering of the groundwater table in a region due to heavy pumping of the wells and the rise in the water table of an irrigated area with poor drainage, are some common examples of the fluctuation of the water table. In a general sense, the water table follows the topographic features of the surface. If the water table intersects the land surface the groundwater comes out to the surface in the form of *springs* or *seepage*.

Sometimes a lens or localised patch of impervious stratum can occur inside an unconfined aquifer in such a way that it retains a water table above the general water table (Fig. 9.3). Such a water table retained around the impervious material is known as *perched water table*. Usually the perched water table is of limited extent and the yield from such a situation is very small. In groundwater exploration a perched water table is quite often confused with a general water table.

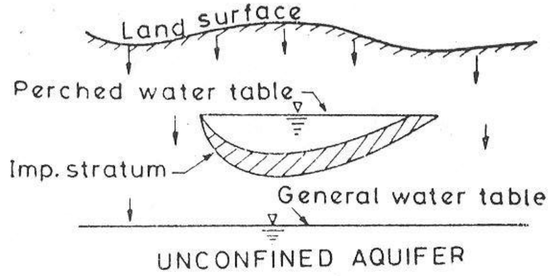

Fig. 9.3 Perched water table

9.3 AQUIFER PROPERTIES

The important properties of an aquifer are its capacity to release the water held in its pores and its ability to transmit the flow easily. These properties essentially depend upon the composition of the aquifer.

Porosity

The amount of pore space per unit volume of the aquifer material is called *porosity*. It is expressed as

$$n = \frac{V_v}{V_o} \tag{9.1}$$

where n = porosity, V_v = volume of voids and V_o = volume of the porous medium. In an unconsolidated material the size distribution, packing and shape of particles determine the porosity. In hard rocks the porosity is dependent on the extent, spacing and the pattern of fracturing or on the nature of solution channels. In qualitative terms porosity greater than 20% is considered as large, between 5 to 20% as medium and less than 5% as small.

Specific Yield

While porosity gives a measure of the water-storage capability of a formation, not all the water held in the pores is available for extraction by pumping or draining by gravity. The pores hold back some water by molecular attraction and surface tension. The actual volume of water that can be extracted by the force of gravity from a unit volume of aquifer material is known as the *specific yield*, S_y. The fraction of water held back in the aquifer is known as *specific retention*, S_r. Thus porosity

$$n = S_y + S_r \tag{9.2}$$

The representative values of porosity and specific yield of some common earth materials are given in Table 9.1.

TABLE 9.1 POROSITY AND SPECIFIC YIELD OF SELECTED FORMATIONS

Formation	Porosity %	Specific yield %
Clay	45–55	1–10
Sand	35–40	10–30
Gravel	30–40	15–30
Sand stone	10–20	5–15
Shale	1–10	0.5– 5
Lime stone	1–10	0.5– 5

It is seen from Table 9.1 that although both clay and sand have high porosity the specific yield of clay is very small compared to that of sand.

Darcy's Law

In 1856 Henry Darcy, a French hydraulic engineer, on the basis of his experimental findings proposed a law relating the velocity of flow in a porous medium. This law, known as Darcy's law, can be expressed as

$$V = K\,i \tag{9.3}$$

where V = Apparent velocity of seepage = Q/A in which Q = discharge and A = cross sectional area of the porous medium. V is sometimes also known as *discharge velocity*. $i = \frac{dh}{ds}$ = hydraulic gradient, in which h = piezometric head and s = distance measured in the general flow direction; the negative sign emphasizes that the piezometric head drops in the direction of flow. K = a coefficient, called *coefficient of permeability*; having the units of velocity.

The discharge Q can be expressed as

$$Q = K\,i\,A \tag{9.3a}$$

$$= K\,A\left(-\frac{\Delta H}{\Delta s}\right)$$

where $(-\Delta H)$ is the drop in the hydraulic grade line in a length Δs of the porous medium.

Darcy's law is a particular case of the general viscous fluid flow. It has been shown valid for laminar flows only. For practical purposes, the limit of the validity of Darcy's law can be taken as Reynolds number of value unity, i.e,

$$\mathrm{Re} = \frac{V d_a}{\nu} = 1 \tag{9.4}$$

where Re = Reynolds number

d_a = representative particle size, usually $d_a = d_{10}$ where d_{10} represents a size such that 10% of the aquifer material is of smaller size.

ν = kinematic viscosity of water

Excepting for flow in fissures and caverns, to a large extent groundwater flow in nature obeys Dacry's law. Further, there is no known lower limit for the applicability of Darcy's law.

It may be noted that the apparent velocity V used in Darcy's law is not the actual velocity of flow through the pores. Owing to irregular pore geometry the actual velocity of flow varies from point to point and the bulk pore velocity (v_a) which represents the actual speed of travel of water in the porous media is expressed as

$$v_a = \frac{V}{n} \tag{9.5}$$

where n = porosity. The bulk pore velocity v_a is the velocity that is obtained by tracking a tracer added to the groundwater.

Coefficient of Permeability

The coefficient of permeability, also designated as *hydraulic conductivity* reflects the combined effects of the porous medium and fluid properties. From an analogy of laminar flow through a conduit (Hagen-Poiseuille flow) the coefficient of permeability K can be expressed as

$$K = C\,d_m^2\,\frac{\gamma}{\mu} \tag{9.6}$$

where d_m = mean particle size of the porous medium $\gamma = \rho g$ = unit weight of fluid. ρ = density of the fluid, g = acceleration due to gravity, μ = dynamic viscosity of the fluid and C = a shape factor which depends on the porosity, packing, shape of grains and grain-size distribution of the porous medium. Thus for a given porous material

$$K :: \frac{1}{\nu}$$

where ν = kinematic viscosity = $\mu/\rho = f$(temperature). The laboratory or *standard value* of the coefficient of permeability (K_s) is taken as that for pure water at a standard temperature of 20°C. The value of K_t, the coefficient of permeability at any temperature t can be converted to K_s by the relation

$$K_s = K_t\,(\nu_t/\nu_s) \tag{9.7}$$

where ν_s and ν_t represent the kinematic viscosity values at 20°C and t°C respectively.

The coefficient of permeability is often considered in two components, one reflecting the properties of the medium only and the other incorporating the fluid properties. Thus, referring to Eq. (9.6), a term K_0 is defined as

$$K = K_0\,\frac{\gamma}{\mu} = K_0\,\frac{g}{\nu} \tag{9.8}$$

where $K_0 = C\, d_m^2$

The parameter K_0 is called *specific* or *intrinsic permeability* which is a function of the medium only. Note that K_0 has dimensions of $[L^2]$. It is expressed in units of cm² or m² or in darcys where 1 darcy $= 9.87 \times 10^{-13}$ m². Where more than one fluid is involved in porous media flow or when there is considerable temperature variation, the coefficient K_0 is useful. However, in groundwater flow problems, the temperature variations are rather small and as such the coefficient of permeability K is more convenient to use. The common units of K are m/day or cm/s. The conversion factor for these two are

$$1 \text{ m/day} = 0.001157 \text{ cm/s}$$

Some typical values of coefficients of permeabilities of some porous media are given in Table 9.2.

TABLE 9.2 REPRESENTATIVE VALUES OF THE PERMEABILITY COEFFICIENTS

No. Material	K (cm/s)	K_0 (darcys)
A. Granular material		
1. Clean gravel	1–100	10^3–10^5
2. Clean coarse sand	0.010–1.00	10–10^3
3. Mixed sand	0.005–0.01	5–10
4. Fine sand	0.001–0.05	1–50
5. Silty sand	1×10^{-4}–2×10^{-3}	0.1–2
6. Silt	1×10^{-5}–5×10^{-4}	0.01–0.5
7. Clay	$< 10^{-6}$	$< 10^{-3}$
B. Consolidated material		
1. Sandstone	10^{-6}–10^{-3}	10^{-3}–1.0
2. Carbonate rock with secondary porosity	10^{-5}–10^{-3}	10^{-2}–1.0
3. Shale	10^{-10}	10^{-7}
4. Fractured and weathered rock (aquifers)	10^{-6}–10^{-2}	10^{-3}–10

At 20°C, for water, $\nu = 0.01$ cm²/s and substituting in Eq. (9.8)
K_0 [darcys] $= 10^3\, K$ [cm/s] at 20°C

Consider an aquifer of unit width and thickness B, (i.e. depth of a fully saturated zone). The discharge through this aquifer under a unit hydraulic gradient is

$$T = K\, B \tag{9.9}$$

This discharge is termed *transmissibility*, T and has the dimensions of $[L^2/T]$. Its units are m²/s or litres per day/metre width (lpd/m). Typical values of T lie in the range 1×10^4 lpd/m to 1×10^6 lpd/m. A well with a value of $T = 1 \times 10^5$ lpd/m is considered satisfactory for irrigation purposes.

The coefficient of permeability is determined in the laboratory by a *permeameter*. For coarse-grained soils a *constant-head permeameter* is used. In this the discharge of water percolating under a constant head difference (ΔH) through a sample of porous material of cross-sectional area A and length L is determined. The coefficient of permeability at the temperature of the experiment is found as

$$K = \frac{Q}{A} \cdot \frac{1}{(\Delta H / L)}$$

For fine grained soils, a *falling-head permeameter* is used. Details of permeameters and their use is available in any good textbook in Soil Mechanics, e.g. Ref. 7. It should be noted that laboratory samples are disturbed samples and a permeameter cannot simulate the field conditions exactly. Hence considerable care in the preparation of the samples and in conducting the tests are needed to obtain meaningful results.

Under field conditions, permeability of an aquifer is determined by conducting pumping tests in a well. One of the many tests available for this purpose consists of pumping out water from a well at a uniform rate till steady state is reached. Knowing the steady-state drawdown and the discharge-rate, transmissibility can be calculated. Information about the thickness of the saturation zone leads one to calculate the permeability. Injection of a tracer, such as a dye and finding its velocity of travel is another way of determining the permeability under field conditions.

EXAMPLE 9.1: *At a certain point in an unconfined aquifer of 3 km² area, the water table was at an elevation of 102.00. Due to natural recharge in a wet season, its level rose to 103.20. A volume of 1.5 Mm³ of water was then pumped out of the aquifer causing the water table to reach a level of 101.20. Assuming the water table in the entire aquifer to respond in a similar way, estimate (a) the specific yield of the aquifer and (b) the volume of recharge during the wet season.*

(a) Volume pumped out = area × drop in water table × S_y

$$1.5 \times 10^6 = 3 \times 10^6 \times 2.0 \times S_y$$

$$S_y = 0.25$$

(b) Recharge volume $= 0.25 \times (103.20 - 102.00) \times 3 \times 10^6$
$= 0.9$ Mm³

EXAMPLE 9.2: *A field test for permeability consists in observing the time required for a tracer to travel between two observation wells. A tracer was found to take 10 h to travel between two wells 50 m apart when the difference in the water-surface elevation in them was 0.50 m. The mean particle size* *of the aquifer was 2 mm and the porosity of the medium 0.3. If $\nu = 0.01$ cm²/s, estimate (a) the coefficients of permeability and intrinsic permeability of the aquifer (b) the Reynolds number of the flow.*

The tracer records the actual velocity of water

$$V_a = \frac{50 \times 100}{10 \times 60 \times 60} = 0.139 \text{ cm/s}$$

Discharge velocity $V = n\, V_a = 0.3 \times 0.139 = 0.0417$ cm/s

Hydraulic gradient $i = \dfrac{0.50}{50} = 1 \times 10^{-2}$

Coefficient of permeability $= \dfrac{4.17 \times 10^{-2}}{1 \times 10^{-2}} = 4.17$ cm/s

Intrinsic permeability, $K_0 = \dfrac{K\nu}{g}$

$$= \frac{4.17 \times 0.01}{981} = 4.25 \times 10^{-5} \text{ cm}^2$$

Since 9.87×10^{-9} cm² = 1 darcy

$K_0 = 4307$ darcys

(b) Reynolds number $\mathbf{Re} = \dfrac{V\, d_a}{\nu}$

Taking d_a = mean particle size, $\mathbf{Re} = \dfrac{0.0417 \times 2}{10} \times \dfrac{1}{0.01} = 0.834.$

9.4 GEOLOGIC FORMATIONS AS AQUIFERS

The identification of a geologic formation as a potential aquifer for ground water-development is a specialized job requiring the services of a trained hydrogeologist. In this section only a few general observations are made and for details the reader is referred to a standard treatise on hydrogeology such as Ref. 3.

The geologic formations of importance for possible use as an aquifer can be broadly classified as (i) unconsolidated deposits and (ii) consolidated rocks. Unconsolidated deposits of sand and gravel form the most important aquifers. They occur as fluvial alluvial deposits, abandoned channel sediments, coastal alluvium and as lake and glacial deposits. The yield is generally good and may be of the order of 50–100 m³/hr. In India, the Gangetic alluvium and the coastal alluvium in the states of Tamil Nadu and Andhra Pradesh are examples of good aquifers of this kind.

Among consolidated rocks, rocks with primary porosity such as sandstones are generally good aquifers. The state of weathering of rocks and occurrence of secondary openings such as joints and fractures enhance the ... aquifers is less than that of alluvial deposits and typically may have a value of 20–50 m³/h. Sandstones of Kathiawar and Kutch areas of Gujarat and of Lathi region of Rajasthan are some examples.

Limestones contain numerous secondary openings in the form of cavities formed by the solution action of flowing subsurface water. Often these form highly productive aquifers. In Jodhpur district of Rajasthan, cavernous limestones of the Vindhyan system are providing very valuable groundwater for use in this arid zone.

The volcanic rock basalt has permeable zones in the form of vesicles, joints and fractures. Basaltic aquifers are reported to occur in confined as well as underunconfined conditions. In the Satpura range some aquifers of this kind give yields of about 20 m³/h.

Igneous and metamorphic rocks with considerable weathered and fractured horizons offer good potentialities as aquifers. Since weathered and fractured horizons are restricted in their thickness these aquifers have limited thickness. Also, the average permeability of these rocks decreases with depth. The yield is fairly low, being of the order of 5–10 m³/h. Aquifers of this kind are found in the hard rock areas of Karnataka, Tamil Nadu, Andhra Pradesh and Bihar.

9.5 COMPRESSIBILITY OF AQUIFERS

In confined aquifers the total pressure at any point due to overburden is borne by the combined action of the pore pressure and intergranular pressure. The compressibility of the aquifer and also that of the pore water causes a readjustment of these pressures whenever there is a change in storage and thus have an important bearing on the storage characteristics of the aquifer. In this section a relation is developed between a defined storage coefficient and the various compressibility parameters.

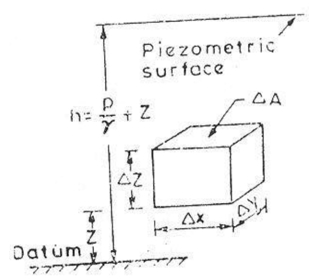

Fig. 9.4 Volume element of a compressible aquifer

Consider an elemental volume $\Delta V = (\Delta X\, \Delta Y)\, \Delta Z = \Delta A\, \Delta Z$ of a compressible aquifer (Fig. 9.4). A cartesian coordinate system with the Z-axis pointing vertically upwards is adopted. Further the following three assumptions are made:

1. The elemental volume is constrained in lateral directions and undergoes change of length in the z-direction only, i.e. ΔA is constant;
2. the pore water is compressible; and
3. the solid grains of the aquifer are incompressible but the pore structure is compressible.

By defining the reciprocal of the bulk modulus of elasticity of water as *compressibility of watar* β, it is written as

$$\beta = -\frac{d\,(\Delta V_w)}{\Delta V_w} \Big/ dp \qquad (9.10)$$

where ΔV_w = volume of water in the chosen element of aquifer, and p = pressure. By conservation of mass

$$\rho . \Delta V_w = \text{constant, where } \rho = \text{density of water.}$$

Thus $\rho\, d(\Delta V_w) + \Delta V_w\, d\rho = 0$

Substituting this relationship in Eq. (9.10),

$$\beta = d\rho/(\rho\, dp) \qquad (9.11a)$$

or

$$d\rho = \rho\, \beta\, dp \qquad (9.11)$$

Similarly by considering the reciprocal of the bulk modulus of elasticity of the pore-space skeleton as the *compressibility of the pores*, α, it is expressed as

$$\alpha = -\frac{d\,(\Delta V)/\Delta V}{d\,\sigma_z} \qquad (9.12)$$

in which σ_z = intergranular pressure. Since $\Delta V = \Delta A . \Delta Z$ with ΔA = constant,

$$\alpha = -\frac{d\,(\Delta Z)/\Delta Z}{d\,\sigma_z} \qquad (9.13)$$

The total overburden pressure $w = p + \sigma_z$ = constant.
Thus $dp = -d\,\sigma_z$, which when substituted in Eq. (9.13) gives

$$d\,(\Delta Z) = \alpha\,(\Delta Z)\, dp \qquad (9.14)$$

As the volume of solids, ΔV_s in the elemental volume is constant,

$$\Delta V_s = (1-n)\, \Delta A .\, \Delta Z = \text{constant}$$

$$d\,(\Delta V_s) = (1-n)\, d(\Delta Z) - \Delta Z .\, dn = 0$$

where n = porosity of the aquifer. Using this relationship in Eq. (9.14),

$$dn = \alpha\,(1-n)\, dp \qquad (9.15)$$

Now, the mass of water in the element of volume, ΔV,

$$\Delta M = \rho n\, \Delta A\, \Delta Z$$

or

$$d\,(\Delta M) = \Delta V \left[n\, d\rho + \rho\, dn + \rho n\, \frac{d\,(\Delta Z)}{\Delta Z} \right]$$

i.e.

$$\frac{d\,(\Delta M)}{\rho\, \Delta V} = n \frac{d\rho}{\rho} + dn + n \frac{d\,(\Delta Z)}{\Delta Z}$$

Substituting from Eqs. (9.11), (9.15) and (9.13) for the terms in the right-hand side respectively

$$\frac{d\,(\Delta M)}{\rho\, \Delta V} = n\, \beta\, dp + \alpha(1-n)\, dp + n\, \alpha\, dp$$

$$= (n\beta + \alpha)\, dp$$

$$= \gamma\,(n\beta + \alpha)\, dh = S_s\, dh \qquad (9.16)$$

where $S_s = \gamma\,(n\beta + \alpha)$ and h = piezometric head $= z + \frac{p}{\gamma}$ and $\gamma = \rho g$ = weight of unit volume of water.

The term S_s is called *specific storage*. It has the dimensions of $[L^{-1}]$ and represents the volume of water released from storage from a unit volume of aquifer due to a unit decrease in the piezometric head. The numerical value of S_s is very small being of the order of $1 \times 10^{-4}\ \text{m}^{-1}$.

By integration of Eq. (9.16) for a confined aquifer of thickness B, a dimensionless *storage coefficient* S can be expressed as

$$S = \gamma\,(n\beta + \alpha)\, B \qquad (9.17)$$

The storage coefficient S represents the volume of water released by a column of a confined aquifer of unit cross-sectional area under a unit decrease in the piezometric head. The storage coefficient S and the transmissibility coefficient T are known as the *formation constant* of an aquifer and play very important role in the unsteady flow through the porous media. Typical values of S in confined aquifers lie in the range 5×10^{-5} to 5×10^{-3}. Values of α for some formation material and β for various temperatures are given in Tables 9.3 and 9.4 respectively.

For an unconfined aquifer, the coefficient of storage is given by

$$S = S_y + \gamma\,(\alpha + n\beta)\, B_s \qquad (9.18)$$

where B_s = saturated thickness of the aquifer. However, the second term on the right-hand side is so small relative to S_y that for practical purposes S is considered equal to S_y, i.e. the coefficient of storage is assumed to have the same value as the specific yield for unconfined aquifers.

The elasticity of the aquifer is reflected dramatically in the response of the water levels in the wells drilled in confined aquifers to changes in the atmospheric pressure. Increase in the atmospheric pressure causes an increase in the loading of the aquifer. The change in the pressure is balanced by a partial compression of the water and partial compression of the pore skeleton. An increase in the atmospheric pressure causes a decrease in the water level in the well. Converse is the case with the decrease in pressure. The ratio of the water level change to pressure head change is called *barometric efficiency* (BE) and is given in terms of the compressibility parameters as

$$\text{BE} = -\left(\frac{n\beta}{\alpha + n\beta}\right) \qquad (9.19)$$

The negative sign indicates the opposite nature of the changes in pressure head and water level. Using (Eq. (9.17), BE $= -n\beta/\gamma\, SB$ and this affords a means of finding S. The barometric efficiency can be expected to be in the range 10–75%. It is apparent that unconfined aquifers have practically no barometric efficiency.

TABLE 9.3 RANGE OF α FOR SOME FORMATION MATERIALS

Material	Bulk modulus of elasticity, E_s (N/cm^2)	Compressibility $\alpha = 1/E_s$ (cm^2/N)
Loose clay	10^2–5×10^2	10^{-2}–2×10^{-3}
Stiff clay	10^3–10^4	10^{-3} $\times 10^{-4}$
Loose sand	10^3–2×10^3	10^{-3}–5×10^{-4}
Dense sand	5×10^3–8×10^3	2×10^{-4}–1.25×10^{-4}
Dense sandy gravel	10^4–2×10^4	10^{-4}–5×10^{-5}
Fissured and jointed rock	1.5×10^4–3×10^5	6.7×10^{-5}–3.3×10^{-6}

TABLE 9.4 VALUES OF β FOR WATER AT VARIOUS TEMPERATURES

Temperature (°C)	Bulk modulus of elasticity E_w (N/cm^2)	Compressibility $\beta = 1/E_w$ (cm^2/N)
0	2.04×10^5	4.90×10^{-6}
10	2.11×10^5	4.74×10^{-6}
15	2.14×10^5	4.67×10^{-6}
20	2.20×10^5	4.55×10^{-6}
25	2.22×10^5	4.50×10^{-6}
30	2.23×10^5	4.48×10^{-6}
35	2.24×10^5	4.46×10^{-6}

A few other examples of compressibility effects causing water-level changes in artesian wells include (i) tidal action in coastal aquifers, (ii) earthquake or underground explosions and (iii) passing of heavy railway trains.

9.6 EQUATION OF MOTION

If the velocities of flow in the cartesian coordinate directions x, y, z of the aquifer element, ΔV, are u, v and w respectively, the equation of continuity for the fluid flow is

$$\frac{\partial(\Delta M)}{\partial t} = -\left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right] \tag{9.20}$$

From Eq. (9.16) considering the differentials with respect to time and taking the limit as ΔV approaches zero

$$\frac{\partial(\Delta M)}{\partial t} = S_s \rho \frac{dh}{dt} \tag{9.16a}$$

Further the aquifer is assumed to be isotropic with permeability coefficient K, so that the Darcy's equation for x, y and z directions can be written as

$$u = -K\frac{\partial h}{\partial x},\ v = -K\frac{\partial h}{\partial y} \text{ and } w = -K\frac{\partial h}{\partial z} \tag{9.21}$$

Using Eqs (9.21) and (9.11) and noting that $h = p/\gamma + z$, the various terms of the right-hand side of Eq. (9.20) are written as

$$\frac{\partial(\rho u)}{\partial x} = \rho\frac{\partial u}{\partial x} + u\frac{\partial \rho}{\partial x} = -K\rho\frac{\partial^2 h}{\partial x^2} - K\rho^2\beta g\left(\frac{\partial h}{\partial x}\right)^2$$

$$\frac{\partial(\rho v)}{\partial y} = \rho\frac{\partial v}{\partial y} + v\frac{\partial \rho}{\partial y} = -K\rho\frac{\partial^2 h}{\partial y^2} - K\rho^2\beta g\left(\frac{\partial h}{\partial y}\right)^2$$

$$\frac{\partial(\rho w)}{\partial z} = \rho\frac{\partial w}{\partial z} + w\frac{\partial \rho}{\partial z} = -K\frac{\partial^2 h}{\partial z^2} - K\rho^2\beta g\left[\left(\frac{\partial h}{\partial z}\right)^2 - \frac{\partial h}{\partial z}\right]$$

Assembling these, Eq. (9.20) can be written as

$$K\rho\left[\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right] + K\rho^2\beta g\left[\left(\frac{\partial h}{\partial x}\right)^2 + \left(\frac{\partial h}{\partial y}\right)^2 + \left(\frac{\partial h}{\partial z}\right)^2 - \frac{\partial h}{\partial z}\right] = \rho S_s \frac{\partial h}{\partial t} \tag{9.22}$$

The second term on the left-hand side is neglected as very small, especially for $\partial h/\partial x << 1$, and Eq. (9.22) is rearranged to yield

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K}\frac{\partial h}{\partial t} \tag{9.23}$$

Defining $S_s B = S$, $K B = T$, and $\nabla^2 h = \left(\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2}\right)$, Eq. (9.23) reads as

$$\nabla^2 h = \frac{S}{T}\frac{\partial h}{\partial t} \tag{9.24}$$

This is the basic differential equation governing unsteady groundwater flow in a homogeneous isotropic confined aquifer. This form of the equation is known as *diffusion equation*.

If the flow is steady, the $\partial h/\partial t$ term does not exist, leading to

$$\nabla^2 h = 0 \tag{9.25}$$

This equation is known as Laplace equation and is the fundamental equation of all potential flow problems. Being linear, the method of superposition is applicable in its solutions.

Equation (9.24) or (9.25) can be solved for suitable boundary conditions by analytical, numerical or analog methods to yield solutions to a variety of groundwater flow problems. The details of solution of the basic differential equation of groundwater are available in standard literature. (Refs 1, 2, 4, 5 and 6).

9.7 WELLS

Wells form the most important mode of groundwater extraction from an aquifer. While wells are used in a number of different applications, they find extensive use in water supply and irrigation engineering practice.

Consider the water in an unconfined aquifer being pumped at a constant rate from a well. Prior to the pumping, the water level in the well indicates the static water table. A lowering of this water level takes place on pumping. If the aquifer is homogeneous and isotropic and the water table horizontal initially, due to the radial flow into the well through the aquifer the water table assumes a conical shape called *cone of depression.* The drop in the water table elevation at any point from its previous static level is called *drawdown.* The areal extent of the cone of depression is called *area of influence* and its radial extent *radius of influence* (Fig. 9.5). At constant rate of pumping, the drawdown curve develops gradually with time due to the withdrawal of water from storage. This phase is called unsteady flow as the water-table elevation at a given location near the well changes with time. On prolonged pumping, an equilibrium state is reached between the rate of pumping and the rate of inflow of groundwater from the outer edges of the zone of influence. The drawdown surface attains a constant position with respect to time when the well is known to operate under *steady-flow* conditions. As soon as the pumping is stopped, the depleted storage in the cone of depression is made good by groundwater inflow into the zone of influence. There is a gradual accumulation of storage till the original (static) level is reached. This stage is called *recuperation* or *recovery* and is an unsteady phenomenon. Recuperation time depends upon the aquifer characteristics.

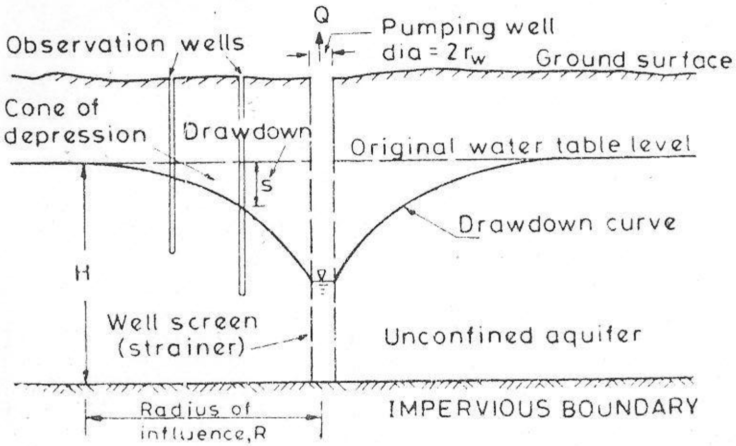

Fig. 9.5 Well operating in an unconfined aquifer, (definition sketch)

Changes similar to the above take place due to a pumping well in a confined aquifer also, but with the difference that, it is the piezometric surface instead of the water table that undergoes drawdown with the development of the cone of depression. In confined aquifers with considerable piezometric head, the recovery into the well takes place at a very rapid rate.

9.8 STEADY FLOW INTO A WELL

Steady-state groundwater problems are relatively simpler. Expressions for steady-state radial flow into a well under both confined and unconfined aquifer conditions are presented below.

Confined Flow

Figure 9.6 shows a well completely penetrating a horizontal confined aquifer of thickness B. Consider the well to be discharging a steady flow, Q. The original piezometric head (static head) was H and the drawdown due to pumping is indicated in Fig. 9.6. The piezometric head at the pumping well is h_w and the drawdown s_w.

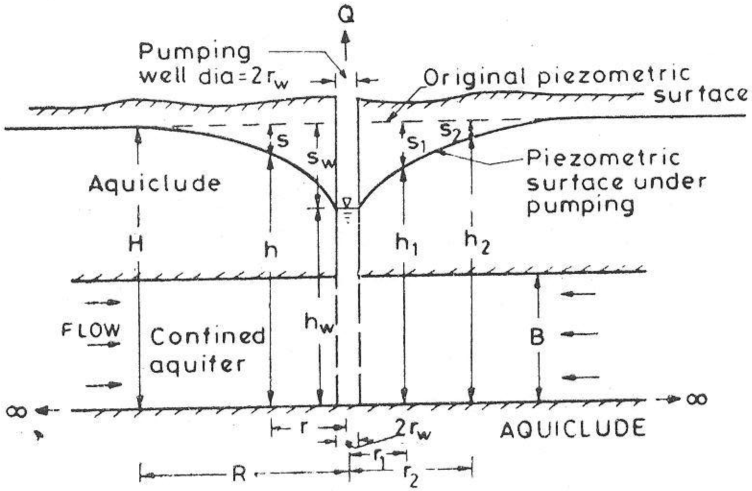

Fig. 9.6 Well operating in a confined aquifer

At a radial distance r from the well, if h is the piezometric head, the velocity of flow by Darcy's law is

$$V_r = K\frac{dh}{dr}$$

The cylindrical surface through which this velocity occurs is $2\pi r B$. Hence by equating the discharge entering this surface to the well discharge,

$$Q = (2\pi r B)\left(K\frac{dh}{dr}\right)$$

$$\frac{Q}{2\pi KB}\,\frac{dr}{r} = dh$$

Integrating between limits r_1 and r_2, with the corresponding piezometric heads being h_1 and h_2 respectively,

$$\frac{Q}{2\pi KB} \ln \frac{r_2}{r_1} = (h_2 - h_1)$$

or

$$Q = \frac{2\pi KB\,(h_2-h_1)}{\ln \frac{r_2}{r_1}} \qquad (9.26)$$

This is the equilibrium equation for the steady flow in a confined aquifer. This equation is popularly known as *Thiem's equation.*

If the drawdown s_1 and s_2 at the observation wells are known, then by noting that

$$s_1 = H-h_1,\ s_2 = H-h_2 \text{ and } KB = T$$

Equation (9.26) will read as

$$Q = \frac{2\pi\, T\,(s_1-s_2)}{\ln r_2/r_1} \qquad (9.27)$$

Further, at the edge of the zone of influence, $s = 0$, $r_2 = R$ and $h_2 = H$; and at the well wall $r_1 = r_w$, $h_1 = h_w$ and $s_1 = s_w$. Equation (9.27) would then be

$$Q = \frac{2\pi T\, s_w}{\ln R/r_w} \qquad (9.28)$$

Equation (9.27) or (9.28) can be used to estimate T, and hence K, from pumping tests. For the use of the equilibrium equation, Eq. (9.26) or its alternative forms, it is necessary that the assumption of complete penetration of the well into the aquifer and steady state of flow are satisfied.

EXAMPLE 9.3: *A 30-cm diameter well completely penetrates a confined aquifer of permeability 45 m/day. The length of the strainer is 20 m. Under steady state of pumping the drawdown at the well was found to be 3.0 m and the radius of influence was 300 m. Calculate the discharge.*

In this problem, referring to Fig. 9.6,

$$r_w = 0.15 \text{ m}$$
$$R = 300 \text{ m}$$
$$s_w = 3.0 \text{ m}$$
$$B = 20 \text{ m}$$
$$K = 45/(60 \times 60 \times 24) = 5.208 \times 10^{-4} \text{ m/s}$$
$$T = KB = 10.416 \times 10^{-3} \text{ m}^2\text{/s}$$

By Eq. (9.28)

$$Q = \frac{2\pi\, T\, s_w}{\ln R/r_w} = \frac{2\pi \times 10.416 \times 10^{-3} \times 3}{\ln \frac{300}{0.15}}$$
$$= 0.02583 \text{ m}^3\text{/s} = 25.83 \text{ lps}$$
$$= 1550 \text{ lpm}$$

EXAMPLE 9.4: *For the well in the previous example, calculate the discharge (a) if the well diameter is 45 cm and all other data remain the same as in Example 9.3, (b) if the drawdown is increased to 4.5 m and all other data remain unchanged as in Example 9.3.*

(a) $$Q = \frac{2\pi\, T\, s_w}{\ln R/r_w}$$

As T and s_w are constants,

$$\frac{Q_1}{Q_2} = \frac{\ln R/r_{w2}}{\ln R/r_{w1}}$$

Putting $R = 300$ m, $Q_1 = 1550$ lpm, $r_{w1} = 0.15$ m and $r_{w2} = 0.225$ m.

$$Q_2 = 1550\, \frac{\ln 300/0.15}{\ln 300/0.225} = 1637 \text{ lpm}$$

[Note that the discharge has increased by about 6% for 50% increase in the well diameter.]

(b) $$Q = \frac{2\pi\, T\, s_w}{\ln R/r_w}$$

$Q \propto s_w$ for constant T, R and r_w. Thus

$$\frac{Q_1}{Q_2} = \frac{s_{w1}}{s_{w2}}$$

$$Q_2 = 1550 \times \frac{4.5}{3.0} = 2325 \text{ lpm}$$

[Note that the discharge increases linearly with the drawdown when other factors remain constant.]

Unconfined Flow

Consider a steady flow from a well completely penetrating an unconfined aquifer. In this case because of the presence of a curved free surface, the streamlines are not strictly radial straight lines. While a streamline at the free surface will be curved, the one at the bottom of the aquifer will be a horizontal line, both converging to the well. To obtain a simple solution a set of two assumptions due to Dupit (1863) known as *Dupit's assumptions* are made. These are:

1. For small inclinations of the free surface, the streamlines can be assumed to be horizontal and the equipotentials are thus vertical.
2. The hydraulic gradient is equal to the slope of the free surface and does not vary with depth. This assumption is satisfactory in most of the flow regions except in the immediate neighbourhood of the well.

Consider the well of radius r_w penetrating completely an extensive unconfined horizontal aquifer as shown in Fig. 9.7 The well is pumping a

From Eq. (9.29),

$$Q = \frac{\pi K \left(h_2^2 - h_1^2\right)}{\ln r_2/r_1}$$

$$0.025 = \frac{\pi K\,[(38)^2-(36.5)^2]}{\ln 75/25}$$

$$K = 7.823 \times 10^{-5} \text{ m/s}$$

$$T = K\,H = 7.823 \times 10^{-5} \times 40 = 3.13 \times 10^{-3} \text{ m}^2\text{/s}$$

(b) At the pumping well, $r_w = 0.15$ m

$$Q = \frac{\pi K \left(h_1^2 - h_w^2\right)}{\ln r_1/r_w}$$

$$0.025 = \frac{\pi \times 7.823 \times 10^{-5}\left[\,(36.5)^2 - h_w^2\,\right]}{\ln 25/0.15}$$

$$h_w^2 = 811.84 \quad \text{and} \quad h_w = 28.49 \text{ m}$$

Drawdown at the well, $s_w = 11.51$ m

9.9 UNSTEADY FLOW IN A CONFINED AQUIFER

When a well in a confined aquifer starts discharging, the water from the aquifer is released resulting in the formation of a cone of depression of the piezometric surface. This cone gradually expands with time till an equilibrium is attained. The flow configuration from the start of pumping till the attainment of equilibrium is in unsteady regime and is described by Eq. (9.24).

In polar coordinates, Eq. (9.24), to represent the radial flow into a well, takes the form

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r}\frac{\partial h}{\partial r} = \frac{S}{T}\frac{\partial h}{\partial t} \qquad (9.31)$$

Making the same assumptions as used in the derivation of the equilibrium formula (Eq. 9.26), Thies (1935) obtained the solution of this equation as

$$s = (H-h) = \frac{Q}{4\pi T}\int_u^{\infty} \frac{e^{-u}}{u}\,du \qquad (9.32)$$

where $s = H-h$ = drawdown at a point distance r from the pumping well, H = initial constant piezometric head, Q = constant rate of discharge, T = transmissibility of the aquifer, u = a parameter $= r^2 S/4T\,t$, S = storage coefficient and t = time from start of pumping. The integral on the right-hand side is called the *well function*, $W(u)$ and is given by

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u}\,du = -0.577216 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} \cdots \qquad (9.33)$$

Table of $W(u)$ are available in literature (e.g. Refs 1, 8 and 9). Values of $W(u)$ can be easily calculated by the series (Eq. 9.33) to the required number of significant digits which rarely exceed 4. For small values of u ($u \leqslant 0.01$), only the first two terms of the series are adequate

The solution of Eq. (9.32) to find the drawdown s for a given S, T, r, T. and Q can be obtained in a straightforward manner. However, the estimation of the aquifer constants S and T from the drawdown vs time data of a pumping well, which involve trial-and-error procedures, can be done either by a digital computer or by semi-graphical methods such as the use of *Type curve* [1, 8, 9] or by Chow's method described in literature[1].

For small values of u ($u \leqslant 0.01$), Jacob (1946, 1950) showed that the calculations can be considerably simplified by considering only the first two terms of the series of $W(u)$, (Eq. 9.33). This assumption leads Eq. (9.32) to be expressed as

$$s = \frac{Q}{4\pi T}\left[-0.5772 - \ln \frac{r^2 S}{4T\,t}\right]$$

i.e.

$$s = \frac{Q}{4\pi T}\ln\left(\frac{2.26\,T\,t}{r^2 S}\right) \qquad (9.34)$$

If s_1 and s_2 are drawdowns at times t_1 and t_2,

$$(s_2 - s_1) = \frac{Q}{4\pi T}\ln\frac{t_2}{t^1} \qquad (9.35)$$

If the drawdown s is plotted against time t on a semi-log paper, the plot will be a straight line for large values of time. The slope of this line enables the storage coefficient S to be determined. From Eq. (9.34), when $s = 0$,

$$\frac{2.25\,T\,t_0}{r^2 S} = 1$$

or

$$S = \frac{2.25\,T\,t_0}{r^2} \qquad (9.36)$$

in which t_0 = time corresponding to "zero" drawdown obtained by extrapolating the straight-line portion of the semi-log curve of s *vs* t (Fig. 9.8). It is important to remember that the above approximate method proposed by Jacob assumes u to be very small.

EXAMPLE 9.6: *A 30-cm well penetrating a confined aquifer is pumped at a rate of a 1200 lpm. The drawdown at an observation well at a radial distance of 30 m is as follows:*

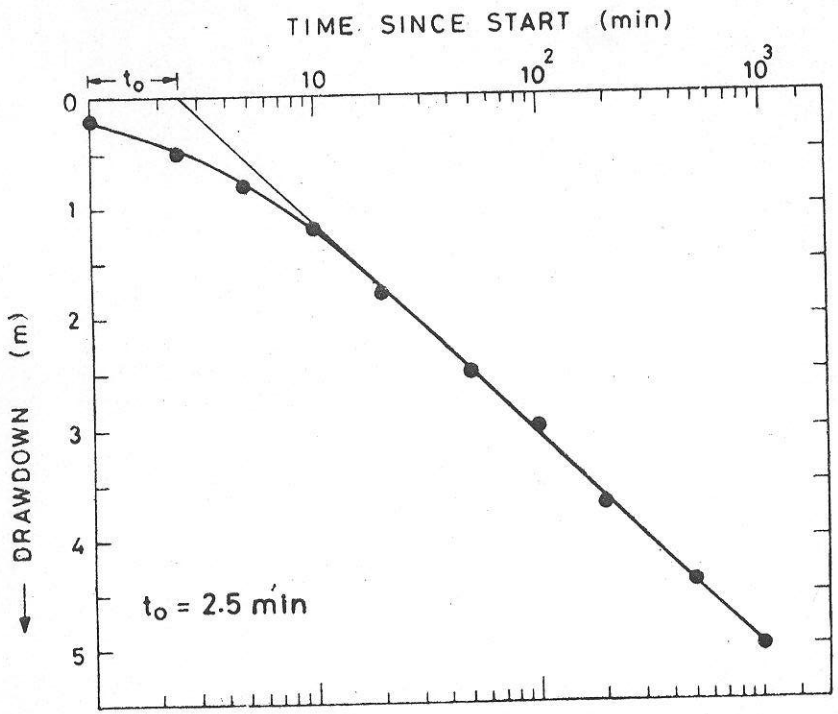

Fig. 9.8 Time-drawdown plot—Example 9.6

Time from start (min)	1.0	2.5	5	10	20	50	100	200	500	1000
Drawdown (m)	0.2	0.5	0.8	1.2	1.8	2.5	3.0	3.7	4.4	5.0

Calculate the aquifer parameters S and T.

The drawdown is plotted against time on a semilog plot (Fig. 9.8). It is seen that for $t > 10$ min, the drawdown values describe a straight line. A best-fitting straight line is drawn for data points with $t > 10$ min. From this line,

when $s = 0,\ t = t_0 = 2.5$ min $= 150$ s

$s_1 = 3.1$ m at $t_1 = 100$ min

$s_2 = 5.0$ m at $t_2 = 1000$ min

Also, $Q = 1200$ lpm $= 0.02\ \text{m}^2/\text{s}$

From Eq. (9.36),

$$s_2 - s_1 = \frac{Q}{4\pi T} \ln \frac{t_2}{t_1}$$

$$(5.0 - 3.1) = \frac{0.02}{4 \times \pi \times T} \ln \frac{1000}{100}$$

$$T = \frac{0.02}{4\pi \times 1.9} \ln 10 = 1.929 \times 10^{-3}\ \text{m}^3/\text{s/m}$$

$$= 1.67 \times 10^5\ \text{lpd/m}$$

From Eq. (9.36),

$$S = \frac{2.25\, T\, t_0}{r^2} = \frac{2.25 \times 1.929 \times 10^{-3} \times 150}{(30)^2}$$

i.e. $S = 7.23 \times 10^{-4}$

Example 9.7: *A well is located in a 25 m confined aquifer of permeability 30 m/day and storage coefficient 0.005. If the well is being pumped at the rate of 1750 lpm, calculate the drawdown at a distance of (a) 100 m and (b) 50 m from the well after 20 h of pumping.*

(a)
$$T = KB = \frac{30}{86400} \times 25 = 8.68 \times 10^{-3}\ \text{m}^2/\text{s}$$

$$u = \frac{r^2 S}{4Tt} = \frac{(100)^2 \times (0.005)}{4 \times (8.68 \times 10^{-3}) \times (20 \times 24 \times 60)}$$

$$= 0.05$$

Using Theis method and calculating $W(u)$ to four significant digits,

$$W(u) = -0.5772 - \ln(0.05) + (0.05) - \frac{(0.05)^2}{2.21} + \frac{(0.05)^3}{3.3!}$$

$$= -0.5772 + 2.9957 + 0.05 - 6.25 \times 10^{-4} - 6.94 \times 10^{-6}$$

$$= 2.468$$

$$s_{100} = \frac{Q}{4\pi T} W(u)$$

$$= \left(\frac{1.750}{60}\right) \times \frac{1}{4\pi\,(8.68 \times 10^{-3})} \times 2.468$$

$$= 0.66\ \text{m}$$

(b) $r = 50$ m, $u = \dfrac{(50)^2 \times (0.005)}{4 \times (8.68 \times 10^{-3})(20 \times 24 \times 60)} = 0.0125$

$$W(u) = -0.5772 + 4.382 + 0.0125 - 3.9 \times 10^{-5}$$

$$= 3.817$$

$$s_{50} = \left(\frac{1.750}{60}\right) \times \frac{1}{4\pi\,(8.68 \times 10^{-3})} \times 3.817 = 1.02\ \text{m}$$

9.10 WELL LOSS

In a pumping artesian well, the total drawdown at the well s_w, can be considered to be made up of three parts:

1. Head drop required to cause laminar porous media flow, called *formation loss*, s_{wL} (Fig. 9.9);

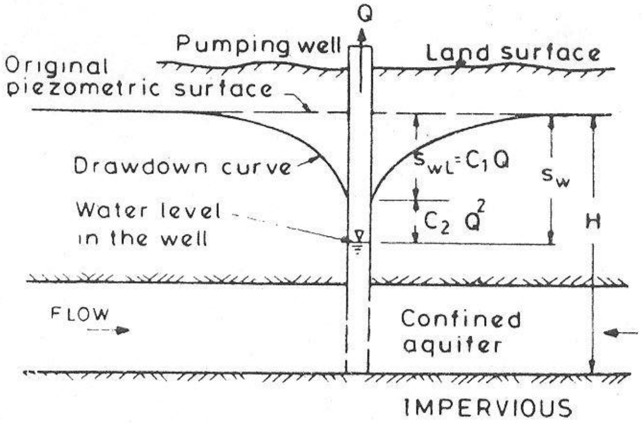

Fig. 9.9 Definition sketch for well loss

2. drop of piezometric head required to sustain turbulent flow in the region nearest to the well where the Reynolds number may be larger than unity, s_{wt}; and
3. head loss through the well screen and casing, s_{wc}.

Of these three,

$$s_{wL} \propto Q$$

and
$$(s_{wt} \text{ and } s_{wc}) \propto Q^2$$

Thus
$$s_w = C_1 Q + C_2 Q^2 \tag{9.37}$$

where C_1 and C_2 are constants for a given well (Fig. 9.9). While the first term $C_1 Q$ is the formation loss the second term $C_2 Q^2$ is termed *well loss.*

The magnitude of a well loss has an important bearing on the pump efficiency. Abnormally high value of well loss indicates clogging of well screens etc. and requires immediate remedial action. The coefficients C_1 and C_2 are determined by pump test data of drawdown for various discharges.

9.11 SPECIFIC CAPACITY

The discharge per unit drawdown at the well (Q/s_w) is known as *specific capacity* of a well and is a measure of the performance of the well. For a well in a confined aquifer under equilibrium conditions and neglecting well losses, by Eq. (9.28)

$$\frac{Q}{s_w} = \frac{2\pi}{\ln R/r_w} T$$

i.e.
$$Q/s_w \propto T$$

However, for common case of a well discharging at a constant rate Q under unsteady drawdown conditions, the specific capacity is given by

$$\frac{Q}{s_w} = \frac{1}{\dfrac{1}{4\pi T} \ln \dfrac{2.25\, T\, t}{r_w^2 \cdot S} + C_2 Q} \tag{9.38}$$

where t = time after the start of pumping. The term $C_2 Q$ is to account for well loss. It can be seen that the specific capacity depends upon T, s, t, r_w and Q. Further, for a given well it is not a constant but decreases with increases in Q and t.

9.12 GROUNDWATER BUDGET

The quantum of groundwater available in a basin is dependent on the inflows and discharges at various points. The interrelationship between inflows, outflows and accumulation is expressed by the water budget equation

$$\Sigma\, I\, \Delta t - \Sigma\, Q\, \Delta t = \Delta S \tag{9.39}$$

where $\Sigma\, I\, \Delta t$ represents all forms of recharge and includes contribution by lakes, streams, canals, precipitation and artificial recharge, if any, in the basin

$\Sigma\, Q\, \Delta t$ represents the net discharge of groundwater from the basin and includes pumping, surface outflows, seepage into lakes and rivers and evapotranspiration

ΔS indicates the change in the groundwater storage in the basin over a time Δt

Considering a sufficiently long time interval, Δt of the order of a year, the capability of the groundwater storage to yield the desired demand and its consequences on the basis can be estimated. It is obvious that too large a withdrawal than what can be replenished naturally leads ultimately to the permanent lowering of the groundwater table. This in turn leads to problems such as drying up of open wells and surface storages like swamps and ponds, change in the characteristics of vegetation that can be supported by the basin. Similarly, too much of recharge and scanty withdrawal or drainage leads to waterlogging and consequent decrease in the productivity of lands.

The maximum rate at which the withdrawal of groundwater in a basin can be carried without producing undesirable results is termed *safe yield.* This is a general term whose implication depends on the desired objective. The "undesirable" results include (i) permanent lowering of the groundwater table or piezometric head, (ii) maximum drawdown exceeding a preset limit leading to inefficient oparation of wells and (iii) salt-water encroachment in a coastal aquifer. Depending upon what undesirable effect is to be avoided, a safe yield for a basin can be identified.

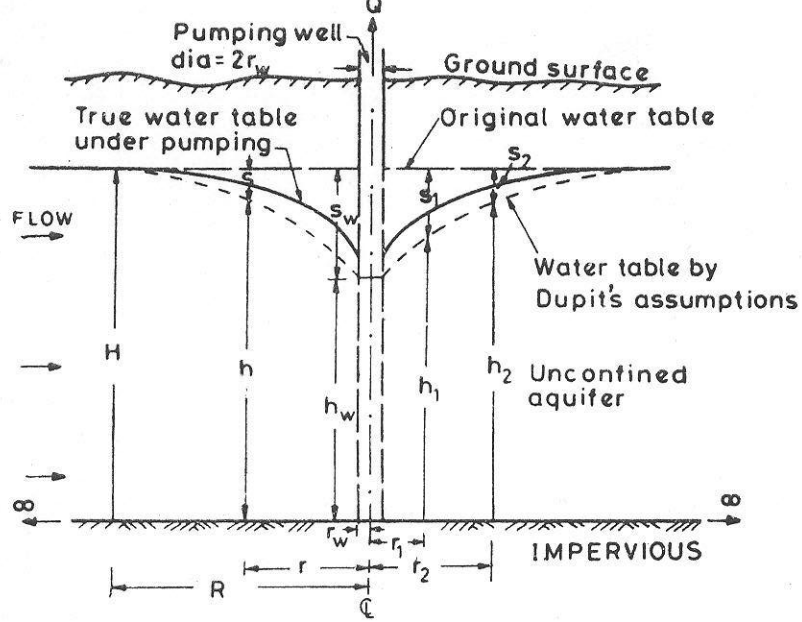

Fig. 9.7 Radial flow to a well in an unconfined aquifer

discharge Q. At any radial distance r, the velocity of radial flow into the well is

$$V_r = K \frac{dh}{dr}$$

where h is the height of the water table above the aquifer bed at that location. For steady flow, by continuity

$$Q = (2\pi rh)\, V_r = 2\pi r \, K \, h \, \frac{dh}{dr}$$

or

$$\frac{Q}{2\pi K} \frac{dr}{r} = h \, dh$$

Integrating between limits r_1 and r_2 where the water-table depths are h_1 and h_2 respectively, and on rearranging

$$Q = \frac{\pi K \left(h_2^2 - h_1^2\right)}{\ln r_2/r_1} \qquad (9.29)$$

This is the equilibrium equation for a well in an unconfined aquifer. As at the edge of the zone of influence of radius R, H = saturated thickness of the aquifer, Eq. (9.29) can be written as

$$Q = \frac{\pi K \left(H^2 - h_w^2\right)}{\ln R/r_w} \qquad (9.30)$$

where h_w = depth of water in the pumping well of radius r_w.

Equations (9.29) and (9.30) can be used to estimate satisfactorily the discharge and permeability of the aquifer by using field data. Calculations of the water-table profile by Eq. (9.29), however, will not be accurate near the well because of Dupit's assumptions. The water-table surface calculated by Eq. (9.29) which involved Dupit's assumption will be lower than the actual surface. The departure will be appreciable in the immediate neighbourhood of the well (Fig. 9.7). In general, values of R in the range 300 to 500 m can be assumed depending on the type of aquifer and operating conditions of a well. As the logarithm of R is used in the calculation of discharge, a small error in R will not seriously affect the estimation of Q. It should be noted that it takes a relatively long time of pumping to achieve a steady state in a well in an unconfined aquifer. The recovery after the cessation of pumping is also slow compared to the response of an artesian well which is relatively fast.

Approximate Equations

If the drawdown at the pumping well $s_w = (H - h_w)$ is small relative to H, then

$$H^2 - h_w^2 = (H + h_w)(H - h_w) \approx 2 H s_w$$

Noting that $T = KH$, Eq. (9.30) can be written as

$$Q = \frac{2\pi T s_w}{\ln R/r_w} \qquad (9.30a)$$

which is the same as Eq. (9.28). Similarly Eq. (9.29) can be written in terms of $s_1 = (H - h_1)$ and $s_2 = (H - h_2)$ as

$$Q = \frac{2\pi \, T (s_1 - s_2)}{\ln r_2/r_1} \qquad (9.29a)$$

Equations (9.29a) and (9.30a) are approximate equations to be used only when Eq. (9.29) or (9.30) cannot be used for lack of data. Equation (9.30a) over estimates the discharge by $[1/2\,(H/s_w - 1)]\%$ when compared to Eq. (9.30).

EXAMPLE 9.5: *A 30-cm well completely penetrates an unconfined aquifer of saturated depth 40 m. After a long period of pumping at a steady rate of 1500 lpm, the drawdown in two observation wells 25 and 75 m from the pumping well were found to be 3.5 and 2.0 m respectively. Determine the transmissivity of the aquifer What is the drawdown at the pumping well?*

(a) $$Q = \frac{1500 \times 10^{-3}}{60} = 0.025 \text{ m}^3/\text{s}$$

$h_2 = 40.0 - 2.0 = 38.0 \qquad h_1 = 40.0 - 3.5 = 36.5$ m

$r_2 = 75$ m $\qquad r_1 = 25$ m

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PROBLEMS

9.1 A confined aquifer is 25 m thick and 2 km wide. Two observation wells located 2 km apart in the direction of flow indicate heads of 45 and 39.5 m. If the coefficient of permeability of the aquifer is 30 m/day, calculate (a) the total daily flow through the aquifer and (b) the piezometric head at an observation well located 300 m from the upstream well.

9.2 In a field test a time of 6 h was required for a tracer to travel between two observation wells 42 m apart. If the difference in water-table elevations in these wells were 0.85 m and the porosity of the aquifer is 20%, calculate the coefficient of permeability of the aquifer.

9.3 A confined aquifer has a thickness of 30 m and a porosity of 32%. If the bulk modulus of elasticity of water and the formation material are 2.2×10^5 and 7800 N/cm² respectively, calculate (a) the storage coefficient and (b) the barometric efficiency of the aquifer.

9.4 A 30-cm well completely penetrates an artesian aquifer. The length of the strainer is 25 m: Determine the discharge from the well when the drawdown at the pumping well is 4.0 m. The coefficient of permeability of the aquifer is 45 m/day. Assume the radius of influence of the well as 350 m.

9.5 A 20-cm dia tubewell taps an artesian aquifer. Find the yield for a drawdown of 3.0 m at the well. The length of the strainer is 30 m and the coefficient of permeability of the aquifer is 35 m/day. Assume the radius of influence as 300 m.

If all other conditions remain same, find the percentage change in yield under the following cases:

(a) The diameter of the well is 40 cm;
(b) the drawdown is 6.0 m;
(c) the permeability is 17.5 m/day.

9.6 The discharge from a fully penetrating well operating under steady state in a confined aquifer of 35 m thickness is 3000 lpm. Values of drawdown at two observation wells 12 and 120 m away from the well are ... respectively. Determine the permeability of the aquifer.

9.7 A 45-cm well penetrates an unconfined aquifer of saturated thickness 30 m completely. Under a steady pumping rate for a long time the drawdowns at two observation wells 15 and 30 m from the well are 5.0 and 4.2 m respectively. If the permeability of the aquifer is 20 m/day, determine the discharge and the drawdown at the pumping well.

9.8 A 30-cm well fully penetrates an unconfined aquifer of saturated depth 25 m. When a discharge of 2100 lpm was being pumped for a long time, observation wells at radial distances of 30 and 90 m indicated drawdown of 5 and 4 m respectively. Estimate the coefficient of permeability and transmissibility of the aquifer. What is the drawdown at the pumping well ?

9.9 A 45-cm well in an unconfined aquifer of saturated thickness of 45 m yields 600 lpm under a drawdown of 3.0 m at the pumping well, (a) What will be the discharge under a drawdown of 6.0 m ? (b) What will be the discharge in a 30-cm well under a drawdown of 3.0 m ? Assume the radius of influence to remain constant at 500 m in both cases.

9.10 For conducting permeability tests in a well penetrating an unconfined aquifer, two observation wells *A* and *B* are located at distances 15 and 30 m respectively from the centre of the well. When the well is pumped at a rate of 5 lps, it is observed that the elevations of the water table above the impervious layer, up to which the well extends are 12.0 and 12.5 m respectively at *A* and *B*. Calculate the permeability of the aquifer in m/day.

9.11 Calculate the discharge in m³/day from a tubewell under the following conditions:

Diameter of the well	=	45 cm
Drawdown at the well	=	12 m
Length of strainer	=	30 m
Radius of influence of the well	=	200 m
Coefficient of permeability	=	0.01 cm/s
Aquifer	=	unconfined

9.12 A fully penerating well of 30-cm diameter in an unconfined aquifer of saturated thickness 50 m was found to give the following drawdown-discharge relations under equilibrium condition.

Drawdown at the pumping well (m)	Discharge (lpm)
3.0	600
11.7	1800

If the radius of influence of the well can be assumed to be proportional to the discharge through the well, estimate the flow rate when the drawdown at the well is 6.0 m.

9.13 A 45-cm well in an unconfined aquifer was pumped at a constant rate of 1500 lpm. At the equilibrium stage the following drawdown values at two observation well were noted:

Observation well	Radial distance from pumping well (m)	Drawdown (m)
A	10	5.0
B	30	2.0

The saturated thickness of the aquifer is 45 m. Assuming the radius of influence to be proportional to the discharge in the pumping well, calculate:

(a) Drawdown at the pumping well;
(b) transmissibility of the aquifer;
(c) drawdown at the pumping well for a discharge of 2000 lpm; and
(d) radius of influence for discharges of 1500 and 2000 lpm.

9.14 A well of 30-cm dia is located in a confined acquifer of transmissibility 500 m^2/day and storage constant of 0.005. What pumping rate will have to be adopted if the drawdown at the well is not to exceed 10 m in 2 days ?

9.15 The drawdown time data recorded at an observation well situated at a distance of 50 m from the pumping well is given below:

Time (min)	1.5	3	4.5	6	10	20	40	100
Drawdown (m)	0.15	0.6	1.0	1.4	2.4	3.7	5.1	6.9

If the well discharge is 1800 lpm, calculate the transmissibility and storage coefficients of the aquifer.

QUESTIONS

9.1 A geological formation which is essentially impermeable for flow of water even though it may contain water in its pores is called

(a) aquifer (b) aquifuge
(c) aquitard (d) aquiclude.

9.2 An aquifer confined at the bottom but not at the top is called

(a) Semiconfined aquifer (b) unconfined aquifer
(c) confined aquifer (d) perched aquifer

9.3 The surface joining the static water levels in several wells penertating a confined aquifer represents

(a) water-table surface (b) capillary fringe
(c) piezometric surface of the aquifer (d) cone of depression.

9.4 The volume of water that can be extracted by force of gravity from a unit volume of aquifer material is called

(a) specific retention (b) specific yield
(c) specific capacity (d) specific storage
(e) porosity.

9.5 The permeability of a soil sample at the standard temperature of 20°C was 0.01 cm/s. The permeability of the same material at a flow temperature of 10°C is in cm/s,

(a) < 0.01 (b) > 0.01 (c) = 0.01 (d) depends upon the porous material.

9.6 A soil has a coefficient of permeability of 0.51 cm/s. If the kinematic viscosity of water is 0.009 cm^2/s, the intrinsic permeability in darcys is

(a) 5.27×10^4 (b) 474 (c) 4.65×10^7 (d) 4000.

9.7 Darcy's law is valid in a porous media flow if the Reynolds number is less than unity. This Reynolds number is defined as

(a) (discharge velocity × maximum grain size)/μ
(b) (actual velocity × average grain size)/ν
(c) (discharge velocity × mean particle size)/ν
(d) (discharge velocity × pore size)/μ.

9.8 A sand sample was found to have a porosity of 40%. For an aquifer of thi material, the specific yield is

(a) = 40% (b) > 40% (c) < 40% (d) data is insufficient.

9.9 An unconfined aquifer of porosity 35%, permeability 35 m/day and specific yield 0.15 has an area of 100 km^2. The water table falls by 0.20 m during a drought. The volume of water lost from storage in Mm^3 is

(a) 7.0 (b) 3.0 (c) 4.0 (d) none of these.

9.10 The unit of intrinsic permeability is

(a) cm/day (b) m/day (c) darcy/day (d) cm^2.

9.11 The dimensions of the storage coefficient S are

(a) L^3 (b) $L T^{-1}$ (c) L^2/T (d) dimensionless.

9.12 The dimensions of the coefficient of transmissibility T are

(a) L^2/T (b) $L^3 T^2$ (c) dimensionless (d) $L T^2$.

9.13 The specific storage is

(a) storage coefficient/ (aquifer depth)
(b) same as specific yield
(c) specific capacity per unit depth of aquifer
(d) (Porosity – specific retention).

9.14 When there is an increase in the atmospheric pressure, the water level in a well penetrating a confined aquifer

(a) decreases (b) increases (c) does not undergo any change
(d) increases or decreases depending on the nature of the aquifer.

9.15 Specific capacity is

(a) a constant for a given well
(b) depends on aquifer characteristics only
(c) increases with discharge rate
(d) decreases with time from the start of pumping.

9.16 The discharge per unit drawdown at a well is known as

(a) specific yield (b) specific storage
(c) safe yield (d) none of these.

7 The specific capacity of a well in a confined aquifer under equilibrium conditions and within the working limits of drawdown

(a) can be taken as constant
(b) decreases as the drawdown is increased
(c) increases as the drawdown is increased
(d) none of these.

APPENDIX—A

CONVERSION FACTORS

A.1 VOLUME

$1\ m^3$ = 35.31 cubic feet
= 264 US gallons
= 220 Imp. gallons
= 1.31 cubic yards
= 8.11×10^{-4} acre feet
= 1000 litres

A.2 FLOW RATE (DISCHARGE)

Unit	Cubic metres per sec (m^3/s)	Litres per minute (lpm)	Litres per second (lps)
1 cft (cusec)	0.02832	1699	28.32
1 Imp.gpm	7.577×10^{-5}	4.546	0.07577
1 US gpm	6.309×10^{-5}	3.785	0.06309
1 Imp.mgd	0.05262	3157	52.62
1 acre ft/day	0.01428	856.6	14.28

A.3 PERMEABILITY

1. *Specific permeability, K_o*
   1 darcy = $9.87 \times 10^{-13}\ m^2 = 9.87 \times 10^{-9}\ cm^2$

2. *Coefficient of permeability, K*

1 lpd/m² = 1.1574 × 10^{-6} cm/s

1 m/day = 1.1574 × 10^{-3} cm/s = 20.44 Imp.gpd/ft²

= 24.53 US gpd/ft²

= 0.017 US gpm/ft²

A.4 TRANSMISSIBILITY

1 m²/day = 67.05 Imp. gpd/ft

= 80.52 US gpd/ft

= 0.056 US gpm/ft

A.5 EQUIVALENTS OF SOME COMMONLY USED UNITS

1 Metre	= 3.28 feet
1 Foot	= 30.48 cm = 0.3048 m
1 Mile	= 1.609 km
1 Acre	= 0.405 ha
1 Hectare	= 2.47 acres
1 Sq. Km	= 100 ha
1 Sq. Mile	= 259 ha = 640 acres
1 Sq. Foot	= 0.093 sq. metres
1 TMC	= 1 thousand million cubic foot
	= 28.317 million cubic metres

1 Million Acre ft = 1233.48 million cubic metres

1 Million Cubic metre = 810.71 Acre ft.

1 Million Gallons (Imperial) = 160.544 Cubic ft

= 4546.09 Cubic metres

APPENDIX B

THE SEQUENT PEAK ALGORITHM

B.1 INTRODUCTION

The mass curve method of estimating the minimum storage capacity to meet a specified demand pattern, described in Sec. 5.6, has a number of different forms of use in its practical application. However, the following basic assumptions are made in all the adaptations of the mass curve method of storage analysis.

1. If N years of data are available, the inflows and demands are assumed to repeat in cyclic progression of N year cycles. It is to be noted that in historical data this leads to an implicit assumption that future flows will not contain a more severe drought than what has already been included in the data.
2. The reservoir is assumed to be full at the beginning of a dry period. Thus, while using the mass curve method the beginning of the dry period should be noted and the minimum storage required to pass each drought period calculated. Sometimes, e.g. in Problem 5.7, it may be necessary to repeat the given data series of N years sequentially for a minimum of one cycle, i.e. for additional N years, to arrive at the desired minimum storage requirement.

The mass curve method is widely used for the analysis of reservoir capacity-demand problems. However, there are many variations of the basic method to facilitate graphical plotting, handling of large data etc. A variation of the arithmetical calculation described in Examples 5.6 and 5.8, called the *sequent peak algorithm* is particularly suited for the analysis of large data with the help of a computer. This procedure, first given by Thomas (1963), is described in the following section.

B.2 SEQUENT PEAK ALGORITHM

Let the data be available for N consecutive periods not necessarily of uniform length. These periods can be year, month, day or hours depending upon the problem. In the ith period let x_i = inflow volume and D_i = demand volume. The surplus or deficit of storage in that period is the *net-flow volume* given by

Net-flow volume = Inflow volume − Outflow volume

$= x_i - D_i$

In the sequent peak algorithm a mass curve of cumulative net-flow volume against chronological time is used. This curve, shown typically in Fig. B.1, will have peaks (local maximums) and troughs (local minimums). For any peak P_i the next following peak of magnitude greater than P_i is called a *sequent peak*. Using two cycles of N periods, where N is the number of periods of the data series, the required storage volume is calculated by the following procedure:

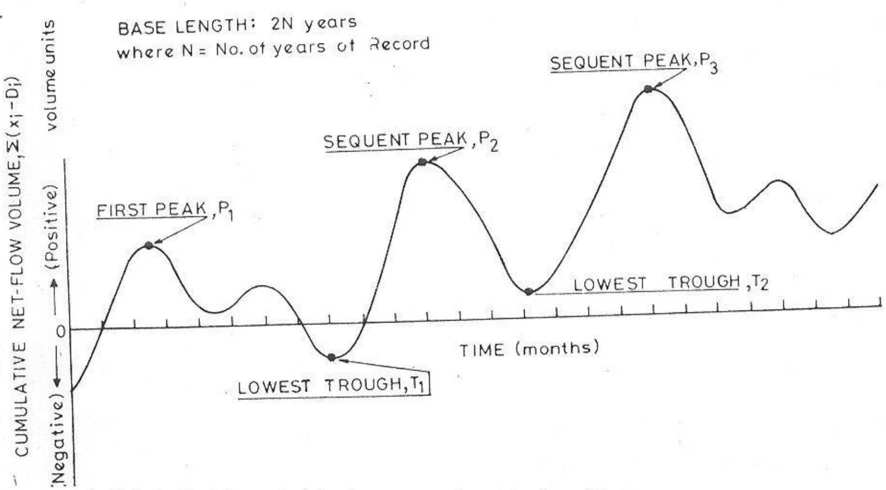

Fig. B-1 Definition sketch for sequent peak algorithm

1. Calculate the cumulative net-flow volumes viz.

$$\sum^{t} (x_i - D_i) \qquad \text{for } t = 1, 2, 3, \ldots, 2N$$

2. Locate the first peak P_1 and the sequent peak P_2 which is the next peak of greater magnitude than P_1 (Fig. B.1).
3. Find the *lowest trough* T_1 between P_1 and P_2 and calculate $(P_1 - T_1)$.
4. Starting with P_2 find the next sequent peak P_3 and the lowest through T_2 and calculate $(P_2 - T_2)$.
5. Repeat the procedure for all the sequent peaks available in the $2N$ periods, i.e. determine the sequent peak P_j, the corresponding T_j and the jth storage $(P_j - T_j)$ for all j values.
6. The required reservoir storage capacity is

$$S = \text{maximum of } (P_j - T_j) \text{ values}$$

EXAMPLE B.1 *The average monthly flows into a reservoir in a period of two consecutive dry years 1981-82 and 1982-83 is given below*

Month	Mean monthly flow (m^3/s)	Month	Mean monthly flow (m^3/s)
1981—June	20	1982—June	15
July	60	July	50
Aug.	200	Aug.	150
Sept.	300	Sept.	200
Oct.	200	Oct.	80
Nov.	150	Nov.	50
Dec.	100	Dec.	110
1982—Jan.	80	1983—Jan.	100
Feb.	60	Feb.	60
March	40	March	45
April	30	April	35
May	25	May	30

If a uniform discharge at 90 m³/s is desired from this reservoir calculate the minimum storage capacity required.

The data is for 2 years. As such, the sequent peak calculations are performed for $2 \times 2 = 4$ years. The calculations are shown in Table B.1.

Scanning the cumulative net-flow volume values (Col. 7) from the start, the first peak P_1 is identified as having a magnitude of 12,200 cumec-day which occurs in the end of the seventh month. The sequent peak P_2 is the peak next to P_1 and of magnitude higher than 12,000. This P_2 is identified as having a magnitude of 13,230 cumec-day and occurs in the end of the 31st month from the start. Between P_1 and P_2 the lowest trough T_1 has a magnitude of (−2,000) cumec-day and occurs at the end of the 26th month. In the present data, run for two cycles of total duration 4 years, no further sequent peak is observed.

The required storage volume is

$$S = 12,000 - (-2,000) = 14,200 \text{ cumec-day}$$

Sl. No.	Month	inflow rate (m^3/s)	volume X_i (cumec-day	rate (m^3/s)	volume D_i (cumec-day	volume $(x_i - D_i)$ (cumec-day	net-flow volume $\sum(x_i - D_i)$ (cumec-day)	
1	June	20	600	90	2700	−2100	−2100	I Cycle
2	July	60	1860	90	2790	− 930	−3030	
3	Aug.	200	6200	90	2790	+3410	+ 380	
4	Sept.	300	9000	90	2700	6300	6680	
5	Oct.	200	6200	90	2790	3410	10,090	
6	Nov.	150	4500	90	2700	1800	11,890	
7	Dec.	100	3100	90	2790	310	12,200	First peak P_1
8	Jan.	80	2480	90	2790	−310	11,890	
9	Feb.	60	1680	90	2520	−840	11,050	
10	March	40	1240	90	2790	−1550	9,500	
11	April	30	900	90	2700	−1800	7,700	
12	May	25	775	90	2790	−2015	5,685	
13	June	15	450	90	2700	−2250	3,435	
14	July	50	1550	90	2790	−1240	2,195	
15	Aug.	150	4650	90	2790	1860	4,055	
16	Sept.	200	6000	90	2700	3300	7,355	
17	Oct.	80	2480	90	2790	− 310	7,045	
18	Nov.	50	1500	90	2700	−1200	5,845	
19	Dec.	110	3410	90	2790	620	6,465	
20	Jan.	100	3100	90	2790	310	6,775	
21	Feb.	60	1680	90	2520	− 840	5,935	
22	March	45	1395	90	2790	−1395	4,540	
23	April	35	1050	90	2700	−1650	2,890	
24	May	30	930	90	2790	−1860	1,030	
25	June	20	600	90	2700	−2100	1,070	II Cycle
26	July	60	1860	90	2790	− 930	−2,000	Lowest
27	Aug.	200	6200	90	2790	3410	1,410	trough T_1
28	Sept.	300	9000	90	2700	6300	7,710	between P_1
29	Oct.	200	6200	90	2790	3410	11,120	and P_2
30	Nov.	150	4500	90	2700	1800	12,920	
31	Dec.	100	3100	90	2790	310	13,230	Sequent
32	Jan.	80	2480	90	2790	− 310	12,920	Peak P_2
33	Feb.	60	1680	90	2520	− 840	12,080	
34	March	40	1240	90	2790	−1550	10,530	
35	April	30	900	90	2700	−1800	8,730	
36	May	25	775	90	2790	−2015	6,715	
37	June	15	450	90	2700	−2250	4,465	
38	July	50	1550	90	2790	−1240	3,225	
39	Aug.	150	4650	90	2790	1860	5,085	
40	Sept.	200	6000	90	2700	3300	8,385	
41	Oct.	80	2480	90	2790	− 310	8,075	
42	Nov.	50	1500	90	2700	−1200	6,875	
43	Dec.	110	3410	90	2790	620	7,495	
44	Jan.	100	3100	90	2790	310	7,805	
45	Feb.	60	1680	90	2520	− 840	6,965	
46	March	45	1395	90	2790	−1395	5,570	
47	April	38	1050	90	2700	−1650	3,920	
48	May	30	930	90	2790	−1860	2,060	

$N = 2$ years — The data is run for 2 cycles of 2 years

$P_1 = 12,00$ cumec-day — Storage $S = 12,200 - (-2,000)$

ANSWERS

CHAPTER 2

Problems

(2.1) 5

(2.2) 12.86 cm

(2.3) (a) 1955
(b) correction ratio = 1.26

(2.4) 7.41 cm

(2.5) 135 cm

(2.6) (1) 10.6 cm
(2) 9.9 cm
(3) 11.1 cm

(2.8) (a) 132.5 cm
(b) 143.0 cm

(2.10) (a) 0.167
(b) 0.0153
(c) 0.183

(2.11) (a) 0.605
(b) 0.01

(2.12) 10 years

(2.13) (a) 0.155
(b) 0.00179
(c) 0.08451

Questions

(2.1) d	(2.2) c	(2.3) b	(2.4) d	(2.5) c
(2.6) a	(2.7) b	(2.8) c, d	(2.9) c	(2.10) a
(2.11) d	(2.12) d	(2.13) b	(2.14) b	(2.15) d
(2.16) b	(2.17) b	(2.18) a	(2.19) a	(2.20) b

CHAPTER 3

Problems

(3.1) 24.5 mm

(3.2) decrease, 48 Mm^3

(3.3) 175 mm/month

(3.4) (a) 24.9 cm
(b) 19.15 cm

(3.5) 23.4 cm

(3.6) 46.8 cm

(3.7) 11.25 cm/month

(3.8) 11.0 cm

(3.9) 2.50 cm

(3.10) 2.24 cm

(3.11) 0.75 cm/h

(3.12) 10.0 cm, 4.2 cm
2.52 cm/h

Questions

(3.1) c	(3.2) b	(3.3) d	(3.4) c	(3.5) d

(3.6) b (3.7) b (3.8) c (3.9) c (3.10) c
(3.11) d (3.12) c

CHAPTER 4

Problems

(4.1) 6.429 m^3/s (4.3) 11.73 m^3/s (4.4) 3460 m^3/s
(4.5) 103 m^3/s (4.6) 500 m^3/s (4.7) 143 m^3/s
(4.8) 11.0 km (4.9) 44.25 m^3/s (4.10) 30.18 m^3/s
(4.11) 427 m^3/s (4.12) 18.60 (4.13) $(G-a)=0.16334\ Q^{0.4666}$; 26.85
(4.14) 164.4 m^3/s

Questions

(4.1) d (4.2) c (4.3) b (4.4) d (4.5) b
(4.6) c (4.7) c (4.8) b (4.9) c (4.10) b

CHAPTER 5

Problems

(5.1) (a) 121 cm
(b) 75.9 cm
(c) 62.7 %
(5.2) 5.06 Mm^3
(5.3) $R = 0.6163\ P - 21.513$; 40.12 cm
(5.4) 2.534 cm; 0.436
(5.5) 0.144
(5.6) $Q_{75} = 14\ m^3/s$
(5.7) 9545 cumec-day
(5.8) (a) 5700 cumec-day
(b) 82 m^3/s
(5.9) 365 Mm^3
(5.10) 387.8 Mm^3

Questions

(5.1) a (5.2) a (5.3) a (5.4) c (5.5) c
(5.6) b (5.7) b (5.8) d (5.9) c (5.10) b
(5.11) a (5.12) b

CHAPTER 6

Problems

(6.1) $K_{rb} = 0.886$; $K_{rs} = 0.204$ (6.2) (given ordinates)/ 4.3

(6.3)

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
6-h UH ord (m^3/s)	0	5	35	64	72	62	46	33	21	11.6	5.6	1.6	0

(6.4) ER = 5.76 cm

Time (days)	0	1	2	3	4	5	6	7	8
Dist. graph (%)	0	10.25	31.50	26.25	15.50	9.25	5.25	2.00	0

(6.5)

Time (h)	0	5	13	21	28	32	35	41
8-h UH ord (m^3/s)	0	8.9	46.7	88.9	133.3	182.2	255.6	320.0
Time (h)	45	55	61	91	98	115	130	
6-h UH ord (m^3/s)	335.0	315.6	264.4	144.4	115.6	64.0	0	

(6.6) 70 m^3/s
(6.7) 1800 m^3/s + base flow
(6.8)

Time (h)	0	3	6	9	12	18	24	30
Q (m^3/s)	30	300	480	1410	2820	4500	6010	6010
Time (h)	36	42	48	54	60	66	72	78
Q (m^3/s)	5080	3996	2866	1866	1060	500	170	30

(6.9)

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
Q (m^3/s)	10	30	90	220	280	220	166	126	92	62	40	20	10

(6.10)

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72
12-h UH ord. (m^3/s)	0	10	40	105	135	105	78	58	41	26	15	5	0

(6.11)

Time (h)	0	2	4	6	8	10	12	14	16
S-curve ord. (m^3/s)	0	25	125	285	475	645	755	825	855
4-h UH ord. (m^3/s)	0	12.5	62.5	130	175	180	140	90	50
Time (h)	18	20	22	24	26				
S-curve ord (m^3/s)	875	881	881	881	881				
4-h UH ord. (m^3/s)	25	13	3	0	0				

(6.12)

Time (h)	0	6	12	18	24	30	36	42	48	54	
6-h UH ord. (m^3/s)	0	20	54	98	124	148	152	154	138	122	
Time	60	66	72	78	84	90	96	102	108	114	120
6-h UH	106	92	76	66	50	42	28	22	12	10	6
Time	126	132	138								
6-h UH ord.	3	1	0								

(6.13)

Time (h)	0	3	6	9	12	15	18	21	24	27			
9-h UH ord. (m^3/s)	0	4	29	73	129	174	191	183	158	135			
Time	30	33	36	39	42	45	48	51	54	57	60	63	66
9-h UH	115	99	83	69	54	43	33	25	18	12	6	2	0

(6.14)

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72	78
Q m^3/s)	20	80	200	550	620	890	698	530	380	280	178	100	60	20

(6.15) $A = 1296$ km²

Time (h)	0	6	12	18	24	30	36	42	48	54	60	66	72	78
Q (m^3/s)	25	75	225	375	525	600	525	450	375	300	225	150	75	25

(6.16) 80.6 m^3/s

(6.17)

Time (h)	0	2	6	10	14	18	22	26	30	34
DRH ord. (m^3/s)	0	4.3	19.4	44.4	39.6	28.1	14.1	4.9	1.4	0

(6.18)

Time (units of 6 h)	1	2	3	4	5	6
Distribution graph (%)	8.33	25.00	29.17	20.83	12.50	4.17

(6.19)

Time (h)	0	3	6	9	12	15	18	21	24	27
3-h UH ord. (m^3/s)	0	60	120	90	50	30	20	10	5	0

(6.20) $t_R = 6.0$ h; $t'_p = 27.75$ h, $Q_p = 126$ m^3/s, $W_{50} = 79$ h, $W_{75} = 45$ h, $t_b = 156$ h

(6.21) $t'_p = 9$ h, $Q_p = 86.5$ m^3/s, $W_{50} = 30.7$ h, $W_{75} = 17.5$ h, $t_b = 52$ h

(6.22) $Q_p = 126$ m^3/s, $t_{pk} = 30.75$ h

(6.23)

Time (h)	0	2	4	6	8	10	12	14
2-h UH ord. (m^3/s)	0	2.50	7.50	12.50	17.50	19.29	17.85	16.42
Time	16	18	20	22	24	26	28	30
2-h UH	14.99	13.56	12.14	10.71	9.28	7.85	6.06	4.99
Time	32	34	36	38				
2-h UH	3.56	2.13	0.71	0				

Questions

(6.1) c	(6.2) a	(6.3) b	(6.4) a	(6.5) c
(6.6) b	(6.7) b	(6.8) c	(6.9) b	(6.10) d
(6.11) c	(6.12) a	(6.13) a	(6.14) c	(6.15) d
(6.16) a	(6.17) b	(6.18) c	(6.19) b	

CHAPTER 7

Problems

(7.1) 4.0 m^3/s (7.2) 10.0 m^3/s (7.3) 19.65 m^3/s

(7.4) (a) 0.025
(b) 0.397
(c) 0.975

(7.5) (a) 0.41
(b) 0.358
(c) 0.02
(d) 8.33×10^{-4}
(e) 0.636

(7.6) 96 years

(7.7)

n	35	45	55	60	65
$K(T, n)$	5.6421	5.5221	5.4420	5.4100	5.3832

(7.8)

T years	50	100	1000
Q_T (Gumbel) m^3/s	5760	6390	8470
Q_T (Log Pearson III) m^3/s	5300	5820	7590

(7.9) $T = 100$ years

(7.10)

T years	50	100	200	1000
Q_T (m^3/s)	790	930	1080	1500

(7.11) Ganga: $Q_{100} = 16360 \pm 2550$ m^3/s
$Q_{1000} = 22020 \pm 3750$ m^3/s

Yamuna: $Q_{100} = 17300 \pm 3890$ m^3/s
$Q_{1000} = 23940 \pm 5720$ m^3/s

(7.12) 567 m^3/s

(7.13) (a) $\bar{x}$ = 385 m³/s, σ_{n-1} = 223 m³/s (b) 1525 m³/s

(7.14)

Time (h)	0	12	24	36	48	60	72	84	96
Q (m³/s)	50	104	482	1669	3138	3699	3358	2603	1929
Time	108	120	132	144	156	168			
Q	1268	753	393	183	51	50			

Questions

(7.1) a (7.2) d (7.3) d (7.4) c (7.5) b
(7.6) b (7.7) a (7.8) b (7.9) c (7.10) c
(7.11) c (7.12) c (7.13) a (7.14) d

CHAPTER 8

Problems

(8.1)

Time (h)	0	3	6	9	12	15	18	21
Q (m³/s)	0	1	10	26	36	43	38	34
Elevation (300.00 +)	0.0	0.30	0.45	0.85	1.03	1.15	1.08	1.00
Time	24	27						
Q	29	22						
Elev.	0.90	0.75						

(8.2)

Time (h)	0	3	6	9	12	15	18
Q (m³/s)	63	54	46	49	53	50	45
Elevation (300.00 +)	0.50	1.38	1.20	1.28	1.35	1.30	1-20
Time	21	24	27				
Q	37	31	24				
Elev.	1.05	0.93	0.80				

(8.3) 62.305, 52.5 m³/s, 7.5 m³/s

(8.4) 2 h 25 min

(8.5)

Time (h)	0	2	4	6	8	10	12	14
Q (m³/s)	0	9.5	20.0	29.6	27.5	23.7	21.5	17.7
Time	16	18	20	22	24	26		
Q (m³/s)	15.4	11.8	9.3	5.9	3.3	0		

(8.6) K = 10.0 h, x = 0.2

(8.7) 21 h, 35 m³/s

(8.8)

Time (h)	0	3	6	9	12	15	18	21
Q (m³/s)	10.0	14.5	25.0	36.8	43.8	47.8	45.2	42.9
Time	24	27	30					
Q	39.2	34.5	29.1					

(8.9)

Time (h)	0	1	2	3	4	5	6	7
IUH ord. (m³/s)	0	2.78	13.06	17.31	28.09	34.18	33.54	30.75
1-h UH ord. (m³/s)	0	1.39	7.92	15.18	22.70	31.14	33.86	32.15
Time	8	9	10	11	12	13	14	
IUH	27.68	24.91	22.42	20.18	18.16	16.34	14.70	so on
1-h UH	29.22	26.30	23.67	21.30	19.17	17.34	15.52	so on

(8.10) 3 h, 0.6 m³/s

(8.11 $Q = \beta t + \left(\frac{\beta}{\alpha} - I_0\right)e^{-\alpha t} - \frac{\beta}{\alpha}$

Questions

(8.1) a (8.2) c (8.3) a (8.4) b (8.5) d
(8.6) d (8.7) c (8.8) d (8.9) b (8.10) b
(8.11) c (8.12) a

CHAPTER 9

Problems

(9.1) (a) 4125 m³/day (b) 44.175 m

(9.2) 1.92 cm/s